Time consistency of preferences

date	0	1	2	3
Plan A	–	x	y	z
Plan B	–	y	z	x

Suppose that you "choose" Plan *B*:

Now when date 1 comes along you re-examine those two plans and are free to change your mind (there was no commitment). Your preferences are **time consistent** if at date 1 you maintain the same ranking that you had at time 0:

Recall

$$U_0(z,t) =$$

Extend this to the preferences at any time *s*:

$$U_s(z,t) =$$ assuming that

$U_s(z,t) =$ assuming that $t \geq s$

	Date 0	Date 1	Date 2	Date 3	Date 4
Plan A	--	--	*x*	*y*	x
Plan B	--	--	y	z	x

$U_0(\text{Plan A}) =$

$U_1(\text{Plan A}) =$

$U_2(\text{Plan A}) =$

And similarly for the utility of Plan B.

Now suppose that at time 0 you prefer Plan A to Plan B:

(**)

Divide both sides of (**) by δ :

Divide both sides of (**) by δ^2 :

The hyperbolic utility model (the β -δ model)

Suppose that on January 1, 2024 you were offered either

- $1,000 to be collected on January 1, 2025 (12 months later), or
- $1,500 to be collected on May 1, 2025 (16 months later).

What would you choose?

Suppose that you are asked again on January 1, 2025: what do you choose:

- $1,000 to be collected now or
- $1,500 to be collected 4 months from now (on May 1, 2025)

Recall that in the **discounted (or exponential) utility** model

$$U_0(z,t) = \delta^t u_t(z) = \begin{cases} \quad & \text{if } t = 0 \\ \quad & \text{if } t > 0 \end{cases} \qquad (*)$$

where $0 < \delta \leq 1$ is the *discount factor*.

In the **hyperbolic utility model**

$$U_0(z,t) = \delta^t u_t(z) = \begin{cases} \quad & \text{if } t = 0 \\ \quad & \text{if } t > 0 \end{cases} \qquad (**)$$

$$\text{discounted utility model:} \quad U_s(z,t) = \begin{cases} & \text{if } t = s \\ & \text{if } t > s \end{cases}$$

$$\text{hyperbolic utility model:} \quad U_s(z,t) = \begin{cases} & \text{if } t = s \\ & \text{if } t > s \end{cases}$$

EXAMPLE 1.

	Date 0	Date 1	Date 2	Date 3
Plan A	--	--	x	y
Plan B	--	--	w	z

Suppose $u_2(x) = 6,\ u_3(y) = 0,\ u_2(w) = 1.5,\ u_3(z) = 9$ $\beta = 0.6$ and $\delta = 0.8$
Then

$U_0(\text{Plan A}) =$

$U_0(\text{Plan B}) =$

Now consider preferences at date 2:

$U_2(\text{Plan A}) =$

$U_2(\text{Plan B}) =$

EXAMPLE 2. Choice is between

- \$100 in 12 months or
- \$160 in 16 months

$u_t(\$x) = \sqrt{x},$ for all t and $\delta = 0.95$

(A) Exponential discounter:

$U_0(\$100,12) =$

$U_0(\$160,16) =$ so that

$U_{12}(\$100,12) =$

$U_{12}(\$160,16) =$ so that

(B) Hyperbolic discounter with $\beta = 0.8$

$U_0(\$100,12) =$

$U_0(\$160,16) =$ so that

$U_{12}(\$100,12) =$

$U_{12}(\$160,16) =$ so that

Interpretation of the parameter β

The parameter β is a measure of the DM's **bias towards the present**: if $\beta = 1$ then there is no present bias, while if $\beta < 1$ there is present bias. **The lower β, the greater the intensity of the present bias.**

F**ocus on date 0** and consider an outcome *z* such that $u_t(z) = u(z) > 0$ for all $t \geq 0$.

For an exponential discounter:

- From the perspective of date 0, what is the cost of delaying *z* from date *t* > 0 to date *t* +1? Measure this cost as the difference between utility of (z,t) and utility of $(z,t+1)$ as a percentage of utility of (z,t):

$$\frac{U_0(z,t) - U_0(z,t+1)}{U_0(z,t)} =$$

- Do the same for the cost of delaying *z* from date 0 to date 1:

$$\frac{U_0(z,0) - U_0(z,1)}{U_0(z,0)} =$$

For a hyperbolic discounter:

- From the perspective of date 0, what is the cost of delaying z from date $t > 0$ to date $t + 1$?

$$\frac{U_0(z,t) - U_0(z,t+1)}{U_0(z,t)} =$$

- Cost of delaying z from date 0 to date 1:

$$\frac{U_0(z,0) - U_0(z,1)}{U_0(z,0)} =$$

Thus **the cost of delaying from today to tomorrow is larger than the cost of delaying from a future date *t* to the successive date *t* + 1**: there is a larger drop in utility in the former case than in the latter.