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HOMEWORK 3 : ANSWERS

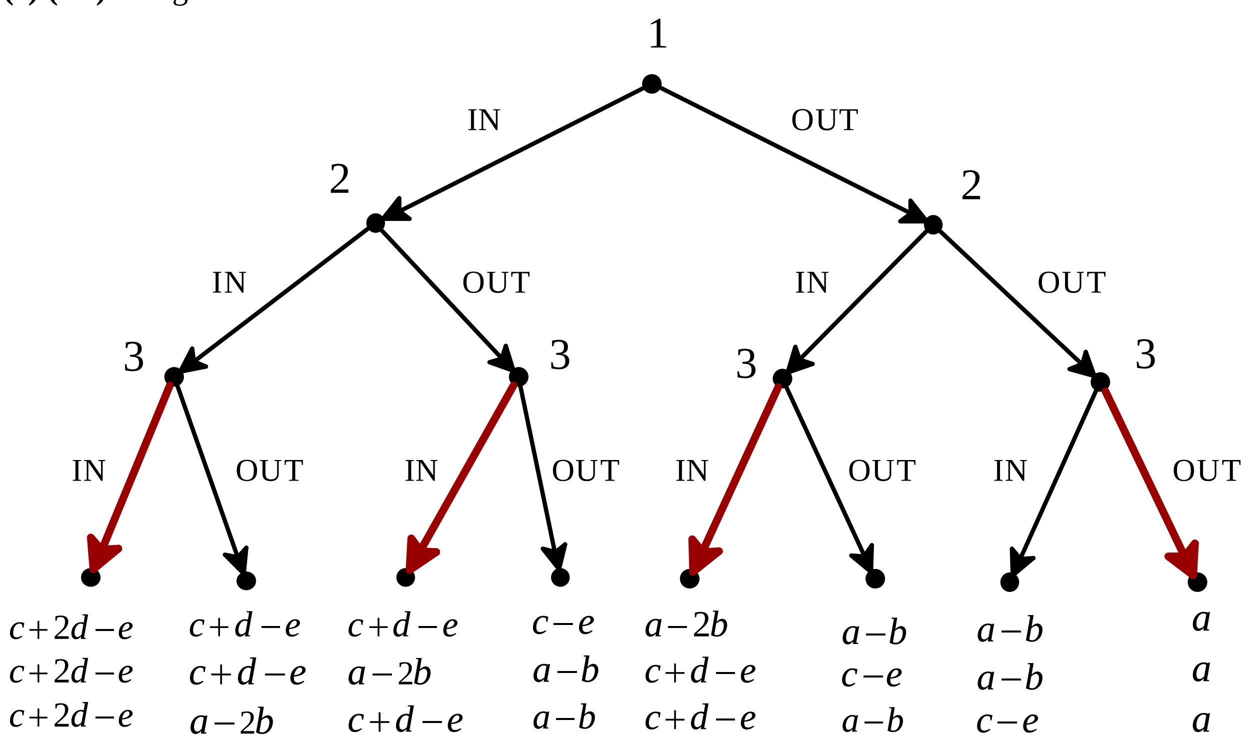
1. **(a)** This is a finite, two-player, perfect-information game with only two outcomes: either Player 1 wins or Player 2 wins. Hence one of the two players has a winning strategy.

(b) Player 1 has a winning strategy, which is as follows:
- start by blowing up one of the bridges between point *B* and point *D*,
- if Player 2 moves to point *B* blow up the second bridge between point *B* and point *D* while if Player 2 moves to point *C* then blow up the bridge between point *C* and point *D*
- after the second move of Player 2, blow up the only remaining bridge that leads to point *D*.

2. **(a)** No. Since the game is symmetric, we can focus on one of the players, say Player 1. Since $a > c - e$, when Players 2 and 3 choose OUT, for Player 1 choosing OUT is strictly better than choosing IN; however, since $c + d - e > a - b$, when at least one of Players 2 and 3 choose IN, for Player 1 choosing IN is strictly better than choosing OUT (since $b, d > 0$, $c + d - e > a - b$ implies that $c + 2d - e > a - 2b$).

(b) (IN,IN,IN) and (OUT,OUT,OUT) are the only two pure-strategy Nash equilibria.

(c) **(c.1)** The game is as follows:



(c.2) Player 1 has 2 strategies, Player 2 has 4 and Player 3 has 16.
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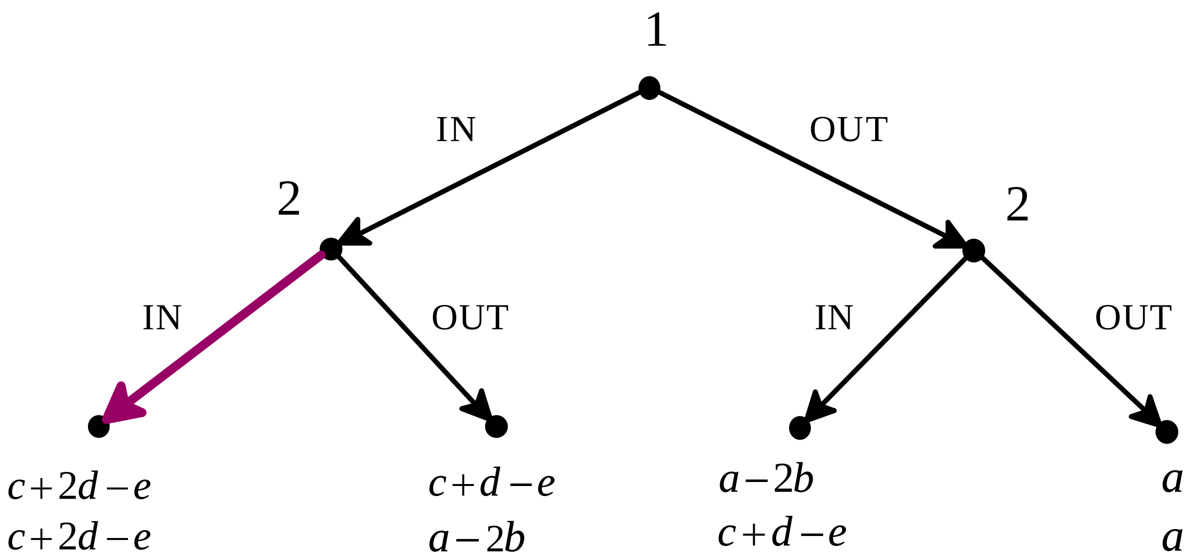
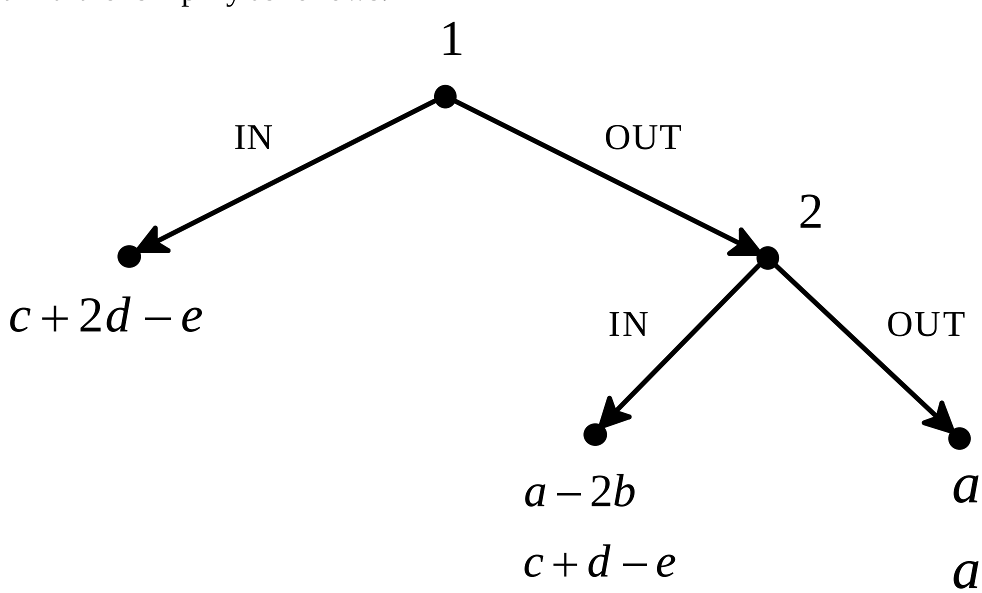
(d) Since $c+d-e>a-b$ (which implies that $c+2d-e>a-2b$) and $a>c-e$, at any backward-induction solution Player 3's strategy is a shown above by thick arrows, namely (IN,IN,IN,OUT). Thus we can simplify the game as follows:



At Player 2's left node backward induction dictates the choice of IN, because $c+2d-e>a-2b$. Thus we can further simplify as follows:



In the simplified game the backward induction solutions depend on the values of the parameters, as follows:

1. If $c+d-e>a$ (which implies that $c+2d-e>a$) then here is a unique backward-induction solution, namely (IN,IN).
2. If $c+2d-e<a$ (which implies that $c+d-e<a$) then here is a unique backward-induction solution given by (OUT,OUT).

3. If $c+d-e<a$ and $c+2d-e>a$ then here is a unique backward-induction solution given by (IN,OUT).
4. If $c+d-e=a$ (which implies that $c+2d-e>a$) then there are two backward-induction solutions (IN,IN) and (IN,OUT).
5. If $c+2d-e=a$ (which implies that $c+d-e<a$) then there are two backward-induction solutions (IN,OUT) and (OUT,OUT).

Thus, the backward-induction solutions of the entire game are as follows:

1. If $c+d-e>a$ (which implies that $c+2d-e>a$) then here is a unique backward-induction solution, namely
   (IN, (IN,IN), (IN,IN,IN,OUT)).

2. If $c+2d-e<a$ (which implies that $c+d-e<a$) then here is a unique backward-induction solution given by (OUT, (IN,OUT), (IN,IN,IN,OUT)).

3. If $c+d-e<a$ and $c+2d-e>a$ then here is a unique backward-induction solution given by (IN, (IN,OUT), (IN,IN,IN,OUT)).

4. If $c+d-e=a$ (which implies that $c+2d-e>a$) then there are two backward-induction solutions (IN, (IN,IN), (IN,IN,IN,OUT)) and (IN, (IN,OUT), (IN,IN,IN,OUT)).

5. If $c+2d-e=a$ (which implies that $c+d-e<a$) then there are two backward-induction solutions (IN, (IN,OUT), (IN,IN,IN,OUT)) and (OUT, (IN,OUT), (IN,IN,IN,OUT)).