University of California, Davis -- Department of Economics

ECON 122 : GAME THEORY Professor Giacomo Bonanno

PRACTICE SECOND MIDTERM EXAM

Answer all questions. Explain your answers.

1. I have the following (and only the following) cards:

an Ace	A
a King	K
a Queen	Q

You and Larry play the following game. I shuffle the cards very well (so that each possible arrangement is equally likely) and place them face down on the table. I then pick the top card and look at it *without showing it to either you or Larry*. If it is an ace, I whisper in your ear (so that Larry cannot hear me) "The top card is an ace". If it is either a king or a queen, I whisper in your ear "The top card is NOT an ace". Then I pick the next card and look at it without showing it to either you or Larry. If this second card is an ace, I whisper in Larry's ear (so that you cannot hear me) "The second card is an ace"; if this second card is either a king or a queen, I whisper in Larry's ear "The second card is NOT an ace". Now it is your turn to move. You have the choice between passing – in which case the game ends without either of you getting anything – or betting $36. If you bet $36, Larry has a choice between passing – in which case, again, the game ends without either of you getting anything – or seeing, in which case the cards are shown and you win (i.e. he gives you $36) if your card was an ace and you lose (i.e. you pay him $36) if his card was an ace (if neither of you has an ace, then nobody pays or receives anything).

(a) Represent this as an extensive game (with imperfect information; **don't include me as a player: the only players are you and Larry**).

(b) What are your strategies in this game? Write them all down explicitly.

(c) What are Larry's strategies in this game? Write them all down explicitly.

(d) Write the strategic form corresponding to this extensive form, assuming that both you and Larry are risk neutral. [Do not forget that payoffs in the strategic form are expected payoffs.]

(e) What are the Nash equilibria of this game?

2. Sam has \$5 to distribute to his children, Ann and Ben. Unfortunately, he does not have any coins and the split has to be \$3 to one and \$2 to the other. Ben is given the option of being chivalrous: he can volunteer to be the one who gets \$2. If Ben volunteers, Ann gets \$3, while Ben gets \$2. If Ben does not volunteer, then Ann must go to the kitchen and write either 'Ann' or 'Ben' on a piece of paper and give it to her father, while Ben must go to the garage and write either 'Ann' or 'Ben' on a piece of paper and give it to his father. If they both wrote 'Ann', then Ann gets \$3 and Ben gets \$2. If they both wrote 'Ben', then Ben gets \$3 and Ann gets \$2. If they wrote different names, Sam keeps the \$5 bill, that is, Ann and Ben get nothing. Both Ann and Ben are selfish and greedy.

(a) Draw an extensive game that represents the situation facing Ann and Ben.

(b) Write the corresponding strategic-form game (let Ann choose rows and Ben choose columns).

(c) What are the pure-strategy Nash equilibria of this game?

(d) What are the pure-strategy subgame-perfect equilibria? Assuming that Ann and Ben are risk neutral, find a subgame-perfect equilibrium where Ann and Ben choose completely mixed strategies in the proper subgame.

3. Suppose an incentive exists for one party to inflict harm on another through dishonest behavior, and that a legal system is in place under which it is possible to detect and punish cheating. It seems reasonable to believe that there is an inverse relationship between the magnitude of the punishment suffered by those caught cheating and the frequency of cheating. Test this intuition in the following "buyer-seller game". The seller knows the quality of his product and can either be "honest" or "cheat" (i.e. claim that the quality is higher than it actually is). The buyer does not know the quality and chooses between "trusting" (i.e. buying without inspection) and "inspecting" (i.e. paying an expert to examine the good). The von Neumann-Morgenstern payoffs are as follows:

		Seller	
		Honest	Cheat
Buyer	Inspect	3 , 2	2 , β
	Trust	4 , 3	1 , 4

where $0 < \beta < 2$. An increase in the fine for cheating can be thought of as a reduction in the value of β. Answer the following questions by referring to the Nash equilibrium of this game:

- Would an increase in the fine for cheating reduce the probability that the Seller cheats?
- Would an increase in the fine for cheating increase or reduce the Seller's expected payoff?
- Would an increase in the fine for cheating increase or reduce the Buyer's expected payoff?