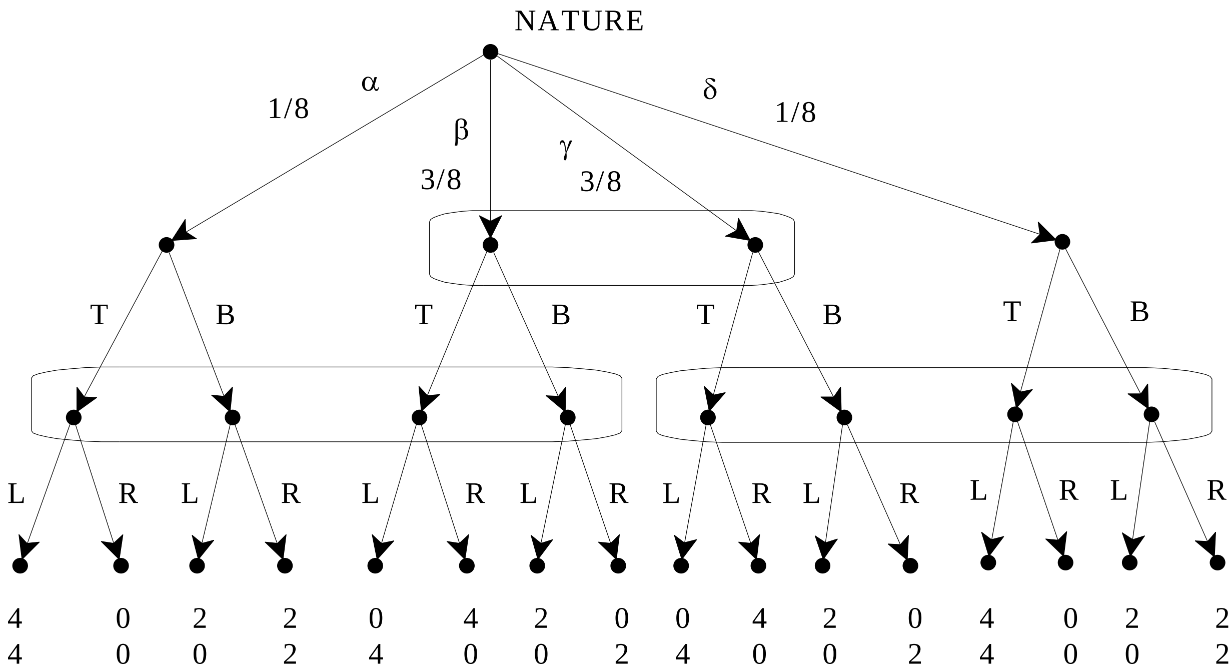
Final Exam ANSWERS

1. **(a)** The imperfect information game is as follows:



(b) Player 1 has 8 strategies (going from left to right): TTT, TTB, TBT, TBB, BTT, BTB, BBT, BBB. Player 2 has four strategies (going from left to right): LL, LR, RL, RR.

(c) No. Player 1 can increase his payoff by switching to TBT: $\pi_1(TTT, LL) = \frac{1}{8}4 + \frac{3}{8}0 + \frac{3}{8}0 + \frac{1}{8}4 = 1$ while $\pi_1(TBT, LL) = \frac{1}{8}4 + \frac{3}{8}2 + \frac{3}{8}2 + \frac{1}{8}4 = 2.5$.

(d) Player 1's beliefs must be $\frac{1}{2}$ at the left node and $\frac{1}{2}$ at the right node of his information set. Player 2's beliefs at his information set on the left must be: $\frac{1}{4}$ at the left-most node and $\frac{3}{4}$ at the third node from the left and his beliefs at the other information set must be $\frac{3}{4}$ at the left-most node and $\frac{1}{4}$ at the third node from the left.

(e) By Nash's theorem, the game has at least one (possibly mixed-strategy) equilibrium. Since the game has no proper subgames, every Nash equilibrium is also subgame-perfect.

(f) No. Sequential rationality fails at player 1's information set in the middle (where, by Bayes' rule his beliefs must be $\frac{1}{2}$ on each node): player 1 would get a higher payoff by choosing T with probability 1.

2. **(a)** There are six inequalities, two for each type (we only need to consider $y = 3$ or $y = y^M$ or $y = y^H$.

For type L: $\begin{cases} 21 - 3(3) = 12 \geq 36 - 3y^M \\ 21 - 3(3) = 12 \geq 40 - 3y^H \end{cases}$ for type M: $\begin{cases} 36 - 2y^M \geq 15 = 21 - 2(3) \\ 36 - 2y^M \geq 40 - 2y^H \end{cases}$

for type H: $\begin{cases} 40 - y^H \geq 18 = 21 - 3 \\ 40 - y^H \geq 36 - y^M \end{cases}$

(b) **(b.1)** When $y^M = 14$ and $y^H = 22$, Type L choose the minimum, namely 3, Type M also choose 3 and Type H choose 14.
(b.2) Thus it is not a signaling equilibrium because the choices ought to be 3, 14 and 22, respectively.

(c) **(b.1)** When $y^M = 9$ and $y^H = 12$, Type L choose 3, Type M choose 9 and Type H choose 12.
(b.2) Thus it is a signaling equilibrium.

(d) When $q_L = \frac{1}{5}$ and $q_M = \frac{1}{2}$ the average productivity is 34.2. [If you answered the question before the correction, that is, if you used $q_H = \frac{1}{2}$, then the average productivity is 35.]

(e) When $q_L = \frac{1}{5}$ and $q_M = \frac{1}{2}$ and $y^L = 8$ and $y^H = 12$, the L types are clearly better off after government intervention because their gross salary is higher and the cost of education is the same.

For the M and H types the net salaries are as follows:

	before	after
Type M	20	$28.2 = 34.2 - 2(3)$
Type H	28	$31.2 = 34.2 - 3$

Thus all the types are better off. [If you answered the question before the correction, that is, if you used $q_H = \frac{1}{2}$, then the average productivity is 35 and the numbers are 35–6 = 29 for Type M and 35–3 = 32 for Type H. The conclusion is the same]

3. The inverse demand functions are: $P_A = 30 - \frac{Q}{4}$, $P_B = 20 - \frac{Q}{6}$ and $P_C = 15 - \frac{Q}{8}$. Let $W_i(Q)$ be the willingness to pay of customer of type $i \in \{A,B,C\}$ for Q units. Then $W_A(Q) = 30Q - \frac{Q^2}{8}$, $W_B(Q) = 20Q - \frac{Q^2}{12}$ and $W_C(Q) = 15Q - \frac{Q^2}{16}$.

(a) Since $W_C(Q_1) - V_1 = -10$, $W_B(Q_1) - V_1 = 156.67$ and $W_A(Q_1) - V_1 = 490$, only customers of type A and B buy. Thus, the firm's revenue is $40V_1 = 40(510) = 20,400$.

(b) Since $W_C(Q_{21}) - V_{21} = 0.75$, $W_C(Q_{22}) - V_{22} = -56.25$, $W_B(Q_{21}) - V_{21} = 132$, $W_B(Q_{22}) - V_{22} = 141.67$, $W_A(Q_{21}) - V_{21} = 394.5$, $W_A(Q_{22}) - V_{22} = 537.5$, C-customers purchase the first package, while the others purchase the second package. Thus the firm's revenue is $20(393) + 40(650) = 33,860$.

(c) $W_C(Q_{31}) - V_{31} = 38.438$, $W_C(Q_{32}) - V_{32} = 15.938$, $W_C(Q_{33}) - V_{33} = -56.25$,
$W_B(Q_{31}) - V_{31} = 221.25$, $W_B(Q_{32}) - V_{32} = 227.917$, $W_B(Q_{33}) - V_{33} = 191.667$,
$W_A(Q_{31}) - V_{31} = 586.875$, $W_A(Q_{32}) - V_{32} = 651.875$, $W_A(Q_{33}) - V_{33} = 687.5$.
Thus type C buy the first package, type B the second and type A the third; hence the firm's revenue is 20(510) + 20(620) + 20(800) = 38,600.