$$\text{operation } O = \begin{pmatrix} \text{cured} & \begin{array}{c}\text{permanent}\\ \text{disability}\end{array} \\ \\ 90\% & 10\% \end{pmatrix}$$

$$\text{drug treatment } D = \begin{pmatrix} \text{cured} & \begin{array}{c}\text{no}\\ \text{benefit}\end{array} & \begin{array}{c}\text{adverse}\\ \text{reaction}\end{array} \\ \\ 75\% & 10\% & 15\% \end{pmatrix}$$

What is the expected value of lottery O?

What is the expected value of lottery D?

Which of the two lotteries is better?

EXPECTED UTILITY THEORY

$Z = \{z_1, z_2, ..., z_m\}$ set of basic outcomes.

A lottery is a probability distribution over Z: $L = \begin{pmatrix} z_1 & z_2 & \ldots & z_m \\ p_1 & p_2 & \ldots & p_m \end{pmatrix}$

Let $\mathcal{L}$ be the set of lotteries. Suppose that the agent has a ranking $\succsim$ of the elements of $\mathcal{L}$:

if $L = \begin{pmatrix} z_1 & z_2 & \ldots & z_m \\ p_1 & p_2 & \ldots & p_m \end{pmatrix}$ and $M = \begin{pmatrix} z_1 & z_2 & \ldots & z_m \\ q_1 & q_2 & \ldots & q_m \end{pmatrix}$ then

$L \succ M$ means that

$L \sim M$ means that

Rationality constraints on $\succsim$ (von Neumann-Morgenstern axioms):

…

Theorem 1 Let $Z=\{z_1,z_2,...,z_m\}$ be a set of basic outcomes and L the set of lotteries over *Z*. If $\succsim$ satisfies the von Neumann-Morgenstern axionm then there exists a function $U:Z\to\mathbb{R}$, called a *von Neumann-Morgenstern utility function*, that assigns a number to every basic outcome and is such that, for any two lotteries $L=\begin{pmatrix} z_1 & z_2 & \dots & z_m \\ p_1 & p_2 & \dots & p_m \end{pmatrix}$ and $M=\begin{pmatrix} z_1 & z_2 & \dots & z_m \\ q_1 & q_2 & \dots & q_m \end{pmatrix}$,

$$L \succ M \quad \text{if and only if} \quad \underbrace{p_1U(z_1)+p_2U(z_2)+...+p_mU(z_m)}_{\text{expected utility of lottery } L} > \underbrace{q_1U(z_1)+q_2U(z_2)+...+q_mU(z_m)}_{\text{expected utility of lottery } M}$$

and

$$L \sim M \quad \text{if and only if} \quad \underbrace{p_1U(z_1)+p_2U(z_2)+...+p_mU(z_m)}_{\text{expected utility of lottery } L} = \underbrace{q_1U(z_1)+q_2U(z_2)+...+q_mU(z_m)}_{\text{expected utility of lottery } M}$$

2.25 3.33

EXAMPLE 1. $Z = \{z_1, z_2, z_3, z_4\}$ $L = \begin{pmatrix} z_1 & z_2 & z_3 & z_4 \\ \frac{1}{8} & \frac{5}{8} & 0 & \frac{2}{8} \end{pmatrix}$ $M = \begin{pmatrix} z_1 & z_2 & z_3 & z_4 \\ \frac{1}{6} & \frac{2}{6} & \frac{1}{6} & \frac{2}{6} \end{pmatrix}$

Suppose we know that $U = \begin{cases} z_1 & z_2 & z_3 & z_4 \\ 6 & 2 & 8 & 1 \end{cases}$

Then

$\mathbb{E}[U(L)] =$

$\mathbb{E}[U(M)] =$

EXAMPLE 2.

$$A = \begin{pmatrix} \text{paid 3-week vacation} & \text{no vacation} \\ 50\% & 50\% \end{pmatrix} \qquad B = \begin{pmatrix} \text{paid 1-week vacation} \\ 100\% \end{pmatrix}$$

Suppose Ann says $\boxed{B \succ A}$ How would she rank

$$C = \begin{pmatrix} \text{paid 3-week vacation} & \text{no vacation} \\ 5\% & 95\% \end{pmatrix} \text{ and } D = \begin{pmatrix} \text{paid 1-week vacation} & \text{no vacation} \\ 10\% & 90\% \end{pmatrix} ?$$

Money lotteries

$$L = \begin{pmatrix} \$17 \\ 1 \end{pmatrix} \qquad M = \begin{pmatrix} \$9 & \$25 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\mathbb{E}[L] = \qquad \mathbb{E}[M] =$$

Suppose Bob's vNM utility function is: $U(\$x) = \sqrt{x}$

$$\mathbb{E}[U(L)] = \qquad \mathbb{E}[U(M)] =$$

$$A = \begin{pmatrix} \$0 & \$100 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad B = \begin{pmatrix} \$40 & \$60 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$$\mathbb{E}[A] = \qquad \mathbb{E}[B] =$$

Suppose Bob's vNM utility function is: $U(\$x) = \sqrt{x}$

$$\mathbb{E}[U(A)] =$$

$$\mathbb{E}[U(B)] =$$

$$A = \begin{pmatrix} \$4 & \$6 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad B = \begin{pmatrix} \$5 \\ 1 \end{pmatrix}$$

$$\mathbf{U(\$x) = x^2}$$

Re-define attitudes to risk in terms of utility:

Risk-averse if

Risk-neutral if

Risk-loving if