University of California, Davis -- Department of Economics
ECN/ARE 200C : **MICRO THEORY**

SPRING 2025 Professor Giacomo Bonanno

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HOMEWORK 2 : ANSWERS

(a) The Cournot-Nash equilibrium is $q_1 = q_2 = \dfrac{a-c}{3b}$.

(b) The profit function of Firm 1 is

$\Pi_1 = \alpha_1\left[q_1\left(a-b(q_1+q_2)\right)-cq_1\right]+(1-\alpha_2)\left[q_2\left(a-b(q_1+q_2)\right)-cq_2\right]$. Solving $\dfrac{\partial \Pi_1}{\partial q_1}=0$ with respect to q_1 gives Firm 1's best reply function as $BR_1 = \dfrac{a-c}{2b} - \dfrac{1+\alpha_1-\alpha_2}{2\alpha_1}q_2$.

$\dfrac{\partial BR_1}{\partial \alpha_1} = q_2\dfrac{1-\alpha_2}{2\alpha_1^2}$ which is well defined (recall that, by assumption, $\alpha_1 > 0$) and positive as long as $\alpha_2 < 1$. Thus BR_1 is increasing in α_1 if $\alpha_2 < 1$ and independent of α_1 if $\alpha_2 = 1$

$\dfrac{\partial BR_1}{\partial \alpha_2} = q_2\dfrac{1}{2\alpha_1}$ which is well defined (since $\alpha_1 > 0$) and positive. Thus BR_1 is increasing also in α_2 .

(c) Solving the first-order conditions we get that the Nash equilibrium is given by

$$q_1 = \frac{(a-c)\alpha_2}{b(1+\alpha_1+\alpha_2)},\quad q_2 = \frac{(a-c)\alpha_1}{b(1+\alpha_1+\alpha_2)}$$

(d) When $\alpha_1 = \alpha_2 = \alpha$ the above expressions become $q_1 = q_2 = \dfrac{(a-c)\alpha}{b(1+2\alpha)}$. The derivative of this expression with respect to α is $\dfrac{(a-c)\alpha}{b(1+2\alpha)^2}$ which is positive. Thus both q_1 and q_2 are increasing in α.

(e) Monopoly output is equal to $\dfrac{a-c}{2b}$. Now $2\dfrac{(a-c)\alpha}{b(1+2\alpha)}$ is equal to this if and only if $\alpha = \dfrac{1}{2}$.
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