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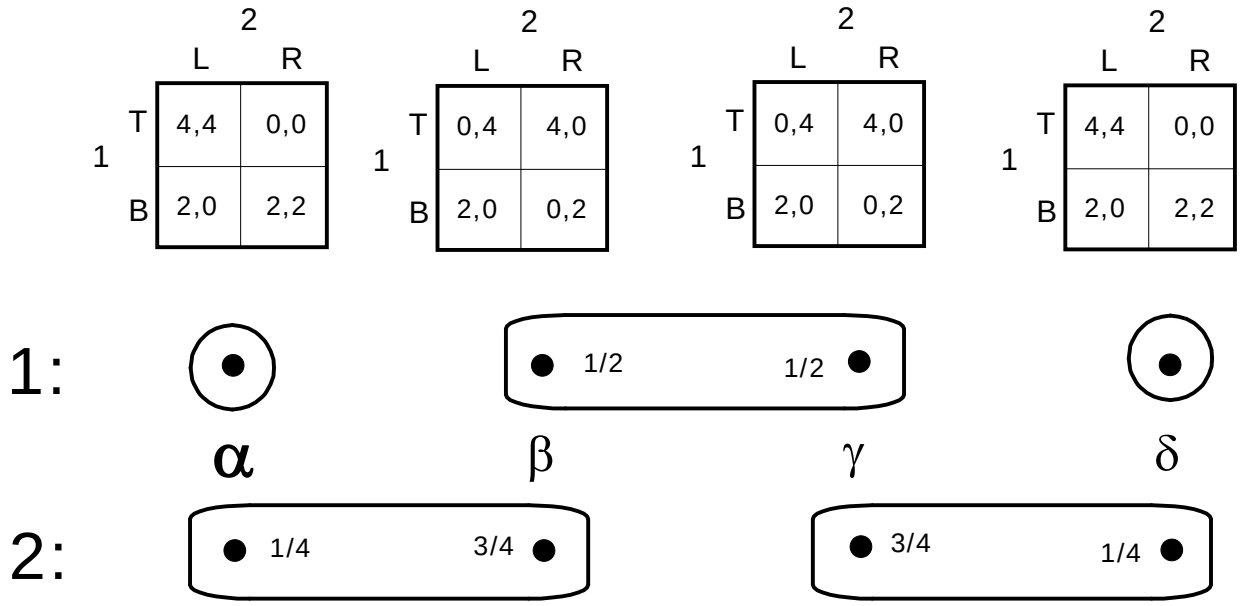
FINAL EXAM

ANSWER ALL QUESTIONS (total 100 points)

1. [36 points] Consider the following two-player situation of incomplete information.



(a) [10 points] Apply the "Harsanyi transformation" to represent this situation of incomplete information as a game with imperfect information.

(b) [4 points] Write all the pure strategies of Player 1 and all the pure strategies of Player 2 in the game with imperfect information of part (a).

(c) [6 points] Is the following a pure-strategy Bayesian Nash (= Nash) equilibrium: Player 1 plays T always and Player 2 plays L always? Explain your answer.

(d) [6 points] What beliefs are required by Bayesian updating (for an assessment) if Player 1's strategy is to always play T?

(e) [4 points] Explain why the above game with imperfect information has at least one subgame-perfect equilibrium (possibly in mixed strategies).

(f) [6 points] In the above game with imperfect information is there a weak sequential equilibrium where each player's behavioral strategy is always to choose between his two actions with equal probability? Justify your answer.

2. [40 points] There are three types of potential workers: those (type L) with productivity **21**, those (type M) with productivity **36** and those (type H) with productivity **40**. The **fraction of type L in the population is q_L** and the **fraction of type M is q_M** (thus the fraction of type H is $1-q_L-q_M$). Each worker knows her own type, while the potential employer does not. The potential employer believes that those applicants with education $y < y^M$ are of type L,

those with education $y^M \le y < y^H$ are of type M and those with education $y \ge y^H$ are of type H and offers each applicant a wage equal to the applicant's estimated productivity. The cost of acquiring y units of education is $\mathbf{C_L(y) = 3y}$ for type L, $\mathbf{C_M(y) = 2y}$ for type M and $\mathbf{C_H(y) = y}$ for type H. **By law, everybody must choose a level of education *y* greater than or equal to 3 (thus, for example, choosing *y* = 0 is not allowed)**. Define a signaling equilibrium as a situation where all the workers make education choices that lead them to being paid an amount equal to their productivity.

(a) [12 points] Write inequalities that must be satisfied in order to have a signaling equilibrium.

(b) **(b.1)** [6 points] What levels of education would the three types choose if $y^M = 14$ and $y^H = 22$? **(b.2)** [3 points] Is this a signaling equilibrium?

(c) **(c.1)** [6 points] What levels of education would the three types choose if $y^M = 9$ and $y^H = 12$? **(c.2)** [3 points] Is this a signaling equilibrium?

(d) [3 points] What is the average productivity in the population if $q_L = \frac{1}{5}$ and $q_M = \frac{1}{2}$?

(e) Assume that $q_L = \frac{1}{5}$ and $q_M = \frac{1}{2}$ and that the government steps in and (1) closes down all the educational institutions beyond the minimum required by law, so that everybody will acquire the same level of education, equal to 3 and (2) requires the employers to pay all the workers the same salary, equal to the average productivity. Suppose that previously the economy was at a signaling equilibrium with $y^M = 8$ and $y^H = 12$.

(e.1) [1 points] Are the L types better off as a consequence of government intervention?
(e.2) [3 points] Are the M types better off as a consequence of government intervention?
(e.3) [3 points] Are the H types better off as a consequence of government intervention?

3. [24 points] A monopolist faces n_A = 20 type *A* consumers, n_B = 20 type B consumers and n_C = 20 type C consumers. Each type A consumer has the demand function $D_A(P) = 120 - 4P$, each type B consumer has the demand function $D_B(P) = 120 - 6P$ and each type C consumer has the demand function $D_B(P) = 120 - 8P$. The monopolist is considering selling the good in packages. The pair (*Q*,*V*) represents a package containing *Q* units at a total price of *V* (thus *V* is the price of the entire package, not the price per unit). The monopolist is considering the following options.

OPTION 1. Sell only one type of package (Q_1 = 40, V_1 = 510).

OPTION 2. Sell two types of packages: (Q_{21} = 30, V_{21} = 393) and (Q_{22} = 50, V_{22} = 650).

OPTION 3. Sell three types of packages: (Q_{31} = 45, V_{31} = 510), (Q_{32} = 55, V_{32} = 620) and (Q_{33} = 70, V_{33} = 800).

(a) [6 points] Calculate the monopolist's revenue (not the profit) for Option 1.

(b) [8 points] Calculate the monopolist's revenue for Option 2.

(c) [10 points] Calculate the monopolist's revenue for Option 3.