# Money lotteries

$$L = \begin{pmatrix} \$17 \\ 1 \end{pmatrix} \qquad M = \begin{pmatrix} \$9 & \$25 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad Z = \{\$9, \$17, \$25\}$$

$$\mathbb{E}[L] = 1 \times 17 = $$
$$= 17$$

$$\mathbb{E}[M] = \frac{1}{2} \times 9 + \frac{1}{2} \times 25 = 17$$

IF DM is <u>risk neutral</u> then $L \sim M$

For a risk neutral person $U(\$x) = x$ identity function

$$E[U(L)] = 1 \times U(\$17) = 1 \times 17 = 17 = E[L]$$

$$E[U(M)] = \frac{1}{2} U(\$9) + \frac{1}{2} U(\$25) = \frac{1}{2} \times 9 + \frac{1}{2} \times 25 = 17 = E[M]$$

# Money lotteries

$$L = \begin{pmatrix} \$17 \\ 1 \end{pmatrix} \qquad M = \begin{pmatrix} \$9 & \$25 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$\mathbb{E}[L] =$ 17 $\qquad \mathbb{E}[M] =$ (17)

4.12

Suppose Bob's vNM utility function is: $U(\$x) = \sqrt{x}$

$\mathbb{E}[U(L)] =$

$1 \times U(\$17) =$

$1 \times \sqrt{17} =$ (4.12)

$>$

$\mathbb{E}[U(M)] =$

$\frac{1}{2} U(\$9) + \frac{1}{2} U(\$25)$

$= \frac{1}{2}\sqrt{9} + \frac{1}{2}\sqrt{25} = \frac{1}{2} 3 + \frac{1}{2} 5 =$ (4)

$L > M$ risk averse

$$A = \begin{pmatrix} \$0 & \$100 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad B = \begin{pmatrix} \$40 & \$60 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

$\mathbb{E}[A] = 50$

$\mathbb{E}[B] = \frac{1}{2} 40 + \frac{1}{2} 60 = 50$

Suppose Bob's vNM utility function is: $U(\$x) = \sqrt{x}$ risk averse

$\mathbb{E}[U(A)] = \frac{1}{2}\sqrt{0} + \frac{1}{2}\sqrt{100} = \frac{1}{2} 10 = 5$

$\wedge$

7.03

$\mathbb{E}[U(B)] = \frac{1}{2}\sqrt{40} + \frac{1}{2}\sqrt{60} = 7.03$ $\quad B > A$

26
25

$E[A] = \frac{1}{2}4 + \frac{1}{2}6 = 5$

$$A = \begin{pmatrix} \$4 & \$6 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad B = \begin{pmatrix} \$5 \\ 1 \end{pmatrix}$$

$E[B] = 5$

$$\mathbf{U(\$x) = x^2}$$

$E[U(A)] = \frac{1}{2}4^2 + \frac{1}{2}6^2 = \frac{1}{2}16 + \frac{1}{2}36 = 26$

$E[U(B)] = 1 \times 5^2 = 25$

$A \succ B$ risk loving

money lottery $L$ $\quad E[L]$

Re-define attitudes to risk in terms of utility:

Risk-averse if : $(E[L]) \succ L \qquad U(E[L]) > E[U(L)]$

Risk-neutral if : $L \sim (E[L]) \qquad E[U(L)] = U(E[L])$

Risk-loving if : $L \succ (E[L]) \qquad E[U(L)] > U(E[L])$

**Theorem 2.** Let $\succsim$ be a von Neumann-Morgenstern ranking of the set of basic lotteries $\mathcal{L}$. Then the following are true.

(A) If $U : Z \to \mathbb{R}$ is a von Neumann-Morgenstern utility function that represents $\succsim$, then, for any two real numbers $a$ and $b$ with $a > 0$, the function $V : Z \to \mathbb{R}$ defined by $V(z_i) = aU(z_i) + b$ $(i = 1,2,\ldots,m)$ is also a von Neumann-Morgenstern utility function that represents $\succsim$.

(B) If $U : Z \to \mathbb{R}$ and $V : Z \to \mathbb{R}$ are two von Neumann-Morgenstern utility functions that represent $\succsim$, then there exist two real numbers $a$ and $b$ with $a > 0$ such that $V(z_i) = aU(z_i) + b$ ( $i = 1,2,\ldots,m$).

$Z = \{z_1, z_2, \ldots, z_6\}$

$$U = \begin{cases} z_1 & z_2 & z_3 & z_4 & z_5 & z_6 \\ 10 & 6 & 16 & 8 & 6 & 14 \end{cases}$$

$a = 2$, $b = -4$

$2 \times 10 - 4 = 16$

$2 \times 6 - 4 = 8$

$2 \times 16 - 4 = 28$

$2 \times 8 - 4 = 12$

$2 \times 14 - 4 = 24$

| $V =$ | $z_1$ | $z_2$ | $z_3$ | $z_4$ | $z_5$ | $z_6$ |
|---|---|---|---|---|---|---|
| | 16 | 8 | 28 | 12 | 8 | 24 |

subtract 4 $(a = 1, b = -4)$

| $W =$ | $z_1$ | $z_2$ | $z_3$ | $z_4$ | $z_5$ | $z_6$ |
|---|---|---|---|---|---|---|
| | 6 | 2 | 12 | 4 | 2 | 10 |

now multiply all by 2 (starting from W) $[a = 2, b = 0]$



$T$ : 12 4 24 8 4 20

$$U = \begin{cases} z_1 & z_2 & z_3 & z_4 & z_5 & z_6 \\ 10 & 6 & 16 & 8 & 6 & 14 \end{cases}$$

subtract 6 $(a=1, b=-6)$

$V \quad 4 \quad 0 \quad 10 \quad 2 \quad 0 \quad 8$

divide by 16 $(a=\frac{1}{10}, b=0)$

$W \quad \frac{2}{5} \quad 0 \quad 1 \quad \frac{1}{5} \quad 0 \quad \frac{4}{5}$

normalized utility function

utility $(z_{best}) = 1$

utility $(z_{worst}) = 0$

| $z_1$ | $z_2$ | $z_3$ | $z_4$ | $z_5$ | $z_6$ |
|---|---|---|---|---|---|
| 0 | −4 | 6 | −2 | −4 | 4 |

add 4 $(a=1, b=4)$

| | | | | | |
|---|---|---|---|---|---|
| 4 | 0 | 10 | 2 | 0 | 8 |

divide by 10 $(a=\frac{1}{10}, b=0)$

| | | | | | |
|---|---|---|---|---|---|
| $\frac{2}{5}$ | 0 | 1 | $\frac{1}{5}$ | 0 | $\frac{4}{5}$ |

$$\text{operation } O = \begin{pmatrix} \overset{z_1}{\text{cured}} & \overset{z_2}{\text{permanent disability}} \\ 90\% & 10\% \end{pmatrix}$$

$$\begin{pmatrix} z_1 & z_2 \\ \frac{90}{100} & \frac{10}{100} \end{pmatrix}$$

$$\text{drug treatment } D = \begin{pmatrix} \overset{z_1}{\text{cured}} & \overset{z_3}{\text{no benefit}} & \overset{z_4}{\text{adverse reaction}} \\ 75\% & 10\% & 15\% \end{pmatrix} = \begin{pmatrix} z_1 & z_3 & z_4 \\ \frac{75}{100} & \frac{10}{100} & \frac{15}{100} \end{pmatrix}$$

**What should you do?**

| | | UTILITY |
|---|---|---|
| best | $z_1$ | 1 |
| | $z_3$ | $\frac{95}{100}$ |
| | $z_4$ | $\frac{1}{2}$ |
| worst | $z_2$ | 0 |

For $z_3$: Question "what value of $p$ is such that $\begin{pmatrix} z_3 \\ 1 \end{pmatrix} \sim \begin{pmatrix} z_1 & z_2 \\ p & 1-p \end{pmatrix}$?

Suppose he says $p = \frac{95}{100}$

$$z_3 \sim \begin{pmatrix} z_1 & z_2 \\ \frac{95}{100} & \frac{5}{100} \end{pmatrix}$$

exp utility of this — exp. utility of this

$$U(z_3) = \frac{95}{100} \times U(z_1) + \frac{5}{100} U(z_2)$$

$$\frac{95}{100} \times 1 + \frac{5}{100} \times 0 = \frac{95}{100}$$

Suppose he says
$p = \frac{95}{100}$

| | | UTILITY |
|---|---|---|
| best | $z_1$ | 100 |
| | $z_3$ | 96 |
| | $z_4$ | 60 |
| worst | $z_2$ | 20 |

then $U(z_3) = \frac{95}{100} \times U(z_1) + \frac{5}{100} U(z_2)$

$= \frac{95}{100} 100 + \frac{5}{100} 20$

$= 95 + 1 = 96$

Last question: what value of $p$ is such that $z_4 \sim \begin{pmatrix} z_1 & z_2 \\ p & 1-p \end{pmatrix}$ ?

Suppose answer is $p = \frac{1}{2}$

In the normalized utility function

$U(z_4) = \frac{1}{2} U(z_1) + \frac{1}{2} U(z_2)$

$= \frac{1}{2} 1 + \frac{1}{2} 0 = \frac{1}{2}$

$U(z_4) = \frac{1}{2} U(z_1) + \frac{1}{2} U(z_2) + \frac{1}{2} 100 + \frac{1}{2} 20 = 60$

$$\text{operation } O = \begin{pmatrix} \overset{z_1}{\text{cured}} & \overset{z_2}{\text{permanent disability}} \\ 90\% & 10\% \end{pmatrix}$$

$$\text{drug treatment } D = \begin{pmatrix} \overset{z_1}{\text{cured}} & \overset{z_3}{\text{no benefit}} & \overset{z_4}{\text{adverse reaction}} \\ 75\% & 10\% & 15\% \end{pmatrix}$$

=92

$$E[U(O)] = \frac{90}{100}\underbrace{\overbrace{U(z_1)}^{100}}_{1} + \frac{10}{100}\underbrace{\overbrace{U(z_2)}^{20}}_{0} = \frac{90}{100}$$

=93.6

$$E[U(D)] = \frac{75}{100}\underbrace{\overbrace{U(z_1)}^{100}}_{1} + \frac{10}{100}\underbrace{\overbrace{U(z_3)}^{96}}_{\frac{95}{100}} + \frac{15}{100}\underbrace{\overbrace{U(z_4)}^{60}}_{\frac{1}{2}}$$

$$= \frac{75}{100} + \frac{10}{100}\,\frac{95}{100} + 100\,\frac{1}{2} =$$ 92/100

So D is better than O for this individual