University of California, Davis -- Department of Economics

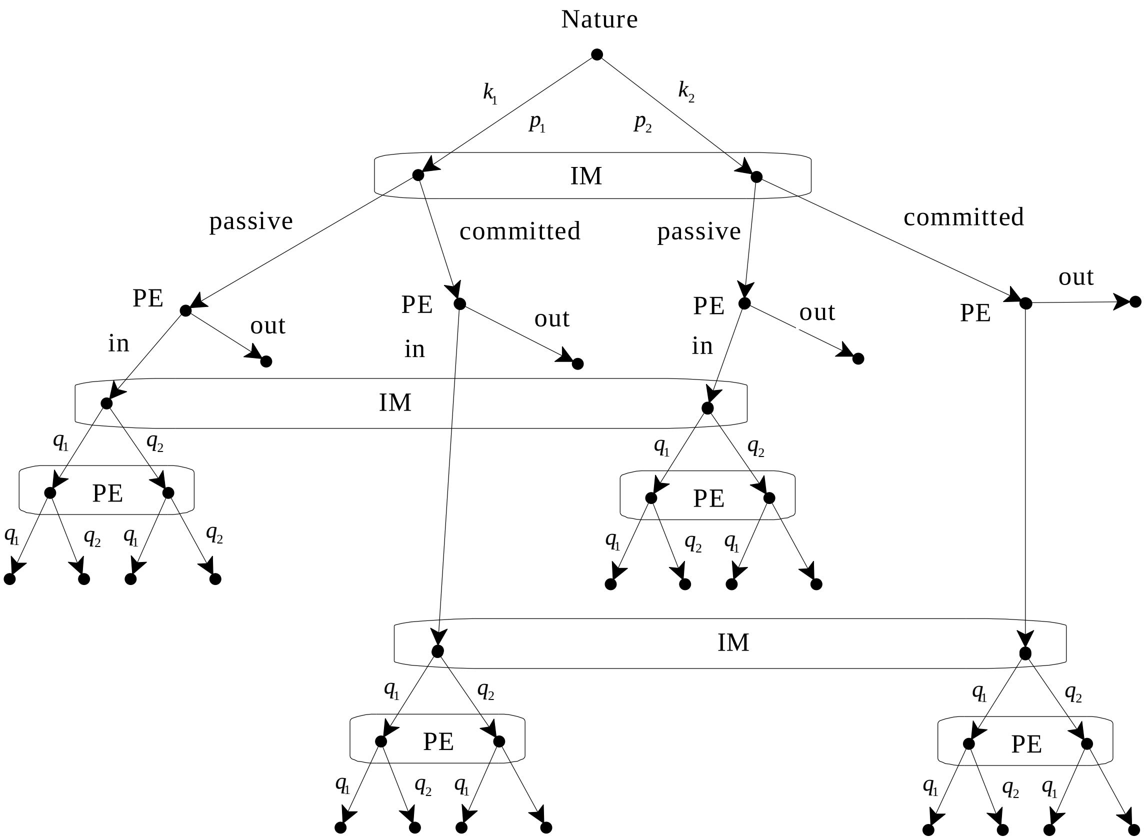
ECN/ARE **200C** : **MICRO THEORY**

SPRING 2025 Professor Giacomo Bonanno

~~~~~~

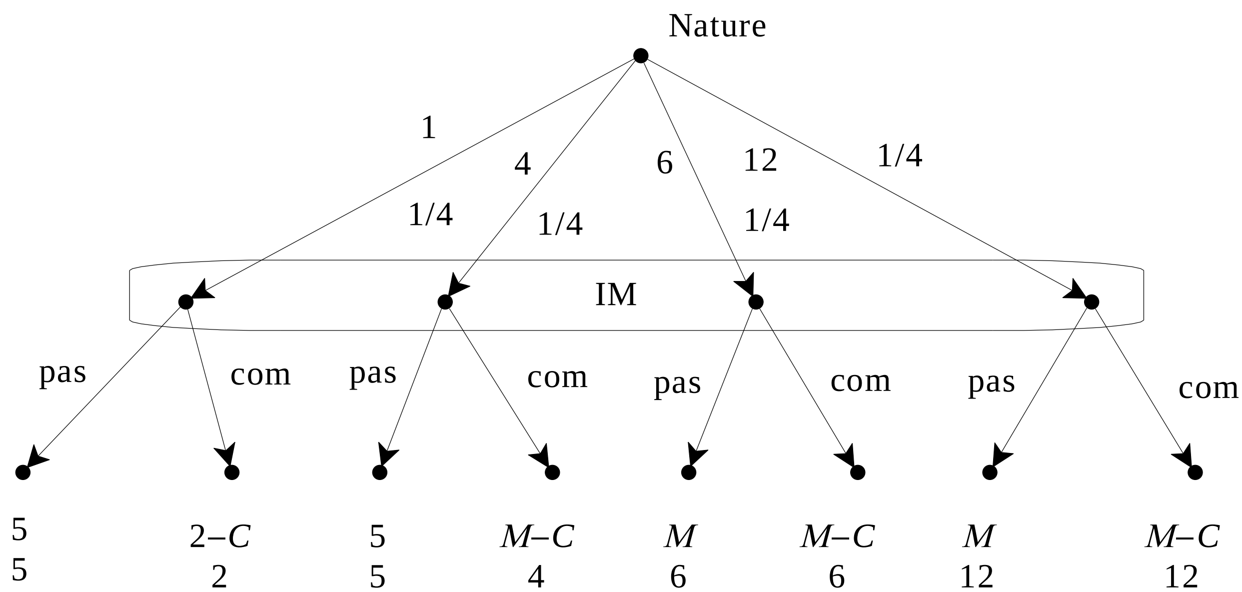
HOMEWORK 4 : $\boxed{\text{ANSWERS}}$

(a) Let p_i be the probability of k_i . Then the extensive form is as follows:



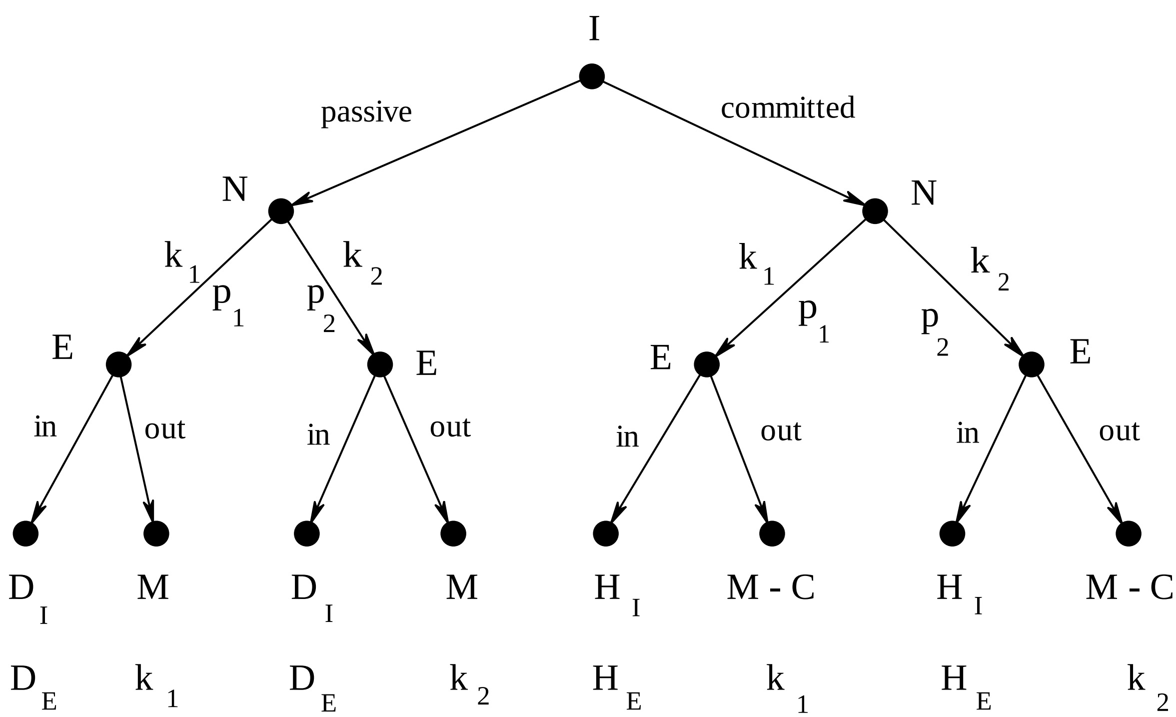
(b) If the PE chooses to enter, the IM will still be uncertain as to the value of k (although this value does not enter the payoffs in the Cournot game). Starting at the node where the IM chooses its level of output and taking everything that follows that node, one would "cut" the IM's information set and therefore not get a subgame.

(c) We can find a Nash equilibrium by pretending that the Cournot games are subgames. In the Cournot game where the IM is **passive** inverse demand is $P = 3 - \frac{Q}{5}$; there is a unique Cournot-Nash equilibrium given by $q_{IM} = q_{PE} = \frac{a}{3} = 5$ with corresponding profits $\pi_{IM} = \pi_{PE} = \frac{a^2}{9b} = 5$. In the Cournot game when the IM is **committed** inverse demand is $P = 3 - \frac{Q}{2}$; there is a unique Cournot-Nash equilibrium given by $q_{IM} = q_{PE} = \frac{c}{3} = 2$ with corresponding profits $\pi_{IM} = \pi_{PE} = \frac{c^2}{9d} = 2$ (so that the payoff of IM is $2 - C$). Thus the PE will enter only in the following cases: (i) the incumbent chose commitment and k = 1, (ii) the incumbent chose to be passive and either k = 1 or k = 4. Hence the game reduces to
~~~~~~



Thus the IM's expected profits if he chooses to be passive are $\frac{2}{4}5+\frac{2}{4}M$ and his expected profits if he chooses commitment are $\frac{1}{4}(2-C)+\frac{3}{4}(M-C)=\frac{1}{2}+\frac{3M}{4}-C$. Thus he will choose commitment if $\frac{1}{2}+\frac{3M}{4}-C>\frac{1}{2}5+\frac{1}{2}M$, that is, if $M>8+4C$, he will choose to be passive if $M<8+4C$ and will be indifferent if $M=8+4C$.

(d) The game is as follows (I denotes the incumbent, N nature and E the potential entrant)



(e) Suppose the IM chose to be passive. Then the entrant will enter if and only if $D_E \geq k$. Thus *ex ante* the probability of entry, if the IM is passive, is Prob$\{k \leq D_E\}$ = $F(D_E)$. Thus the incumbent's expected profits if he chooses to be passive are:

$$F(D_E)\ D_I + [1 - F(D_E)]\ M \qquad (3).$$

Similarly, if the IM is committed, entry occurs with probability $F(H_E)$ and the IM's expected profits are:

$$F(H_E)\ H_I + [1 - F(H_E)]\ (M - C) \qquad (4).$$

Thus the incumbent will choose to be passive if (3) > (4), and will choose to commit if (4) > (3) and be indifferent otherwise.

(f) In this case we have that F(1)=1/5, F(2)=2/5, F(4)=3/5 and F(7)=1. Thus $F(D_E)$ = F(7/2) = F(2) = 2/5 and $F(H_E)$ = F(3/2) = F(1) = 1/5. Thus (3) of part (e) becomes:

$$(2/5)\ (7/2) + [1 - 2/5]\ 8 = 17/2 = 31/5 = 6.2$$

while (4) becomes:

$$(1/5)\ (3/2) + [1 - 1/5]\ (8 - 2) = 51/10 = 5.1$$

Hence the Incumbent will choose **to be passive**.