You go to a dealer, intending to buy a second-hand car. He shows you a car. He knows what the quality of the car is. Let $\theta$ denote the quality of the car. You think that there is a probability of $\frac{1}{3}$ that $\theta = 5,000$ and a probability of $\frac{2}{3}$ that $\theta = 3,000$. This belief of yours is common knowledge (the dealer knows it, you know that he knows, etc.). The dealer makes a claim about the quality of the car, which can be either truthful or false; he can either claim that the car is of quality $\theta = 5,000$ or that it is of quality $\theta = 3,000$. You hear the dealer’s claim and then decide whether to offer to pay $6,000 or $4,000. The dealer can either accept your offer or reject it. If he rejects it, you go back home without the car. The utility functions are as follows, where $U$ is your utility and $V$ is the utility of the dealer:

- if you buy a car of quality $\theta$ at price $p$:
  \[
  U = \frac{\theta}{1000} - \frac{p}{1000} + 2
  \]
- if you don't buy the car:
  \[
  U = 0
  \]
- if he sells the car to you at price $p$:
  \[
  V = \frac{p - \theta}{1000}
  \]
- if he does not sell the car to you:
  \[
  V = 0.
  \]

(a) Use states and information partitions to represent this situation of incomplete information. Be clear about what game is associated with each state.

(b) Apply the Harsanyi transformation to represent the situation of part (a) as an extensive-form game.

(c) What are the pure-strategy subgame-perfect equilibria of the game of part (b)?

(d) Is it possible, at a pure-strategy subgame-perfect equilibrium, that you will buy a car of quality 5,000?