Hurwicz index

	s_1	s_2	s_3
a_1	8	1	0
a_2	6	2	3
a_3	0	3	4

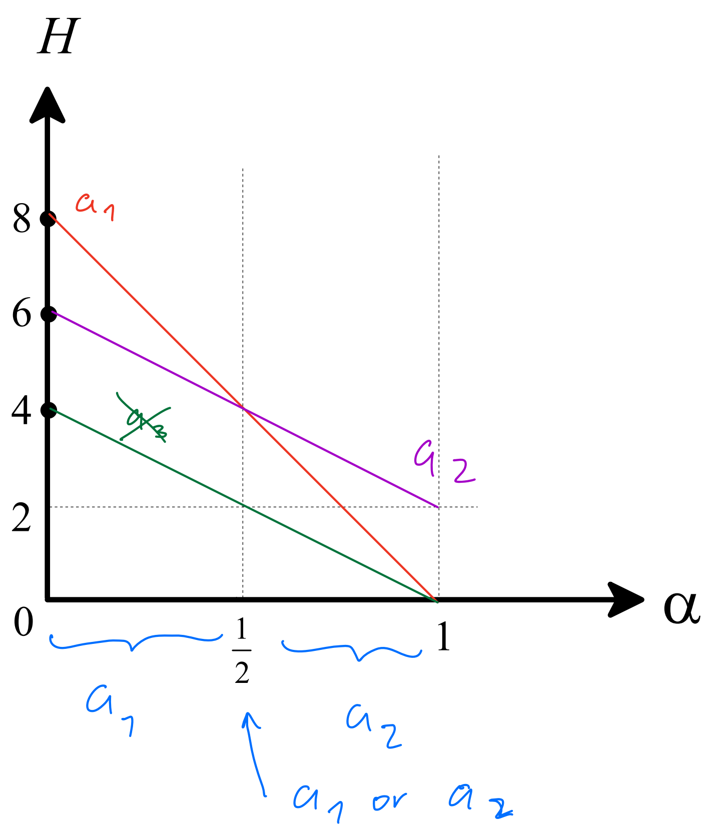
$U(\cdot)$

$V(\cdot) = a\,U(\cdot) + b$

$$H_\alpha(a_1) = 0\alpha + 8(1-\alpha) = 8 - 8\alpha$$

$$H_\alpha(a_2) = 2\alpha + 6(1-\alpha) = 6 - 4\alpha$$

$$H_\alpha(a_3) = 0\alpha + 4(1-\alpha) = 4 - 4\alpha$$



Note: the Hurwicz index is invariant to allowed transformations of the utility function.

MinMax REGRET

	s_1	s_2	s_3
a_1	8	1	0
a_2	6	2	3
a_3	0	3	4

Define the **regret of taking action *a* under state *s*** as the difference between the maximum utility you could have got under state *s* (by taking the best action for that state) and the utility that you get with action *a*. We can then construct a **regret table:**

	s_1	s_2	s_3
a_1	$8-8=0$	$3-1=2$	$4-0=4$
$\boxed{a_2}$	$8-6=\boxed{2}$	$3-2=1$	$4-3=1$
a_3	$8-0=8$	$3-3=0$	$4-4=0$

MinMax regret

Highlight maximum regrets

Choose action that minimizes these highlighted numbers

If I had chosen an alternative utility function, would I have reached the same conclusion in terms of MinMaxRegret? Consider a new decision problem:

	s_1	s_2	s_3
a_1	0	2	4
a_2	2	1	1
a_3	8	0	0

	s_1	s_2	s_3
a_1	z_1	z_2	z_3
a_2	z_4	z_5	z_6
a_3	z_7	z_8	z_9

we infer that the ranking is

$V(z) = a\,U(z) + b$

$a > 0$

Choose $a = 2$

$b = -1$

		U	V
best	z_7	8	15
	z_3	4	7
	z_2, z_4	2	3
	z_5, z_6	1	1
worst	z_1, z_8, z_9	0	−1

utility table

Using U:

	s_1	s_2	s_3
a_1	0	2	4
a_2	2	1	1
a_3	8	0	0

utility table ~~regret:~~ using V

	s_1	s_2	s_3
a_1	−1	3	7
a_2	3	1	1
a_3	15	−1	−1

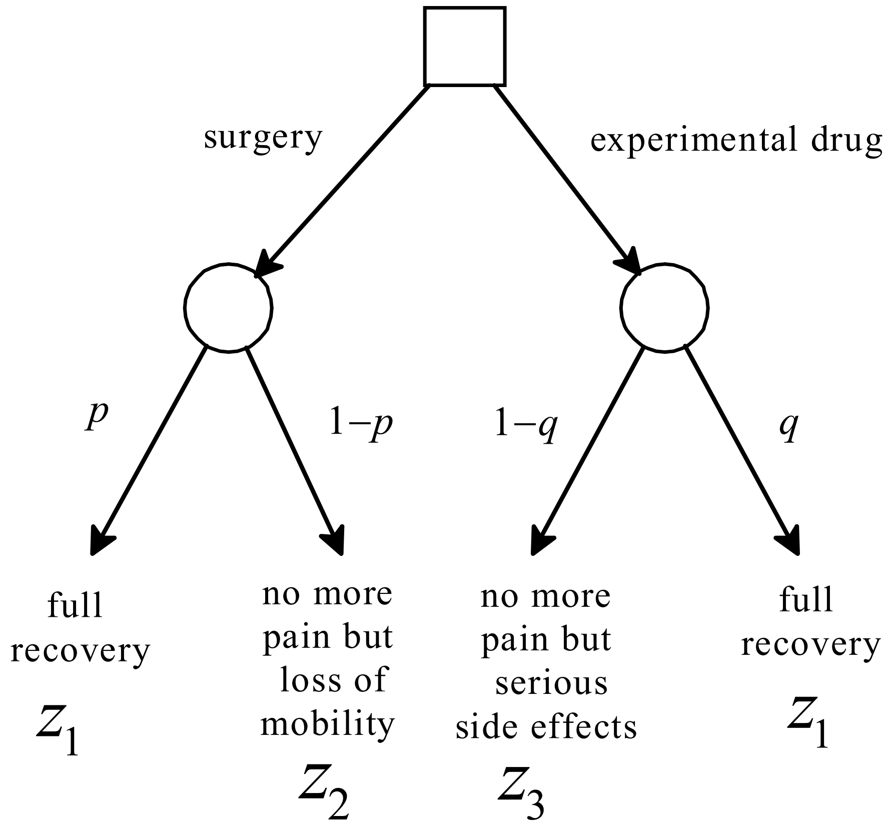
regret using U ~~using V~~ :

	s_1	s_2	s_3
a_1	8	0	0
a_2	6	1	3
a_3	0	2	4

regret: using V

	s_1	s_2	s_3
a_1	16	0	0
a_2	12	2	6
a_3	0	4	8

Example: knee injury



Suppose that:

normalized			U
1	best	z_1	100
$\frac{3}{4}$		z_3	75
0	worst	z_2	0

What value of s is such that

$z_3 \sim \begin{pmatrix} z_1 & z_2 \\ s & 1-s \end{pmatrix}$

Answer is $s = \frac{3}{4}$

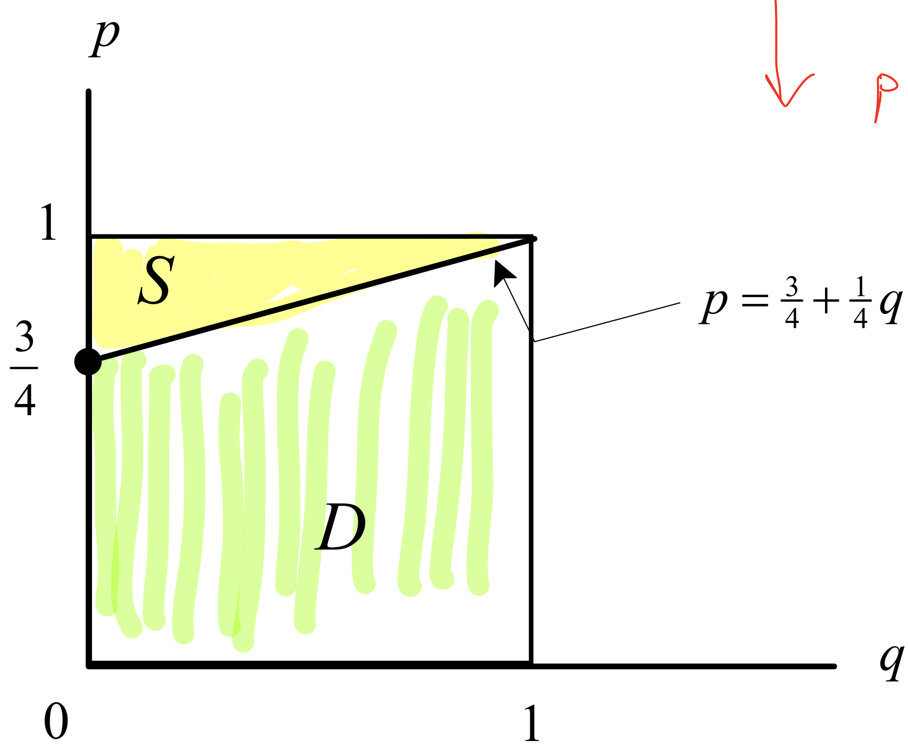
The expected utility of surgery is $p\,100 + (1-p)0 = 100p$

the expected utility of taking the drug is $q\,100 + (1-q)75 = 75 + 25q$

So if you know the values of p and q then your optimal decision is:

- surgery if $100p > 75 + 25q$
- drug if $75 + 25q > 100p$
- either surgery or drug is $\underbrace{100p = 75 + 25q}$

$p = \frac{3}{4} + \frac{1}{4} q$



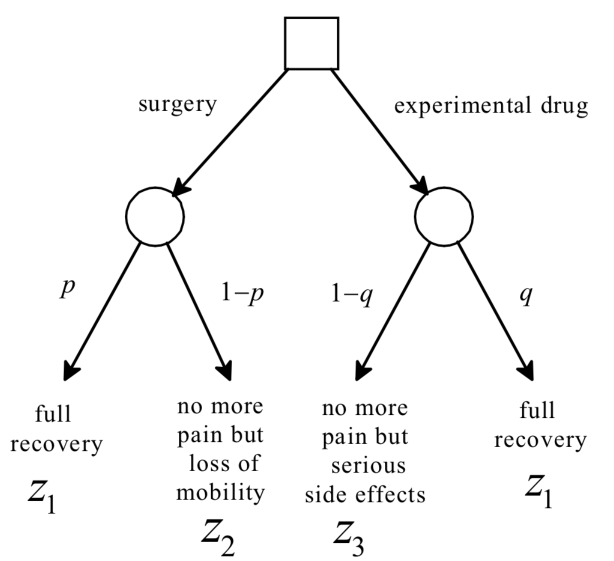
indifferent between Surgery and drug

Suppose that the values of p and q are not available

(S, D) : surgery would be successful and drug would be successful

$(\neg S, D)$: surgery would NOT be successful, drug would be

	(S, D)	$(S, \neg D)$	$(\neg S, D)$	$(\neg S, \neg D)$
Surgery	z_1	z_1	z_2	z_2
Drug	z_1	z_3	z_1	z_3



	U
best z_1	100
z_3	75
worst z_2	0

Replacing outcomes with utilities:

	(S, D)	$(S, \neg D)$	$(\neg S, D)$	$(\neg S, \neg D)$
Surgery	100	100	0	0
Drug	100	75	100	75

The corresponding regret table is:

	(S, D)	$(S, \neg D)$	$(\neg S, D)$	$(\neg S, \neg D)$
Surgery	0	0	100	75
Drug	0	25	0	0

What about the Hurwicz index?

$$H_\alpha(Drug) = 75\alpha + 100(1-\alpha) = 100 - 25\alpha$$

$>$

$$H_\alpha(Surgery) = 0 \cdot \alpha + 100(1-\alpha) = 100 - 100\alpha$$