CASE 2.2: risk aversion

Smaller example.

Changes in wealth:

probability	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$21	\$0	\$156
b	\$0	\$125	\$0
c	\$96	\$0	\$69

Assume: $U(\$x) = \sqrt{x}$ and initial wealth is \$100. Then

probability	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$121	\$100	\$256
b	\$100	\$225	\$100
c	\$196	\$100	\$169

probability	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$121	\$100	\$256
b	\$100	\$225	\$100
c	\$196	\$100	\$169

STEP 1. If she does **not purchase** information.

$$\mathbb{E}[U(a)] =$$

$$\mathbb{E}[U(b)] =$$

$$\mathbb{E}[U(c)] =$$

Thus she will choose with an expected utility of

probability	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state $\rightarrow$	s_1	s_2	s_3
act $\downarrow$			
a	\$121	\$100	\$256
b	\$100	\$225	\$100
c	\$196	\$100	\$169

STEP 2. If she purchases information $\{\{s_1, s_2\}, \{s_3\}\}$ at price p.

- If informed that $\{s_1, s_2\}$ then the revised decision problem is:

probability		
state $\rightarrow$	s_1	s_2
act $\downarrow$		
a	\$121	\$100
b	\$100	\$225
c	\$196	\$100

$$\mathbb{E}[U(a)] =$$

$$\mathbb{E}[U(b)] =$$

$$\mathbb{E}[U(c)] =$$

Thus she will choose with an expected utility of

probability	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state →	s_1	s_2	s_3
act ↓			
a	\$121	\$100	\$256
b	\$100	\$225	\$100
c	\$196	\$100	\$169

- If informed that $\{s_3\}$ then she will choose with a utility of

Given the initial probabilities:

probability	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{3}{9}$
state →	s_1	s_2	s_3

the probability of receiving information $\{s_1, s_2\}$ is $\frac{6}{9} = \frac{2}{3}$ and the probability of receiving information $\{s_3\}$ is $\frac{1}{3}$.
Thus the expected utility of purchasing information at price p is:

For example, if p = \$30 then

The maximum price the DM is willing to pay for information is given by the solution to:

Which is

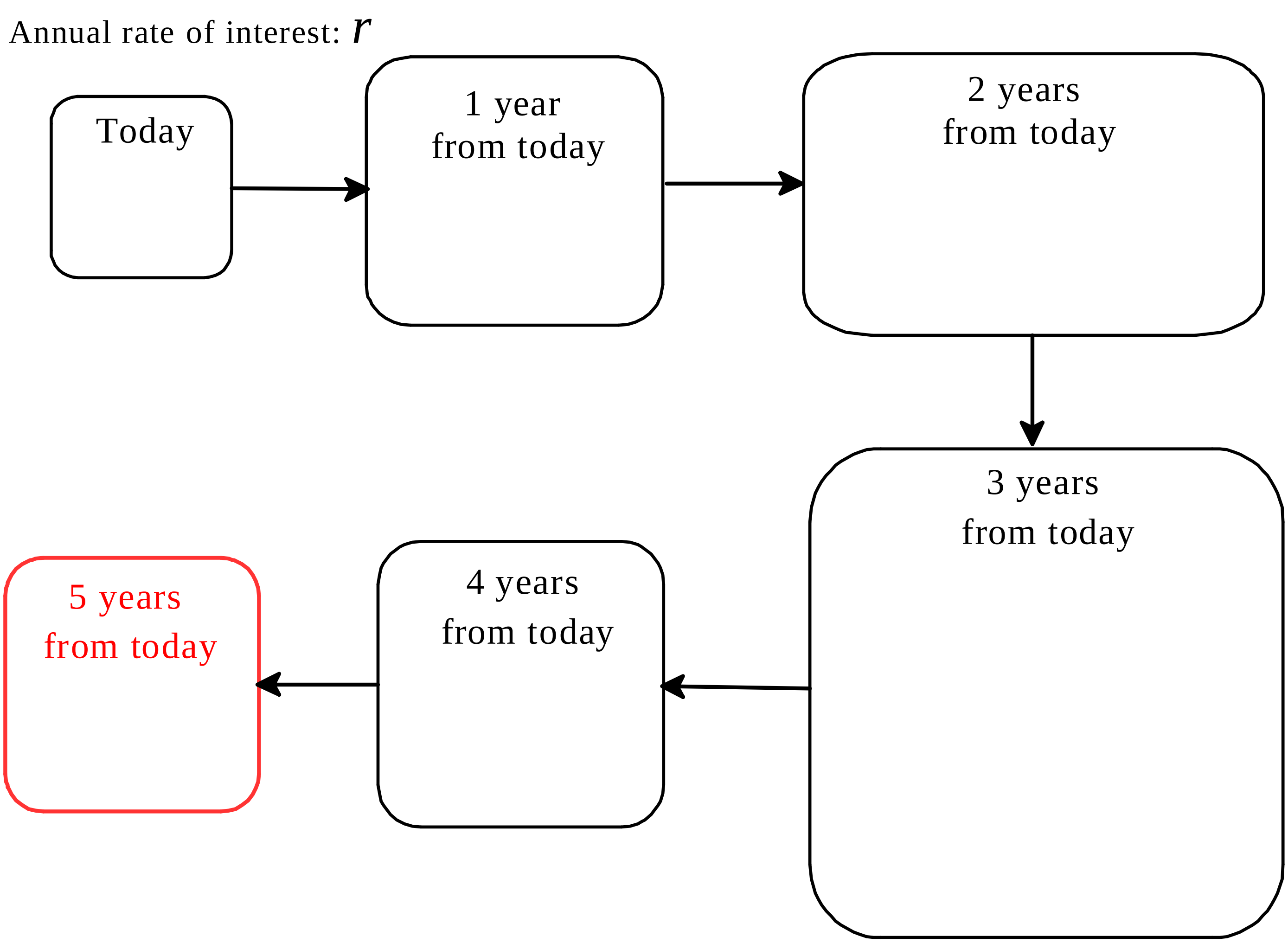
Future Value and Present Value

- **$100 today**, or
- **$200, 5 years from now**

Reasons for preferring $100 today:

Rephrase the choice as:

- $100 today, **but cannot be spent until 5 years from now**, or
- $200, 5 years from now



Definition: the ***future value*** of \$*x*, *n* periods from now is

$$x(1+r)^n$$

where *r* is the interest rate per period *r* .

- If *r* = 0.10 (i.e. 10%) then the future value of \$100 five years from now is

- If *r* = 0.15 (i.e. 15%) then the future value of \$100 five years from now is

Definition: the ***present value y*** of \$*x* available *n* periods from now is the solution to

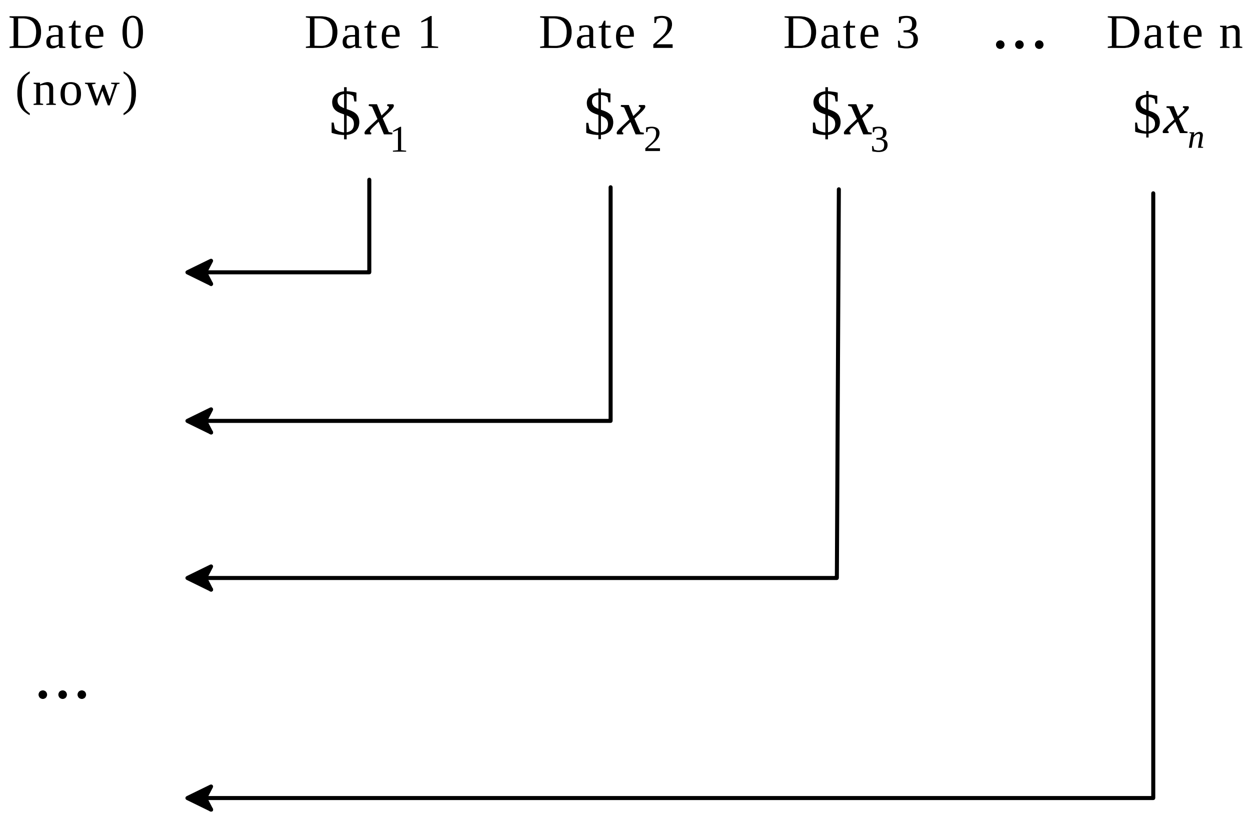
- If *r* = 0.10 (i.e. 10%) then the present value of \$200 five years from now is

- If *r* = 0.15 (i.e. 15%) then the present value of \$200 five years from now is

r is the interest **rate**, $\delta = \frac{1}{1+r}$ is the discount **factor**. Thus the present value of \$*x* available *n* periods from now is also denoted by $x\delta^n$.

Note that $\frac{1}{(1+r)^n} = \left(\frac{1}{1+r}\right)^n = \delta^n$.

Above we calculated the present value of a sum of money. We can also calculate the present value of a **stream of payments**:



So the present value of that income stream is

This is a sum of money that is **equivalent to that income stream**. Equivalent in what sense?

Suppose that $r = 12\%$ (the present is date 0):

date 2	date 3	date 5
\$2,000	\$3,000	\$3,500

The present value of \$2,000 available at date 2 is

the present value of \$3,000 available at date 3 is

the present value of \$3,500 available at date 5 is

Put these three sums of money in three different accounts

CD1 (principal: $1,594.39)

CD2 principal: $2,135.34)

CD3 (principal: $1,985.99).

After two years (at date 2) close account CD1: the balance is

After three years (at date 3) close account CD2: the balance is

After five years (at date 5) close account CD3: the balance is

What if instead of sums of money we are considering other outcomes? For example, your boss might offer you a **1-week vacation now** or a **2-week vacation a year from now**. Can we compute the "present value" of a 2-week vacation a year from now? The answer is obviously No.

Then how useful is the notion of present value in allowing us to think about intertemporal choices? The answer is: it merely suggests an ***analogy***.

The discounted utility model

$Z = \{z_1, z_2, ..., z_m\}$ set of basic outcomes $T = \{0, 1, 2, ..., n\}$ a set of dates

$t = 0$ is now, $t = 1$ is one period from now …

(z,t) : **outcome *z* experienced at date *t***

Preferences over the set of dated outcomes: indexed by the date at which the preferences are being considered:

$(z,1) \succ_0 (z',2)$ means:

RESTRICTION: $(z,t) \succsim_s (z',t')$ implies that

U_s utility function that represents the preferences at date *s*:

When the preferences at time *s* are restricted to outcomes to be experienced at time *s* then simpler notation $u_s(z)$:

$$u_s(z) =$$

Call $u_s(z)$ the *instantaneous utility of z at time s.*

Begin with preferences at time 0 (the present): $\succsim_0$ represented by $U_0(\bullet)$. The **discounted or exponential utility model** assumes that these preferences have the following form:

(*)

$(z,t) \succsim_0 (z',s)$ if and only if

Example 1. z = take online yoga class, z' = take in-person yoga class

$$(z,1) \sim_0 (z',3)$$

If her preferences satisfy the discounted utility model then

Suppose that $u_1(z) = 4$ and $u_3(z') = 6$.

1. Then what is her discount factor?

2. What is her discount rate?

$$U_0(z,t) = \delta^t u_t(z)$$

Suppose you have a choice between $(z',0)$, $(z,0)$ and $(z,1)$

z' = do nothing and z = carry out a particular activity

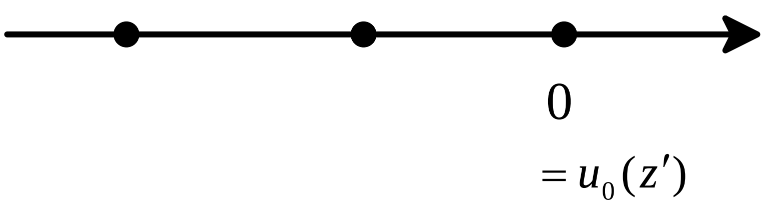
$U_0(z',0) =$

$U_0(z,0) =$

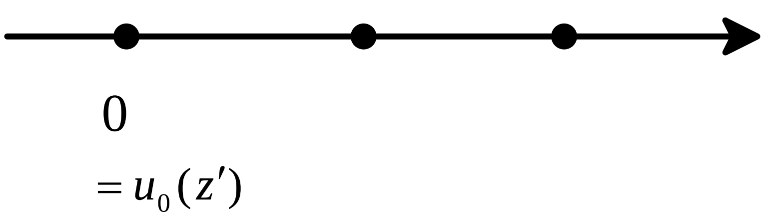
$U_0(z,1) =$

Suppose that $u_0(z') = 0$ and $u_1(z) = u_0(z)$ so that $U_0(z,1) =$

- $u_0(z) < \underbrace{0}_{=u_0(z')}$



- $u_0(z) > \underbrace{0}_{=u_0(z')}$



Ranking sequence of outcomes

EXAMPLE 2.

	Today	*Tomorrow*
date	0	1
Plan A	x	y
Plan B	y	x

Suppose: $u_0(x) = u_1(x) = 4$ $u_0(y) = u_1(y) = 6$ $\delta = 0.8$.

	Today	*Tomorrow*
date	0	1
Plan A		
Plan B		

Extension of the discounted utility:

$U_0(\text{Plan A}) =$

$U_0(\text{Plan B}) =$

EXAMPLE 3.

$date$	0	1	2
$Plan\,A$	x	y	z
$Plan\,B$	y	z	x

$U_0(\text{Plan A}) =$

$U_0(\text{Plan B}) =$

Suppose $\begin{cases} \delta = 0.9, \\ u_0(x) = 0,\ u_1(y) = 4,\ u_2(z) = 2 \\ u_0(y) = 3,\ u_1(z) = 1,\ u_2(x) = 1 \end{cases}$, then

$date$	0	1	2
$Plan\,A$			
$Plan\,B$			

$U_0(\text{Plan A}) =$

$U_0(\text{Plan B}) =$

Time consistency of preferences

date	0	1	2	3
Plan A	–	x	y	z
Plan B	–	y	z	x

Suppose that you "choose" Plan *B*:

Now when date 1 comes along you re-examine those two plans and are free to change your mind (there was no commitment). Your preferences are **time consistent** if at date 1 you maintain the same ranking that you had at time 0:

Recall

$$U_0(z,t) =$$

Extend this to the preferences at any time s:

$$U_s(z,t) =$$ assuming that

$U_s(z,t) =$ assuming that $t \geq s$

	Date 0	Date 1	Date 2	Date 3	Date 4
Plan A	--	--	*x*	*y*	x
Plan B	--	--	y	z	x

$U_0(\text{Plan A}) =$

$U_1(\text{Plan A}) =$

$U_2(\text{Plan A}) =$

And similarly for the utility of Plan B.

Now suppose that at time 0 you prefer Plan A to Plan B:

(**)

Divide both sides of (**) by δ :

Divide both sides of (**) by δ^2 :