(a) The game is as follows:

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<table>
<thead>
<tr>
<th></th>
<th>U</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>R</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
<th></th>
<th>U</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>R</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
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(b) There is no pure-strategy separating weak sequential equilibrium. The only separating pure strategies for Player 1 are \((D,U)\) (that is, \(D\) if of type \(A\) and \(U\) if of type \(B\)) and \((U,D)\).

- If Player 1’s strategy is \((D,U)\) then Player 2’s beliefs must assign probability 1 to the right node of the top information set and to the left node of the bottom information set so that the best reply of Player 2 is to choose \(R\) at both information sets, but then \(U\) is not sequentially rational for type \(B\) of Player 1.

- If Player 1’s strategy is \((U,D)\) then Player 2’s beliefs must assign probability 1 to the left node of the top information set and to the right node of the bottom information set so that the best reply of Player 2 is to choose \(L\) at both information sets, but then \(D\) is not sequentially rational for type \(B\) of Player 1.

(c) The only pooling pure strategies of Player 1 are \((D,D)\) and \((U,U)\).

- If Player 1’s strategy is \((D,D)\) then, by Bayes’ rule Player 2’s beliefs must assign probability \(p\) to the left node of the bottom information set and probability \((1-p)\) to
the right node. Then her sequentially rational choice there is
\[ \begin{cases} L & \text{if } p \leq \frac{1}{3} \\ R & \text{if } p \geq \frac{1}{3} \end{cases} \].

If \( p \leq \frac{1}{3} \) then \(((D,D),(R,L))\) is a weak sequential equilibrium provided that Player 2’s beliefs attach probability not greater than \( \frac{1}{3} \) to the left node of the top information set.
If \( p \geq \frac{1}{3} \) then \(((D,D),(R,R))\) is a weak sequential equilibrium provided that Player 2’s beliefs attach probability not greater than \( \frac{1}{3} \) to the left node of the top information set.

- If Player 1’s strategy is \((U,U)\) then, by Bayes’ rule Player 2’s beliefs must assign probability \( p \) to the left node of the top information set and probability \( (1-p) \) to the right node. Then her sequentially rational choice is
\[ \begin{cases} L & \text{if } p \geq \frac{1}{3} \\ R & \text{if } p \leq \frac{1}{3} \end{cases} \].

If \( p \geq \frac{1}{3} \) then \(((U,U),(L,L))\) is a weak sequential equilibrium provided that Player 2’s beliefs attach probability not greater than \( \frac{1}{3} \) to the left node of the bottom information set.
If \( p \leq \frac{1}{3} \) then \(((U,U),(R,L))\) is a weak sequential equilibrium provided that Player 2’s beliefs attach probability not greater than \( \frac{1}{3} \) to the left node of the bottom information set.

(d) If Type B plays \( U \) with probability 1 and Type A plays \( D \) with positive probability, then Player 2 must assign probability 1 to the left node of the bottom information set, thus making \( R \) the only sequentially rational choice there, and must assign probability
\[
\frac{pr}{pr + 1-p}
\]
to the left node of the top information set, where \( r \) is the probability with which Type A plays \( U \). In order for Type A to be sequentially rational in playing both \( D \) and \( U \) with positive probability, her payoff must be the same with \( U \) and with \( D \). Since Player 2 is playing \( R \) at the bottom information set, Type A gets a payoff of 3 by playing \( D \) thus the payoff from playing \( U \) must also be 3, which requires Player 2 to play \( L \) with probability \( \frac{2}{3} \) and \( R \) with probability \( \frac{1}{3} \) at the top information set (note that this makes playing \( U \) with probability 1 indeed sequentially rational for Type B). Mixing between \( L \) and \( R \) at the top information set is sequentially rational for Player 2 if and only if his expected payoff from \( L \) is equal to his expected payoff from \( R \), that is, if and only if
\[
4 \frac{pr}{pr + 1-p} = 1 ,
\]
that is, if and only if \( r = \frac{1-p}{3p} \). Thus the following is a weak sequential equilibrium:

1’s behavioral strategy:
\[ \begin{pmatrix} U \\ \frac{1-p}{3p} \end{pmatrix} \quad \begin{pmatrix} D \\ 1 - \frac{1-p}{3p} \end{pmatrix} \] if Type A and
\[ \begin{pmatrix} U \\ 1 \end{pmatrix} \quad \begin{pmatrix} D \\ 0 \end{pmatrix} \] if Type B

2’s behavioral strategy:
\[ \begin{pmatrix} L \\ \frac{2}{3} \end{pmatrix} \quad \begin{pmatrix} R \\ \frac{1}{3} \end{pmatrix} \] at the top information set and
\[ \begin{pmatrix} L \\ 0 \end{pmatrix} \quad \begin{pmatrix} R \\ 1 \end{pmatrix} \] at the bottom information set,

2’s beliefs:
\[ \begin{pmatrix} \text{left} \\ 1 \end{pmatrix} \quad \begin{pmatrix} \text{right} \\ 0 \end{pmatrix} \] at the bottom information set and
\[ \begin{pmatrix} \text{left} \\ \frac{1}{4} \end{pmatrix} \quad \begin{pmatrix} \text{right} \\ \frac{3}{4} \end{pmatrix} \] at the top information set.