UNCERTAINTY, INFORMATION and BELIEFS

uncertainty = set of possibilities

Murder suspects:

Ann	Amy	Arthur	Jane	Jim	John
Boone	Bloom	Bragg	Singer	Shore	Smith

information = reduction of uncertainty = shrinking of the set of possibilities

A handkerchief with initials was found at the murder scene:

Ann	Amy	Arthur	Jane	Jim	John
Boone	Bloom	Bragg	Singer	Shore	Smith

Add a witness who can tell if the murderer was a man or a woman:

Ann	Amy	Arthur	Jane	Jim	John
Boone	Bloom	Bragg	Singer	Shore	Smith

Think of information not as a particular item of information but as the list of possible items of information that one might receive

Why? Because when you contemplate seeking or purchasing information you do not know yet what specific item of information you might receive.

INFORMATION = PARTITION OF THE SET OF STATES

- Initial state of uncertainty: $S = \{s_1, s_2, ..., s_n\}$
- Information: partition of *S* into two or more subsets (information sets): $\mathcal{I} = \{S_1, S_2, ..., S_m\}$

An eye witness can tell if it was a man or a woman

Ann	Amy	Arthur	Jane	Jim	John
Boone	Bloom	Bragg	Singer	Shore	Smith

Initial state of uncertainty: $S = \{s_1, s_2, \ldots, s_n\}$

- PERFECT INFORMATION: $\mathcal{I} = \{\{s_1\}, \{s_2\}, \ldots, \{s_n\}\}$ you learn what the state is

- IMPERFECT information: $\mathcal{I} = \{S_1, S_2, \ldots, S_m\}$ where at least one of the information sets S_i is not a singleton.

Theorem: FREE INFORMATION IS ALWAYS VALUABLE

Free information will always make you better off or leave you just as well off.

EXAMPLE

Your symptoms are compatible with three diseases: A, B and C. For each disease there is a drug that is effective only for that disease. Only two outcomes: you heal (H) or you remain sick (S). The doctor gives you the following probabilistic assessment of the three diseases:

	A	B	C
	70%	20%	10%
TA (treat A)	H	S	S
TB (treat B)	S	H	S
TC (treat C)	S	S	H

Suppose you have von Neumann-Morgenstern preferences.

	A	B	C
	70%	20%	10%
TA (treat A)			
TB (treat B)			
TC (treat C)			

$\mathbb{E}[U(TA)] =$

$\mathbb{E}[U(TB)] =$

$\mathbb{E}[U(TC)] =$

thus your best course of action is and your expected utility is

Suppose that there is a **free** blood test to check if you have disease C, that is, the blood test gives you information that corresponds to the following partition:

A	B	C
70%	20%	10%

1. Suppose I take the test and it comes back **negative**. What would I do then?

- Update the probabilities:

	A	B	C
$P(\bullet \mid negative)$:			

	A	B	C
probability			
TA (treat A)			
TB (treat B)			
TC (treat C)			

$\mathbb{E}[U(TA) \mid negative] =$

$\mathbb{E}[U(TB) \mid negative] =$

$\mathbb{E}[U(TC) \mid negative] =$

thus your best course of action is and your expected utility is

$$\begin{array}{ccc} A & B & C \end{array}$$

$P(\bullet \mid positive):$

	A	B	C
probability			
TA (treat A)			
TB (treat B)			
TC (treat C)			

$\mathbb{E}[U(TA) \mid positive] =$

$\mathbb{E}[U(TB) \mid positive] =$

$\mathbb{E}[U(TC) \mid positive] =$

thus your best course of action is and your expected utility is

Positive test: take action TA with expected utility

negative test: take action TC with expected utility

- Based on the initial assessment: $\begin{array}{ccc} A & B & C \\ 70\% & 20\% & 10\% \end{array}$, how likely is it that you will get a negative result and how likely is it that you will get a positive result?

$$P(negative) =$$

$$P(positive) =$$

- Thus your expected utility if you take the blood test is:

WHAT IF INFORMATION IS COSTLY?

How much should one be prepared to pay for information?

CASE 1: monetary outcomes and perfect information

CASE 1.1: risk neutrality

CASE 1.2: risk aversion

CASE 2: monetary outcomes and imperfect information

CASE 2.1: risk neutrality

CASE 2.2: risk aversion

CASE 3: general outcomes and perfect information

CASE 4: general outcomes and imperfect information

Let us go back to the previous example, where the amounts are **changes** in wealth.

probability	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$4	\$36	\$244
b	\$8	\$201	\$18
c	\$124	\$12	\$24

Suppose that the DM's initial wealth is \$140 and her utility function is $U(\$x) = \sqrt{x}$. How much would she be willing to pay for perfect information?

STEP 1. First of all: expected utility is if she does not purchase information.

probability	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$144	\$176	\$384
b	\$148	\$341	\$158
c	\$264	\$152	\$164

$$\mathbb{E}[U(a)] =$$

$$\mathbb{E}[U(b)] =$$

$$\mathbb{E}[U(c)] =$$

STEP 2. Calculate her expected utility if she purchases perfect information at price p.

• If I am told that the state is s_1 then I will choose and get a utility of	probability	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$
• If I am told that the state is s_2 then I will choose and get a utility of	state →	s_1	s_2	s_3
• If I am told that the state is s_3 then I will choose and get a utility of	act ↓			
	a	\$144	\$176	\$384
	b	\$148	\$341	\$158
	c	\$264	\$152	\$164

Expected utility if I purchase information is:

How much should one be prepared to pay for information?

CASE 2: monetary outcomes and IMPERFECT information

CASE 2.1: risk neutrality

The amounts are **changes** in her wealth.

probability	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{4}$
state $\rightarrow$	s_1	s_2	s_3	s_4
act $\downarrow$				
a	\$16	\$36	\$100	\$12
b	\$10	\$64	\$18	\$120
c	\$104	\$12	\$24	\$0

STEP 0. Change the probabilities so that they have the same denominator:

probability				
state $\rightarrow$	s_1	s_2	s_3	s_4
act $\downarrow$				
a	\$16	\$36	\$100	\$12
b	\$10	\$0	\$18	\$120
c	\$104	\$12	\$24	\$0

$\mathbb{E}[a] =$

$\mathbb{E}[b] =$

$\mathbb{E}[c] =$

Thus she will choose and expect

Suppose now that Ann is offered, at price *p*, the following imperfect information:

$$\{\{s_1, s_2\}, \{s_3, s_4\}\}$$

probability	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{3}{12}$
state →	s_1	s_2	s_3	s_4
act ↓				
a	\$16	\$36	\$100	\$12
b	\$10	\$0	\$18	\$120
c	\$104	\$12	\$24	\$0

- If informed that $\{s_1, s_2\}$ then

probability		
state →	s_1	s_2
act ↓		
a	\$16	\$36
b	\$10	\$0
c	\$104	\$12

$$\mathbb{E}[a] =$$

$$\mathbb{E}[b] =$$

$$\mathbb{E}[c] =$$

Thus she will choose and expect

probability	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{3}{12}$
state $\rightarrow$	s_1	s_2	s_3	s_4
act $\downarrow$				
a	\$16	\$36	\$100	\$12
b	\$10	\$0	\$18	\$120
c	\$104	\$12	\$24	\$0

- If informed that $\{s_3, s_4\}$ then

probability		
state $\rightarrow$	s_3	s_4
act $\downarrow$		
a	\$100	\$12
b	\$18	\$120
c	\$24	\$0

$$\mathbb{E}[a] =$$

$$\mathbb{E}[b] =$$

$$\mathbb{E}[c] =$$

Thus she will choose and expect

probability	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{2}{12}$	$\frac{3}{12}$
state $\rightarrow$	s_1	s_2	s_3	s_4
act $\downarrow$				
a	\$16	\$36	\$100	\$12
b	\$10	\$0	\$18	\$120
c	\$104	\$12	\$24	\$0

The probability of $\{s_1, s_2\}$ is and the probability of $\{s_3, s_4\}$ is

Thus the expected change in wealth with perfect information at price *p* is

Thus as long as

it is worth paying for the information.