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HOMEWORK 5 : ANSWERS

(a) For a U type the expected utility of not insuring is $\frac{9}{10}100\sqrt{193,600}+\frac{1}{10}100\sqrt{193,600-17,200}=43,800$. Thus the maximum premium that she is willing to pay for full insurance is given by the solution to $100\sqrt{193,600-h}=43,800$ which is h = \$1,756.

(b) If the premium is \$3,450, it follows from part (a) that the U types will not apply for insurance. For a V type the expected utility of not insuring is $\frac{4}{5}100\ln(193,600)+\frac{1}{5}100\ln(193,600-17,200)=1{,}215.49$. The utility from the contract is $100\ln(193,600-3,450)=1215.556851$. Thus the V types will buy insurance and the monopolist's expected profits are $\pi_1(n_v)=\left[3,450-\frac{1}{5}(17,200)\right]n_v=10n_v$.

(c) We already know that the U types will not consider the full insurance contract. The partial insurance contract gives them an expected utility of $\frac{9}{10}100\sqrt{193,600-225}+\frac{1}{10}100\sqrt{193,600-225-15000}=43{,}800.43$, higher than no insurance. Thus they will buy the partial insurance contract. For the V type the partial insurance contract gives an expected utility of $\frac{4}{5}100\ln(193,600-225)+\frac{1}{5}100\ln(193,600-225-15,000)=1{,}215.62$, higher than the expected utility from the full-insurance contract. Thus everybody will buy the partial insurance contract and the monopolist's expected profits are
$\pi_2(n_u,n_v)=\left[225-\frac{1}{10}(17,200-15,000)\right]n_u+\left[225-\frac{1}{5}(17,200-15,000)\right]n_v=5n_u-215n_v$

(d) $\pi_1(100)=\$1,000$ and $\pi_2(4400,100)=500$. Thus option (b) is better.

(e) $\pi_1(100)=\$1,000$ and $\pi_2(4700,100)=\$2,000$. Thus option (c) is better.

(f) First some notation. For an arbitrary contract $C=(h,d)$,
let $EU(C)=\frac{1}{10}100\sqrt{193600-h-d}+\frac{9}{10}100\sqrt{193600-h}$,
$EV(C)=\frac{1}{5}100\ln(193600-h-d)+\frac{4}{5}100\ln(193600-h)$.
Recall that the expected utility of no insurance is 43,800 for the U type and 1215.49 for the V type. The profit maximization problem is as follows, where $C_U=(h_U,d_U)$ is the contract targeted to the U type and $C_V=(h_V,d_V)$ is the contract targeted to the V type:

$$\max_{h_U,h_V,d_U,d_V}\quad n\left[h_U-\tfrac{1}{10}(17200-d_U)\right]+n\left[h_V-\tfrac{1}{5}(17200-d_V)\right]$$

subject to:

$(IR_U)\quad EU(C_U)\geq 43800$

$(IR_V)\quad EV(C_V)\geq 1215.49$

$(IC_U)\quad EU(C_U)\geq EU(C_V)$

$(IC_V)\quad EV(C_V)\geq EV(C_U)$
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