CASE 2.2: risk aversion

Smaller example.

Changes in wealth:

probability	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$21	\$0	\$156
b	\$0	\$125	\$0
c	\$96	\$0	\$69

Assume: $U(\$x) = \sqrt{x}$ and initial wealth is \$100. Then

probability	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$121	\$100	\$256
b	\$100	\$225	\$100
c	\$196	\$100	\$169

add initial wealth to each outcome

probability	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$121	\$100	\$256
b	\$100	\$225	\$100
c	\$196	\$100	\$169

STEP 1. If she does **not purchase** information.

$$\mathbb{E}[U(a)] = \frac{2}{9}\sqrt{121} + \frac{4}{9}\sqrt{100} + \frac{3}{9}\sqrt{256} = 12.22$$

$$\mathbb{E}[U(b)] = \frac{2}{9}\sqrt{100} + \frac{4}{9}\sqrt{225} + \frac{3}{9}\sqrt{100} = 12.22$$

$$\mathbb{E}[U(c)] = \frac{2}{9}\sqrt{196} + \frac{4}{9}\sqrt{100} + \frac{3}{9}\sqrt{169} = 11.89$$

Thus she will choose either a or b with an expected utility of $\boxed{12.22}$

		$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
probability		$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state $\rightarrow$		s_1	s_2	s_3
act $\downarrow$				
	a	\$121	\$100	\$256
	b	\$100	\$225	\$100
	c	\$196	\$100	\$169

STEP 2. If she purchases information $\{\{s_1, s_2\}, \{s_3\}\}$ at price p.

- If informed that $\{s_1, s_2\}$ then the revised decision problem is:

probability		$\frac{2}{6}$	$\frac{4}{6}$	$\left(\begin{matrix} 0 \\ s_3 \end{matrix}\right)$
state $\rightarrow$		s_1	s_2	
act $\downarrow$				
	a	\$121	\$100	
	b	\$100	\$225	
	c	\$196	\$100	

$2+4=6$

$$\mathbb{E}[U(a)] = \frac{1}{3}\sqrt{121-p} + \frac{1}{3}\sqrt{100-p} + \cancel{\frac{1}{3}\sqrt{100-p}}$$

$$\mathbb{E}[U(b)] = \cancel{\frac{1}{3}\sqrt{100-p}} + \frac{1}{3}\sqrt{225-p} + \frac{1}{3}\sqrt{225-p}$$

$$\mathbb{E}[U(c)] = \frac{1}{3}\sqrt{196-p} + \frac{1}{3}\sqrt{100-p} + \cancel{\frac{1}{3}\sqrt{100-p}}$$

Thus she will choose b with an expected utility of $\frac{1}{3}\sqrt{100-p} + \frac{2}{3}\sqrt{225-p}$

probability	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$121	\$100	\$256
b	\$100	\$225	\$100
c	\$196	\$100	\$169

- If informed that $\{s_3\}$ then she will choose a with a utility of $\sqrt{256-p}$

Given the initial probabilities:

probability	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state $\rightarrow$	s_1	s_2	s_3

the probability of receiving information $\{s_1, s_2\}$ is $\frac{6}{9} = \frac{2}{3}$ and the probability of receiving information $\{s_3\}$ is $\frac{1}{3}$.

Thus the expected utility of purchasing information at price p is:

$$f(p) = \frac{2}{3}\left[\frac{1}{3}\sqrt{100-p} + \frac{2}{3}\sqrt{225-p}\right] + \frac{1}{3}\sqrt{256-p}$$

For example, if p = \$30 then $f(30) = 13.08 > 12.22$

Yes
to
information

The maximum price the DM is willing to pay for information is given by the solution to:

$$\frac{2}{3}\left[\frac{1}{3}\sqrt{100-p} + \frac{2}{3}\sqrt{225-p}\right] + \frac{1}{3}\sqrt{256-p} = 12.22$$

Which is

Future Value and Present Value

- **$100 today**, or
- **$200, 5 years from now**

Reasons for preferring $100 today:

Rephrase the choice as:

- $100 today, **but cannot be spent until 5 years from now**, or
- $200, 5 years from now

Annual rate of interest: r

Today

100

1 year from today

$100 + 100r$
$= 100(1+r)$

2 years from today

$100(1+r) + r\,100(1+r)$
$= 100(1+r)^2$

3 years from today

$100(1+r)^3$

4 years from today

$100(1+r)^4$

5 years from today

$100(1+r)^5$

Future value of $100 today 5 years from now

Compare $100(1+r)^5$ to 200

Definition: the ***future value*** of $\$x$, n periods from now is

$$x(1+r)^n$$

where r is the interest rate per period r .

- If $r = 0.10$ (i.e. 10%) then the future value of \$100 five years from now is

\$161.05 So will choose \$200 in 5 years

- If $r = 0.15$ (i.e. 15%) then the future value of \$100 five years from now is

\$201.14 So will choose \$100 today

Definition: the ***present value y*** of $\$x$ available n periods from now is the solution to

$\delta = \frac{1}{1+r}$ discount factor

$$y(1+r)^n = x \qquad y = \frac{x}{(1+r)^n} = \underbrace{\left(\frac{1}{1+r}\right)}_{\delta}^n x$$

- If $r = 0.10$ (i.e. 10%) then the present value of \$200 five years from now is

$$\frac{200}{(1+0.1)^5} = 124.18 \qquad (124.18)(1+0.1)^5 = 200$$

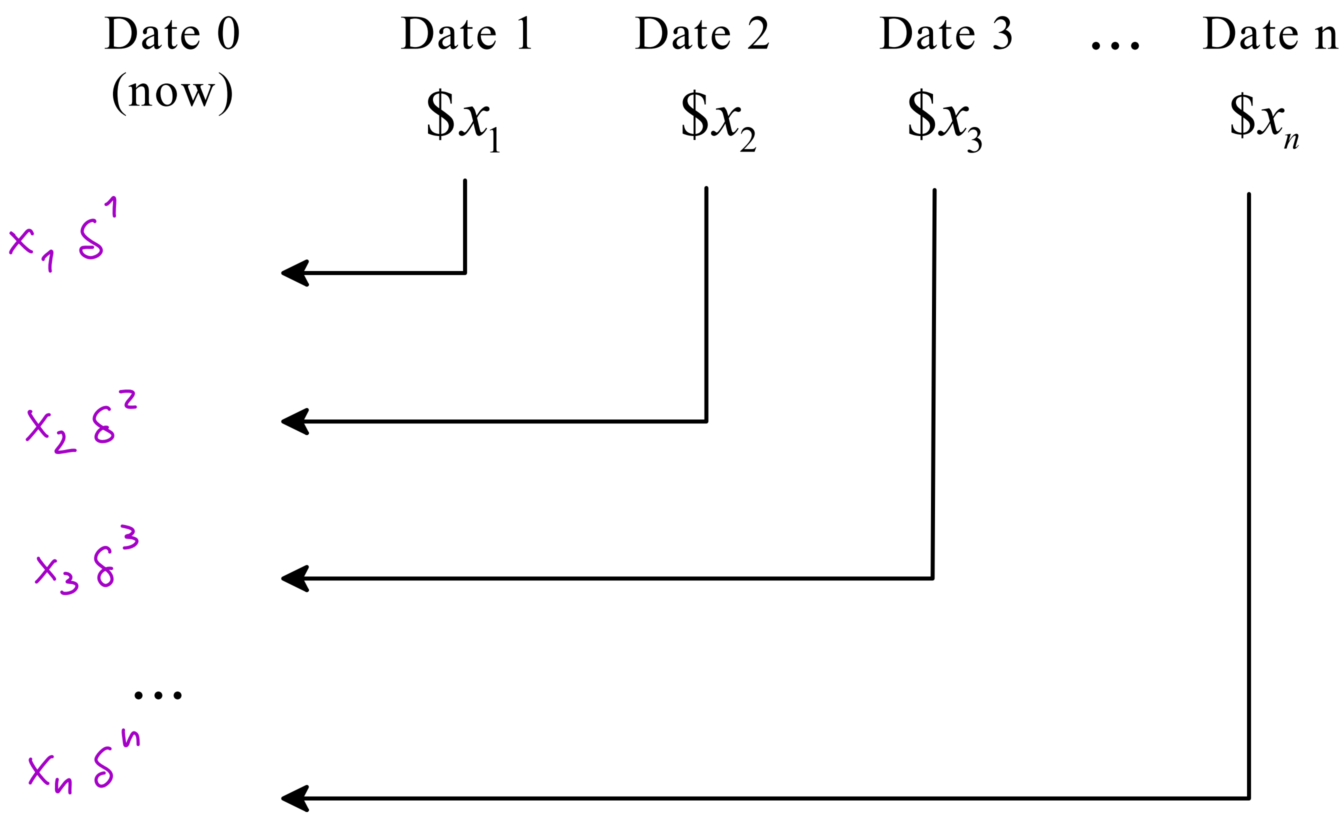
- If $r = 0.15$ (i.e. 15%) then the present value of \$200 five years from now is

$$\frac{200}{(1+0.15)^5} = 99.44 \qquad (99.44)(1+0.15)^5 = 200$$

r is the interest **rate**, $\delta = \dfrac{1}{1+r}$ is the discount **factor**. Thus the present value of $\$x$ available n periods from now is also denoted by $x\delta^n$.

Note that $\dfrac{1}{(1+r)^n} = \left(\dfrac{1}{1+r}\right)^n = \delta^n$.

Above we calculated the present value of a sum of money. We can also calculate the present value of a **stream of payments**:



So the present value of that income stream is

$$T = x_1 \delta + x_2 \delta^2 + x_3 \delta^3 + \cdots + x_n \delta^n$$

This is a sum of money that is **equivalent to that income stream**. Equivalent in what sense?

Suppose that $r = 12\%$ (the present is date 0):

date 2	date 3	date 5
\$2,000	\$3,000	\$3,500

The present value of \$2,000 available at date 2 is $2000\left(\frac{1}{1.12}\right)^2 = 1,594.39$

the present value of \$3,000 available at date 3 is $3000\left(\frac{1}{1.12}\right)^3 = 2,135.34$

the present value of \$3,500 available at date 5 is $3300\left(\frac{1}{1.12}\right)^5 = 1,985.99$

total …

$$2000 = (1,594.39)(1.12)^2$$

Put these three sums of money in three different accounts

balance

CD1 (principal: $1,594.39) matures in 2 years 2,000 date 2

CD2 principal: $2,135.34) " 3 years 3,000 date 3

CD3 (principal: $1,985.99). " 5 years 3,500 date 5

After two years (at date 2) close account CD1: the balance is

After three years (at date 3) close account CD2: the balance is

After five years (at date 5) close account CD3: the balance is

What if instead of sums of money we are considering other outcomes? For example, your boss might offer you a **1-week vacation now** or a **2-week vacation a year from now**. Can we compute the "present value" of a 2-week vacation a year from now? The answer is obviously No.

Then how useful is the notion of present value in allowing us to think about intertemporal choices? The answer is: it merely suggests an ***analogy***.