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HOMEWORK 7 : ANSWERS

(a)(a.1) Whenever $p_G R > (1+r)X > p_B R$ (so that there does not exist a $w \le R$ such that $p_B w - (1+r)X \ge 0$ but there exists a $w \le R$ such that $p_G w - (1+r)X > 0$).

(a.2) Each bank would offer any type *G* entrepreneur a loan of $*X* with repayment $w = \frac{X(1+r)}{p_G}$ (the expected profit of a bank is $p_G\left(w - X(1+r)\right) - X(1+r)\left(1-p_G\right) = p_G w - X(1+r)$ and, because of the zero-expected-profit condition, this must be equal to 0). No bank would offer a loan to type *B* entrepreneurs. The expected utility of a type *G* entrepreneur would be $U_G = p_G\left(R - \frac{X(1+r)}{p_G}\right) = p_G R - X(1+r) > 0$ and the expected utility of a type *B* entrepreneur would be $U_B = 0$.

(b) A bank's expected profit from a loan is $w\left[p_B\lambda + (1-\lambda)p_G\right] - X(1+r)$. The zero profit condition yields $w\left[p_B\lambda + (1-\lambda)p_G\right] - X(1+r) = 0$, that is, $w = \frac{X(1+r)}{p_B\lambda + (1-\lambda)p_G}$. So $U_G = p_G\left(R - \frac{X(1+r)}{p_B\lambda + (1-\lambda)p_G}\right)$ and $U_B = p_B\left(R - \frac{X(1+r)}{p_B\lambda + (1-\lambda)p_G}\right)$. In order for this to be an equilibrium it must be that $U_B \ge 0$ (which implies $U_G \ge 0$), that is, $\lambda \le \frac{p_G R - X(1+r)}{R(p_G - p_B)}$: the fraction of high-risk borrowers should not be too high.

(c) $U_G(c, w) = p_G(R-w) - (1+\rho)c$. $U_B(c, w) = p_B(R-w) - (1+\rho)c$

(d) First of all, note that nobody would offer a collateral $c > c_0$. Furthermore, the zero-profit condition requires $w = \hat{w} \equiv \frac{\left(X - c_0\right)(1+r)}{p_G}$. The *B* types will not apply if and only if $p_B\left(R - \hat{w}\right) - (1+\rho)c_0 \le 0$. The *G* types will apply if and only if $p_G\left(R - \hat{w}\right) - (1+\rho)c_0 > 0$. Thus we need

$$\frac{p_B(R-\hat{w})}{1+\rho} \le c_0 < \frac{p_G(R-\hat{w})}{1+\rho} \quad (\blacklozenge)$$

(e) **(e.1)** For part (b): $w = 500/7 = 71.429$, $U_G = 200/7 = 28.57$, $U_B = 20/7 = 2.857$.

For part (c): $U_G(c, w) = 100 - w - \frac{11}{10}c$; $U_B(c, w) = \frac{1}{10}(100 - w) - \frac{11}{10}c = 10 - \frac{1}{10}w - \frac{11}{10}c$.

For part (d): $w = 50 - c_0$, $5 \le c_0 \le X = 50$ $(< R = 100)$ [the RHS of $(\blacklozenge)$ would give $c_0 < 500$, thus the binding constraint becomes $c_0 \le X$].

(e.2) The lowest value of c_0 is 5. In the separating equilibrium $w = 50 - 5 = 45$ and thus $U_G = 100 - 45 - (1+\frac{1}{10})5 = 49.5$ and $U_B = 0$. So, relative to pooling, signaling helps the *G* types and hurts the *B* types. It is also more efficient: under pooling the expected average utility is $\lambda(2.857) + (1-\lambda)28.57 = 20$ while with signaling it equals $\lambda(0) + (1-\lambda)49.5 = 33$, close to the average expected utility without asymmetric information, which is $(1-\lambda)(100-50) = \frac{2}{3}50 = 33.33$.
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