- The discounted utility model
- Time consistency
- The hyperbolic utility model
- Dealing with time inconsistency

The discounted utility model

$Z = \{z_1, z_2, ..., z_m\}$ set of basic outcomes $T = \{0,1,2,...,n\}$ a set of dates

$t = 0$ is now, $t = 1$ is one period from now …

(z,t) : **outcome *z* experienced at date *t***

Preferences over the set of dated outcomes: indexed by the date at which the preferences are being considered:

$(z,1) \succ_0 (z',2)$ means:

RESTRICTION: $(z,t) \succsim_s (z',t')$ implies that

U_s utility function that represents the preferences at date *s*:

When the preferences at time *s* are restricted to outcomes to be experienced at time *s* then simpler notation $u_s(z)$:

$$u_s(z) =$$

Call $u_s(z)$ the *instantaneous utility of z at time s.*

Begin with preferences at time 0 (the present): $\succsim_0$ represented by $U_0(\bullet)$. The **discounted or exponential utility model** assumes that these preferences have the following form:

(*)

$(z,t) \succsim_0 (z',s)$ if and only if

Example 1. z = take online yoga class, z' = take in-person yoga class

$$(z,1) \sim_0 (z',3)$$

If her preferences satisfy the discounted utility model then

Suppose that $u_1(z) = 4$ and $u_3(z') = 6$.

1. Then what is her discount factor?

2. What is her discount rate?

$$U_0(z,t) = \delta^t u_t(z)$$

Suppose you have a choice between $(z',0)$, $(z,0)$ and $(z,1)$

z' = do nothing and z = carry out a particular activity

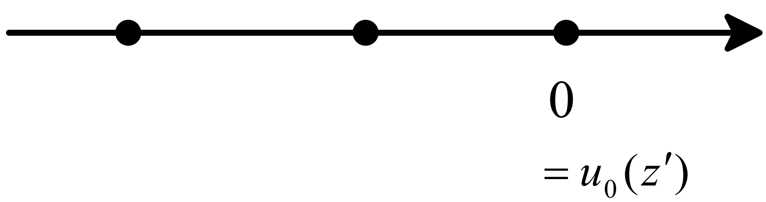
$U_0(z',0) =$

$U_0(z,0) =$

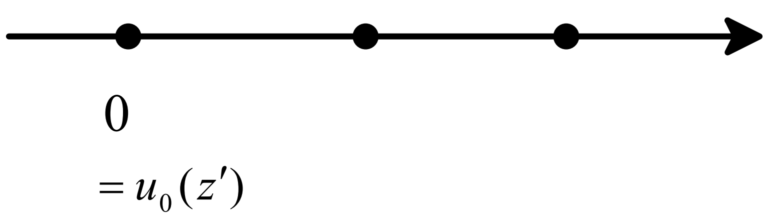
$U_0(z,1) =$

Suppose that $u_0(z') = 0$ and $u_1(z) = u_0(z)$ so that $U_0(z,1) =$

- $u_0(z) < \underbrace{0}_{=u_0(z')}$



- $u_0(z) > \underbrace{0}_{=u_0(z')}$



Ranking sequence of outcomes

EXAMPLE 2.

	Today	*Tomorrow*
date	0	1
Plan A	x	y
Plan B	y	x

Suppose: $u_0(x) = u_1(x) = 4$ $\quad u_0(y) = u_1(y) = 6$ $\quad \delta = 0.8.$

	Today	*Tomorrow*
date	0	1
Plan A		
Plan B		

Extension of the discounted utility:

$U_0(\text{Plan A}) =$

$U_0(\text{Plan B}) =$

EXAMPLE 3.

date	0	1	2
Plan A	x	y	z
Plan B	y	z	x

$U_0(\text{Plan A}) =$

$U_0(\text{Plan B}) =$

Suppose $\begin{cases} \delta = 0.9, \\ u_0(x) = 0,\ u_1(y) = 4,\ u_2(z) = 2 \\ u_0(y) = 3,\ u_1(z) = 1,\ u_2(x) = 1 \end{cases}$, then

date	0	1	2
Plan A			
Plan B			

$U_0(\text{Plan A}) =$

$U_0(\text{Plan B}) =$

Time consistency of preferences

date	0	1	2	3
Plan A	–	x	y	z
Plan B	–	y	z	x

Suppose that you "choose" Plan *B*:

Now when date 1 comes along you re-examine those two plans and are free to change your mind (there was no commitment). Your preferences are **time consistent** if at date 1 you maintain the same ranking that you had at time 0:

Recall

$$U_0(z,t) =$$

Extend this to the preferences at any time s:

$U_s(z,t) =$ assuming that

$U_s(z,t) =$ assuming that $t \geq s$

	Date 0	Date 1	Date 2	Date 3	Date 4
Plan A	--	--	*x*	*y*	x
Plan B	--	--	y	z	x

$U_0(\text{Plan A}) =$

$U_1(\text{Plan A}) =$

$U_2(\text{Plan A}) =$

And similarly for the utility of Plan B.

Now suppose that at time 0 you prefer Plan A to Plan B:

(**)

Divide both sides of (**) by δ :

Divide both sides of (**) by δ^2 :

The hyperbolic utility model (the β -δ model)

Suppose that on January 1, 2024 you were offered either

- \$1,000 to be collected on January 1, 2025 (12 months later), or
- \$1,500 to be collected on May 1, 2025 (16 months later).

What would you choose?

Suppose that you are asked again on January 1, 2025: what do you choose:

- \$1,000 to be collected now or
- \$1,500 to be collected 4 months from now (on May 1, 2025)

Recall that in the **discounted (or exponential) utility** model

$$U_0(z,t) = \delta^t u_t(z) = \begin{cases} & \text{if } t = 0 \\ & \text{if } t > 0 \end{cases} \qquad (*)$$

where $0 < \delta \leq 1$ is the *discount factor*.

In the **hyperbolic utility model**

$$U_0(z,t) = \delta^t u_t(z) = \begin{cases} & \text{if } t = 0 \\ & \text{if } t > 0 \end{cases} \qquad (**)$$

discounted utility model: $U_s(z,t) = \begin{cases} & \text{if } t = s \\ & \text{if } t > s \end{cases}$

hyperbolic utility model: $U_s(z,t) = \begin{cases} & \text{if } t = s \\ & \text{if } t > s \end{cases}$

EXAMPLE 1.

	Date 0	Date 1	Date 2	Date 3
Plan A	--	--	x	y
Plan B	--	--	w	z

Suppose $u_2(x) = 6,\ u_3(y) = 0,\ u_2(w) = 1.5,\ u_3(z) = 9\ \ \beta = 0.6$ and $\delta = 0.8$
Then

$U_0(\text{Plan A}) =$

$U_0(\text{Plan B}) =$

Now consider preferences at date 2:

$U_2(\text{Plan A}) =$

$U_2(\text{Plan B}) =$

EXAMPLE 2. Choice is between

- \$100 in 12 months or
- \$160 in 16 months

$u_t(\$x) = \sqrt{x},$ for all t and $\delta = 0.95$

(A) Exponential discounter:

$U_0(\$100,12) =$

$U_0(\$160,16) =$ so that

$U_{12}(\$100,12) =$

$U_{12}(\$160,16) =$ so that

(B) Hyperbolic discounter with $\beta = 0.8$

$U_0(\$100,12) =$

$U_0(\$160,16) =$ so that

$U_{12}(\$100,12) =$

$U_{12}(\$160,16) =$ so that

Interpretation of the parameter β

The parameter β is a measure of the DM's **bias towards the present**: if $\beta = 1$ then there is no present bias, while if $\beta < 1$ there is present bias. **The lower β, the greater the intensity of the present bias.**

F**ocus on date 0** and consider an outcome *z* such that $u_t(z) = u(z) > 0$ for all $t \geq 0$.

For an exponential discounter:

- From the perspective of date 0, what is the cost of delaying *z* from date $t > 0$ to date $t + 1$? Measure this cost as the difference between utility of (z, t) and utility of $(z, t+1)$ as a percentage of utility of (z, t):

$$\frac{U_0(z,t) - U_0(z,t+1)}{U_0(z,t)} =$$

- Do the same for the cost of delaying *z* from date 0 to date 1:

$$\frac{U_0(z,0) - U_0(z,1)}{U_0(z,0)} =$$

For a hyperbolic discounter:

- From the perspective of date 0, what is the cost of delaying *z* from date $t > 0$ to date $t + 1$?

$$\frac{U_0(z,t) - U_0(z,t+1)}{U_0(z,t)} =$$

- Cost of delaying *z* from date 0 to date 1:

$$\frac{U_0(z,0) - U_0(z,1)}{U_0(z,0)} =$$

Thus **the cost of delaying from today to tomorrow is larger than the cost of delaying from a future date *t* to the successive date *t* + 1**: there is a larger drop in utility in the former case than in the latter.

EXAMPLE 2. Choice is between

- \$100 in 12 months or
- \$160 in 16 months

$u_t(\$x) = \sqrt{x}$, for all t and $\delta = 0.95$

(A) Exponential discounter:

$U_0(\$100,12) =$
$U_0(\$160,16) =$ so that

$U_{12}(\$100,12) =$
$U_{12}(\$160,16) =$ so that

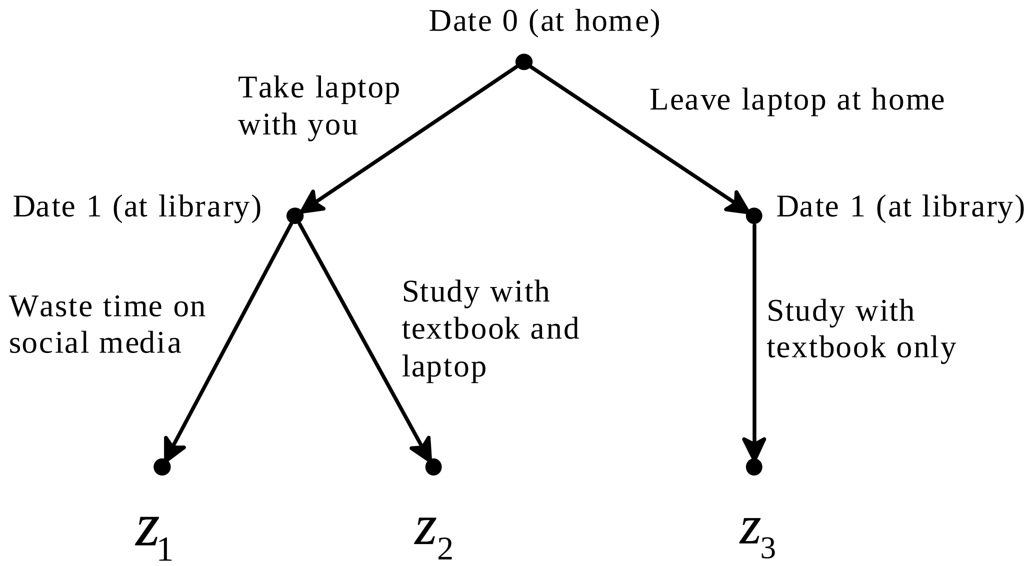
(B) Hyperbolic discounter with $\beta = 0.8$

$U_0(\$100,12) =$
$U_0(\$160,16) =$ so that

$U_{12}(\$100,12) =$
$U_{12}(\$160,16) =$ so that

Dealing with time inconsistency

EXAMPLE 1. You have a final tomorrow. You are going to the library to study.



Utility:

best

Your ranking at Date 0 is:

worst

Utility:

best

You realize that your ranking at Date 1 will be:

worst

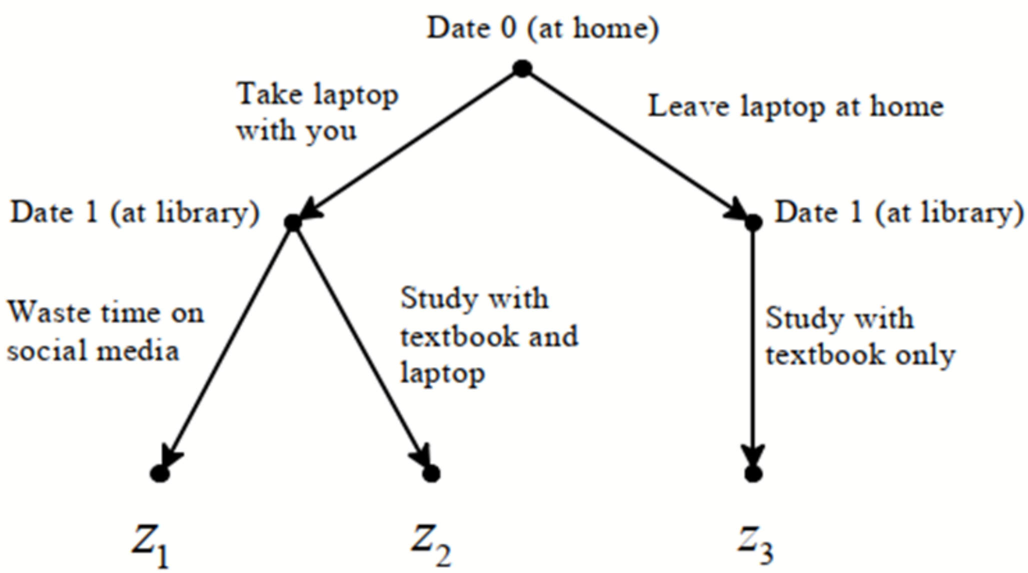
Utility:

best

Your ranking at Date 0 is:

worst

IF you can commit:



HOW CAN YOU COMMIT?

Once started, **it cannot be undone by the application**, by deleting the application, or by restarting the computer – you must wait for the timer to run out.

Concentrate

When I activate "Writing," the app automatically closes my email client and Internet Browser; blocks me from Twitter, Facebook, and YouTube; launches Microsoft Word; and sets my instant messaging status to "away". *For Macs only.*

FocusWriter

FocusWriter re-creates a word processor-like environment, blocking out absolutely everything on your screen except for the words you type on a simple grey background – all menus (date, timer, dock, etc) are tucked away until rollover. *For Macs and PCs.*

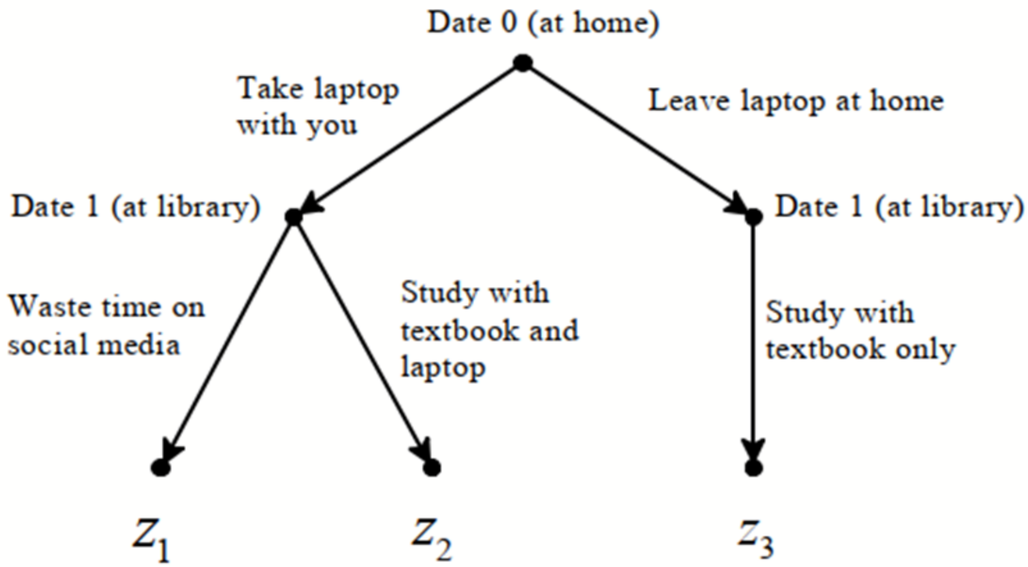
Anti-Social

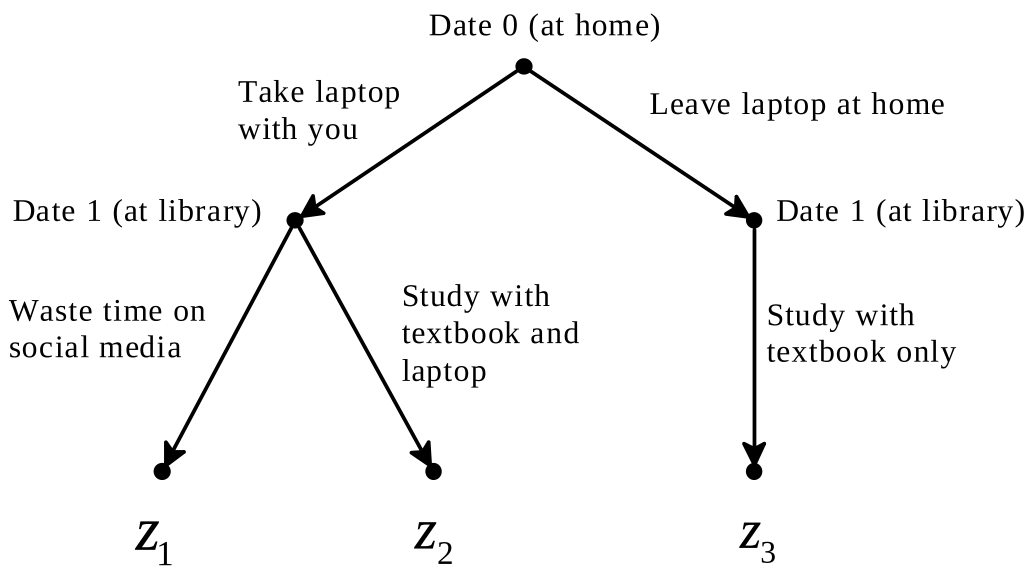
Rather than blocking the Internet in its entirety, Anti-Social automatically blocks all of the known time-sinks for a set period of time. Sites that are off-limits include Twitter, Facebook, Flickr, Digg, Reddit, YouTube, Hulu, Vimeo, and all standard web email programs. *For Macs and PCs.*

StayFocusd

This extension, for users of Google's Chrome browser, works in the reverse manner to Anti-Social or Self-Control. Rather than setting a period of time for which you CANNOT use the Internet, it allows you to set a period of time to indulge in time-wasting sites. Only want to give yourself 60 minutes a day for Twitter, vanity Googling, and updating your Netflix queue? This is your app. Rather like when you were a kid and only allowed to watch 2 hours of TV a day. For Firefox users, LeechBlock performs a similar function. *For Macs and PCs.*

What is commitment? Elimination of options:





Utility:

best

Your ranking at Date 0 is:

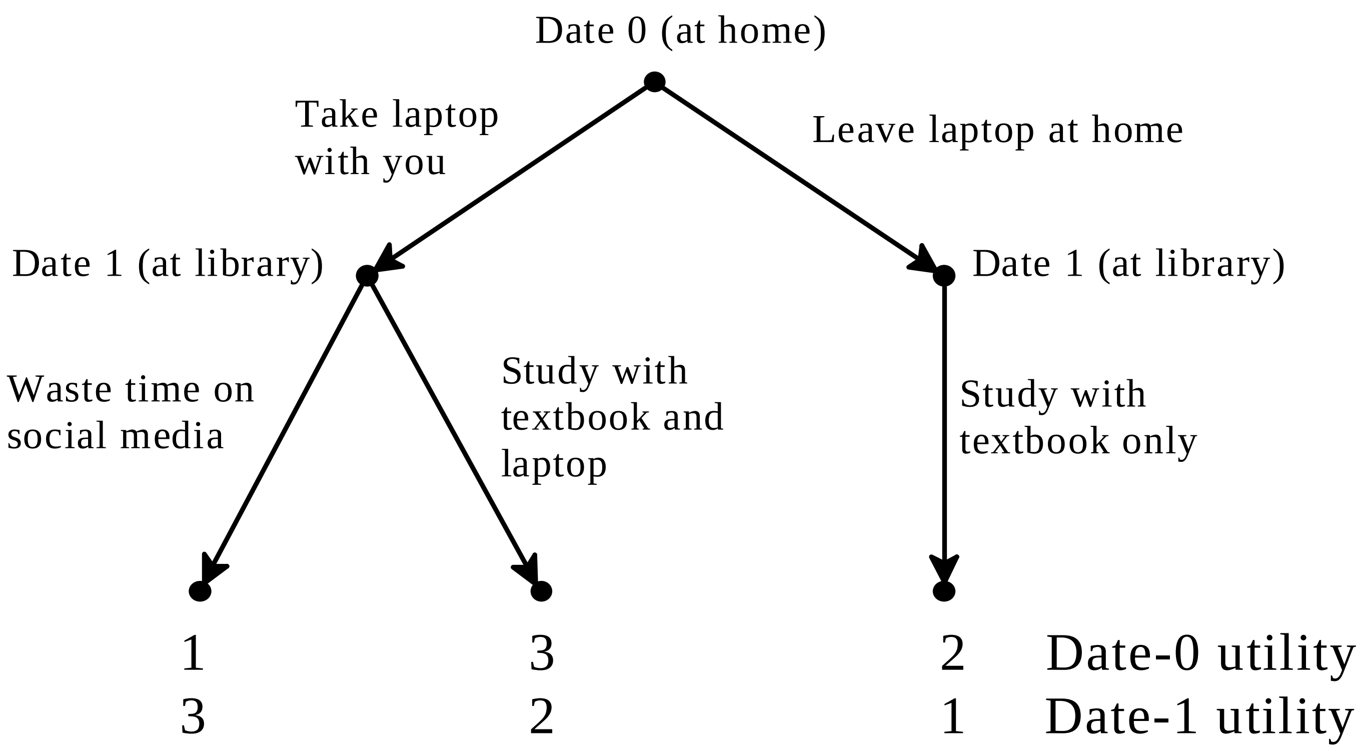
worst

Utility:

best

You realize that your ranking at Date 1 will be:

worst


Date 0 (at home)
Take laptop with you
Leave laptop at home
Date 1 (at library)
Date 1 (at library)
Waste time on social media
Study with textbook and laptop
Study with textbook only
1
3
3
2
2
1
Date-0 utility
Date-1 utility

EXAMPLE 2.

You have promised to help a friend paint her house (activity *x*) either this weekend (Date 1) or the next (Date 2) or the following one (Date 3). The instantaneous utility of *x* is the same at every date: $u_t(x) = 1$, for every t = 1,2,3. You are also a member of the snowboarding club which has trips planned for all three weekends. Call *y* the activity of joining the trip and suppose that

$$u_1(y) = 6 \qquad u_2(y) = 8 \qquad u_3(y) = 12$$

So you have three possible plans:

Choice	First weekend (Date 1)	Second weekend (Date 2)	Third weekend (Date 3)
A	*x*	*y*	*y*
B	*y*	*x*	*y*
C	*y*	*y*	*x*

Replacing outcomes with instantaneous utilities:

Choice	First weekend (Date 1)	Second weekend (Date 2)	Third weekend (Date 3)
A			
B			
C			

Choice	First weekend (Date 1)	Second weekend (Date 2)	Third weekend (Date 3)
A	1	8	12
B	6	1	12
C	6	8	1

Suppose that your preferences are represented by the hyperbolic utility model with discount factor $\delta = 0.85$ and present-bias parameter $\beta = 0.7$.

$U_1(A) =$

$U_1(B) =$

$U_1(C) =$

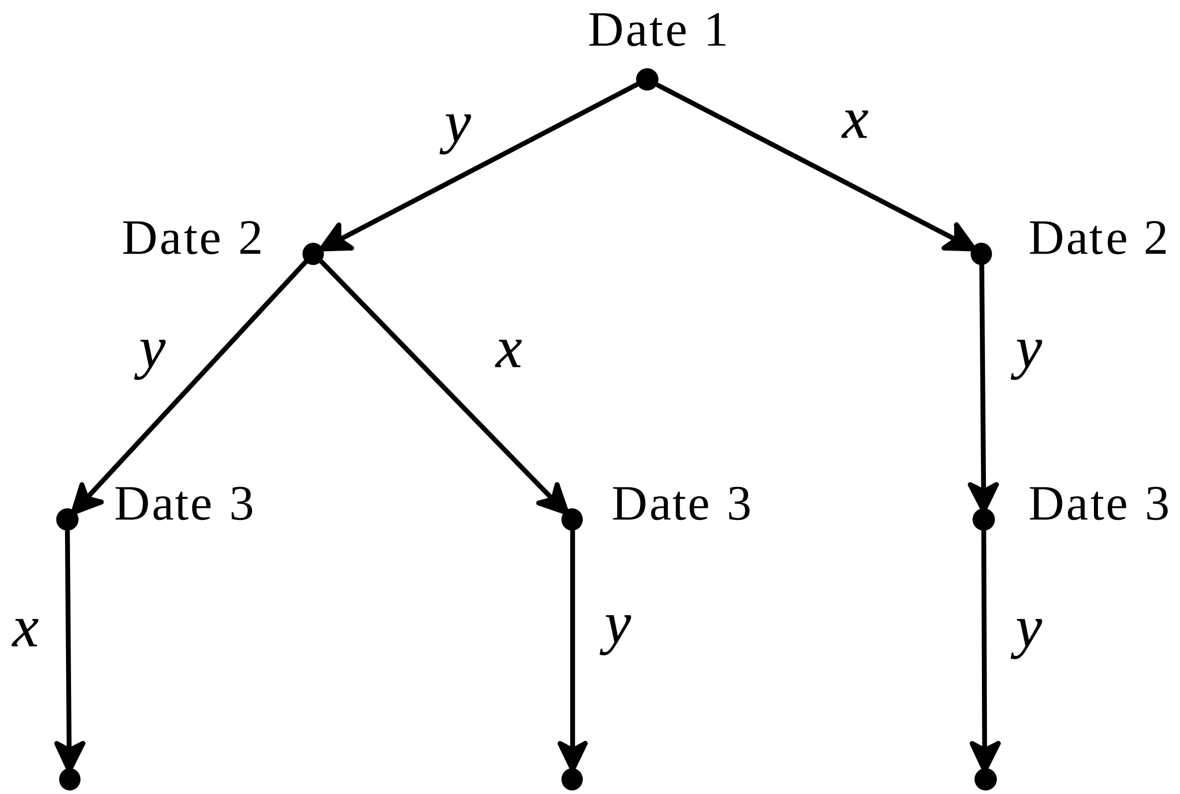
So your ranking at Date 1 is:

However, if you know your own preferences you know that

$U_2(B) =$

$U_2(C) =$

So that you understand that your ranking at Date 2 will be:



$U_1(Plan\ C) =$ $\qquad$ $U_1(Plan\ B) =$ $\qquad$ $U_1(Plan\ A) =$

$U_2(Plan\ C) =$ $\qquad$ $U_2(Plan\ B) =$