You are bidding against a computer for an item that you **value at $30**. The allowed bids are $10, $20, $30, $40 and $50. The computer will pick one of these bids randomly. Let x be the bid generated by the computer. If your bid is greater than or equal to x then you win the object and you **pay** not your bid but **the computer's bid**. If your bid is less than x then you get nothing and pay nothing.

computer's bid → / your bid ↓	$10	$20	$30	$40	$50
$10					
$20					
$30					
$40					
$50					
	pay 10	pay 20	pay 30	pay 40	pay 50

pay

$$U = \begin{cases} 0 & \text{if lose} \\ 30-p & \text{if win and have to pay } p \end{cases}$$

In case of ties you win.

~~Now same as above, but if you win the object and **pay your own bid**.~~

computer's bid → / your bid ↓	$10	$20	$30	$40	$50
$10	20	0	0	0	0
$20	20	10	0	0	0
$30	20	10	0	0	0
$40	20	10	0	-10	0
$50	20	10	0	-10	-20

VALUE: $30

bidding your true value is a weakly dominant act

(so is bidding 20)

Now same as above, but if you win the object and **pay your own bid**.

VALUE: \$30

your bid ↓ / computer's bid →	\$10	\$20	\$30	\$40	\$50
\$10	20				
\$20	10				
\$30	0				
\$40	-10				
\$50	-20				

state → act ↓	s_1	s_2	s_3
~~a_1~~	~~4~~	~~3~~	~~1~~
a_2	6	2	2
a_3	5	3	2
~~a_4~~	~~6~~	~~1~~	~~0~~
a_5	3	2	5

Dominance:

→ a_1 is weakly dominated by a_3

→ a_4 is weakly dominated by a_2

So we can simplify

state → act ↓	s_1	s_2	s_3
a_2	6	2	2
a_3	5	3	2
a_5	3	2	5

no dominance here

What then?

First a different example:

state → / act ↓	s_1	s_2	s_3
a_1	4	3	1
a_2	3	2	2
a_3	5	3	2
a_4	6	1	0
a_5	3	3	4

Worst-case scenario

Pick the action that gives the best of the worst:

a_5 because $3 > 1$

$3 > 2$

$3 > 0$

MaxiMin = $\{a_5\}$

One criterion that can be used is the **MaxiMin** criterion.

Now back to the previous problem:

state → / act ↓	s_1	s_2	s_3
a_2	6	$\cancel{2}$	2
a_3	5	$\cancel{3}$	$\cancel{2}$
a_5	$\cancel{3}$	$\cancel{2}$	5

MaxiMin = $\{a_2, a_3, a_5\}$

MaxiMin =

LexiMin = $\{a_3, a_5\}$

A refinement is the **LexiMin**

state $\rightarrow$ / act $\downarrow$	s_1	s_2	s_3
a_2	6	2	2
a_3	5	3	2
a_5	3	2	5

Here the LexiMin picks

One more example:

state $\rightarrow$ / act $\downarrow$	s_1	s_2	s_3	s_4
a_1	2	3	1	5
a_2	6	2	2	3
a_3	5	3	2	4
a_4	6	1	0	7
a_5	3	2	5	1

MaxiMin = $\{a_2, a_3\}$

LexiMin = $\{a_3\}$

	s_1	s_2	s_3	s_4
a	\$1,000	\$1,000	\$1,000	\$0
b	\$1	\$1	\$1	\$1

Max. Min = $\{b\}$

Special case: outcomes are sums of money

state $\rightarrow$ / act $\downarrow$	s_1	s_2	s_3	s_4
a_1	\$12	\$30	\$0	\$18
a_2	\$36	\$6	\$24	\$12
a_3	\$6	\$42	\$12	\$0

no dominance

Suppose that we are able to assign probabilities to the states:

state $\rightarrow$	s_1	s_2	s_3	s_4
	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{5}{12}$	$\frac{1}{12}$

sum = 1

RISK NEUTRALITY

a_1 is the lottery $\begin{pmatrix} \$12 & \$30 & \$0 & \$18 \\ \frac{1}{3} & \frac{1}{6} & \frac{5}{12} & \frac{1}{12} \end{pmatrix} = \frac{1}{3}12 + \frac{1}{6}30 + \frac{5}{12}0 + \frac{1}{12}18 = 10.5$

a_2 is the lottery $\begin{pmatrix} \$36 & \$6 & \$24 & \$12 \\ \frac{1}{3} & \frac{1}{6} & \frac{5}{12} & \frac{1}{12} \end{pmatrix} = \frac{1}{3}36 + \frac{1}{6}6 + \frac{5}{12}24 + \frac{1}{12}12 = 24$

a_3 is the lottery $\begin{pmatrix} \$6 & \$42 & \$12 & \$0 \\ \frac{1}{3} & \frac{1}{6} & \frac{5}{12} & \frac{1}{12} \end{pmatrix} = \frac{1}{3}6 + \frac{1}{6}42 + \frac{5}{12}12 + \frac{1}{12}0 = 14$

The expected values are:

$E[a_1] = 10.5$

$E[a_2] = 24$ $\leftarrow$ A risk-neutral person picks a_2

$E[a_3] = 14$

Definition of attitude to risk ….

Given a money lottery L, imagine giving the individual a choice between L and the expected value of L for sure, that is, the choice

$$\text{between } \begin{pmatrix} \mathbb{E}[L] \\ 1 \end{pmatrix} \text{ and } L \quad \text{or, written more simply, between } \mathbb{E}[L] \text{ and } L$$

If she says that

- $\mathbb{E}[L] \succ L$ we say that she is **risk** AVERSE **relative to** $\boldsymbol{L}$
- $\mathbb{E}[L] \sim L$ we say that she is **risk** NEUTRAL **relative to** $\boldsymbol{L}$ (indifferent)
- $L \succ \mathbb{E}[L]$ we say that she is **risk** LOVING **relative to** $\boldsymbol{L}$

So in the above example, if we assume that the agent is risk neutral relative to every lottery and her preferences are transitive, then, since

$$\mathbb{E}[a_1] = 10.5$$
$$\mathbb{E}[a_2] = 24$$
$$\mathbb{E}[a_3] = 14$$

$$\begin{pmatrix} \$12 & \$30 & \$0 & \$18 \\ \frac{1}{3} & \frac{1}{6} & \frac{5}{12} & \frac{1}{12} \end{pmatrix} = a_1$$

$$\begin{pmatrix} \$10.5 \\ 1 \end{pmatrix} \text{ versus } a_1$$

$$\$10.5 \succ a_1$$

Can we infer risk attitudes from choices?

Hypothetical choice:
$50 and L

Let $L = \begin{pmatrix} \$40 & \$60 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ Then $\mathbb{E}[L] = \frac{1}{2}\,40 + \frac{1}{2}\,60 = 50$

Suppose Ann's preferences are transitive, she prefers more money to less and she says that she prefers \$49 to L.

$\$50 \succ \$49$

$\$49 \succ L$

$\$50 \succ L$ So Ann is risk averse

Suppose Bob's preferences are transitive, he prefers more money to less and he says that he prefers \$51 to L.

Bob could be risk neutral: $\$51 \succ \$50 \sim L$ by transitivity $\$51 \succ L$

Bob could be risk averse: $\$51 \succ \$50 \succ L$ "

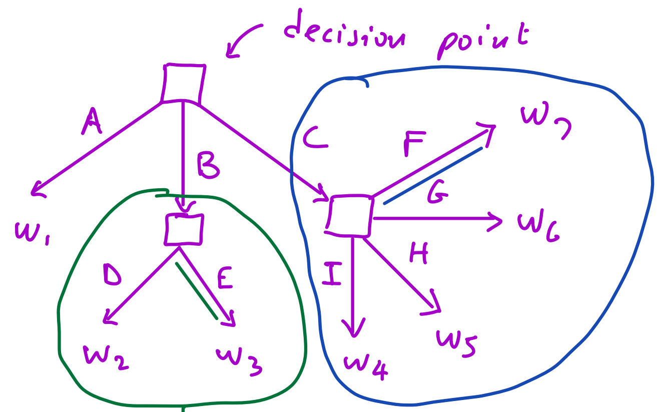
Bob could be risk loving $L \succ \$50$

$\wr$ ← possibility

$\$51 \succ \$50.50$

	s_1	s_2	s_3	s_4
a	z_1	z_2	z_3	z_4
b	z_5	z_6	z_7	z_8

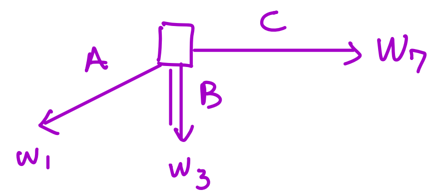
DECISION TREES



if $w_7 > w_4 \sim w_5 > w_6$

Backward induction

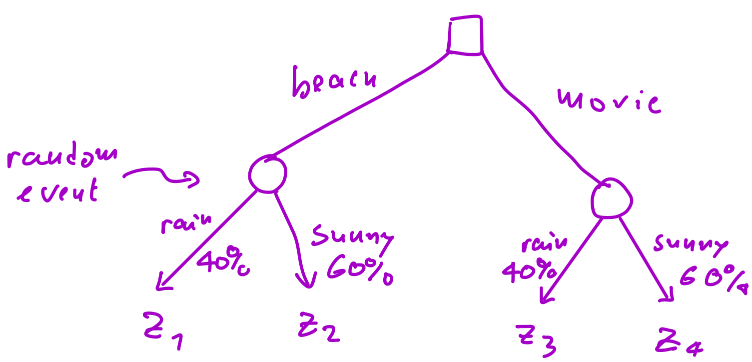
if $w_3 > w_2$



if $w_3 > w_1$

$w_3 > w_7$

Best course of action : first B, then E
(if by mistake I chose C then F)



$z_2 > z_3 > z_4 > z_1$