University of California, Davis -- Department of Economics
ECN 122 : Game Theory  Professor Giacomo Bonanno
Spring 2022 - FIRST MIDTERM EXAM  Version 1

Answer all questions. If you don’t explain (= show your work for) your answers you will get no credit.

NAME:_______________________________ University ID:___________________

CIRCLE THE NAME OF YOUR TA:  Kalyani Chauduri  or  Hyunseo Park

If you don’t know the name of your TA, then write your Section Number: ________________

- By writing your name on this exam you certify that you have not violated the University’s Code of Academic Contact (for example, you have not copied from the work of another student and you have not knowingly facilitated cheating by another student).

- If you submit the exam without writing your name and ID, you will get a score of 0 for this exam.

- If you do not stop writing when told so (at the end), a penalty of 10 points will be deducted from your score.
1. [7 points] There are 4 students in a class. The instructor is lazy and instead of preparing a final exam tells the students: “On the last day each of you should give me a written note, requesting a grade and your request can be either an A or a B; if 2 or fewer people request an A, then I will give to each student the grade that he/she requested, otherwise I will give a C to everybody”. Assuming that each student only cares about his/her own grade and prefers an A to a B and a B to a C (and, by transitivity, an A to a C), list all the Nash equilibria of this game.

2. [10 points] Consider the following two-player game. Each player chooses an integer from the set \{0,1,2,…,49,50\} (that is, an integer between 0 and 50, including 0 and 50). The payoffs are sums of money and each player cares only about how much money she herself gets (and prefers more money to less). Let \( n_1 \) be the number chosen by player 1 and \( n_2 \) the number chosen by player 2. Player 1 gets \( |n_1 - n_2| \) (where \( |x| \) denotes the absolute value of \( x \); recall that \( |x| = x \) if \( x \geq 0 \) and \( |x| = -x \) if \( x < 0 \); thus \( 2 = |-2| = 2 \)); for example, if \( n_1 = 24 \) and \( n_2 = 44 \) then player 1 gets $20. Player 2, on the other hand, gets \( 150 - |n_1 + n_2| \).

(a) [4 points] Does Player 1 have a dominant strategy? If Yes, name the strategy and state whether it is weak or strict dominance, if No, explain why not.

(b) [3 points] Does player 2 have a dominant strategy? If Yes, name the strategy and state whether it is weak or strict dominance, if No explain why not.

(c) [3 points] Does this game have any Nash equilibria? If your answer is ‘Yes’ then give all the Nash equilibria, if your answer is ‘No’ then explain why not.
3. [36 points] Consider the following game $(x$ and $y$ can be any non-negative real numbers):

$$
\begin{array}{|c|c|c|c|c|c|}
\hline
   & d & e & f & g & h \\
\hline
a & 3, 2 & 1, 2 & 2, 3 & 3, y & 2, 1 \\
\hline
\text{Player 1} & b & 3, 2 & 4, 3 & 2, 4 & 4, 3 & 2, 3 \\
\hline
 & c & 3, 2 & 0, 1 & x, 2 & 4, 1 & 1, 0 \\
\hline
\end{array}
$$

(a) [4 points] Are there values of $y$ for which \textbf{Player 2} has a \textbf{strictly} dominant strategy? If Yes, say what values and name the strategy, if No, explain why not.

(b) [6 points] For which of the following values of $y$ does \textbf{Player 2} have a \textbf{weakly} dominant strategy? Circle the relevant values and name the strategy.

$$
y = 1, \quad 1 = 1.5, \quad y = 3, \quad y = 3.5, \quad y = 4, \quad y = 5
$$

Name of strategy: ___________________

(c) [4 points] Are there values of $x$ for which \textbf{Player 1} has a \textbf{strictly} dominant strategy? If Yes, say what values and name the strategy, if No, explain why not.
<table>
<thead>
<tr>
<th></th>
<th>Player 1</th>
<th></th>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td></td>
<td>3, 2</td>
<td>3, 2</td>
<td>3, 2</td>
</tr>
<tr>
<td>Player 1</td>
<td>4, 3</td>
<td>4, 3</td>
<td>0, 1</td>
</tr>
</tbody>
</table>

(d) [6 points] Let $x = 5$ and $y = 3$. What do you get when you apply the procedure of iterative deletion of strictly dominated strategies (IDSDS)?

(e) [8 points] Let $x = 5$ and $y = 3$. Apply the iterative deletion of weakly dominated strategies (IDWDS). **State clearly what strategies are deleted at each step.**

(f) [8 points] Find all the Nash equilibria when $x = 2$ and $y = 2$. 
4. [40 points] Three salesmen (Drew, Jeff, and Luke) are deciding which of two clients (X or Y) to pursue. There are two sign-up sheets (one for each client), and a salesman makes his decision by writing his name on the sign-up sheet for his chosen client. At least one salesman must pursue each client, so once two have signed up for a given client, the third is automatically assigned to the other client (no action is necessary on the part of the third salesman in this case). Drew is the most senior salesman, so he gets to make his decision first; Jeff sees what Drew chose and makes his choice next; finally Luke sees what Drew and Jeff chose and makes his decision (unless both Drew and Jeff signed up for the same client). For example, if Drew and Jeff sign up for Y then Luke has no choice and is assigned to X; if Drew signs up for X and Jeff for Y then Luke has a choice between signing up for X or signing up for Y. The two clients are identical, so each salesman cares only about which other salesman (if any) he is partnered with. It is common knowledge among the three of them that they rank the possible outcomes as follows:

<table>
<thead>
<tr>
<th></th>
<th>Drew</th>
<th>Jeff</th>
<th>Luke</th>
</tr>
</thead>
<tbody>
<tr>
<td>best</td>
<td>working with Jeff</td>
<td>working with Luke</td>
<td>working with Drew</td>
</tr>
<tr>
<td>middle</td>
<td>working alone</td>
<td>working with Drew</td>
<td>working with Jeff</td>
</tr>
<tr>
<td>worst</td>
<td>working with Luke</td>
<td>working alone</td>
<td>working alone</td>
</tr>
</tbody>
</table>

(a) [14 points] Draw the extensive-form game, using utility values from the set \{0,1,2\}.

(b) [10 points] Find the backward-induction solution(s). You can mark the solution(s) in the tree of part (a) as long as you do so very clearly.
(c) [16 points] Write the strategic-form game associated with the extensive-form game of Part (a). You don’t need to write all the payoffs: fill in only 8 cells (any 8 of your choice) and leave the other cells blank. Let Drew choose the rows, Jeff the columns, etc. [Note: if the game that you drew in Part (a) is wrong, but the strategic-form that you write here is correct for that game, then you will get full credit for this question, unless the game of Part (a) is too simple or trivial, in which case you will get a maximum of 8 points for this question.]
5. [7 points] Consider the following game-frame:

(a) [4 points] How many strategies does Player 1 have? [Use the definition of strategy given in class and in the textbook.]

(b) [3 points] How many strategies does Player 2 have? [Use the definition of strategy given in class and in the textbook.]