ECN/ARE 200C : MICRO THEORY

SPRING 2025

Professor Giacomo Bonanno

## **HOMEWORK 1** (for due date see the web page)

**1.** You are probably familiar with the Rock-Paper-Scissors game. It is a two-player, simultaneous, strategic-form game, in which each player forms one of three shapes with an outstretched hand; these shapes are "Rock" (a closed fist), "Paper" (a flat hand), and "Scissors" (a fist with the index finger and middle finger extended, forming a V). Paper beats Rock, Rock beats Scissors and Scissors beats Paper. Let (x, y) denote the strategy profile where Player 1 chooses *x* and Player 2 chooses *y*. The outcomes are as follows:

$$\begin{cases} \text{if } (P,R) \text{ or } (R,S) \text{ or } (S,P) & \text{then } W_1 = \text{Player 1 wins} \\ \text{if } (R,P) \text{ or } (S,R) \text{ or } (P,S) & \text{then } W_2 = \text{Player 2 wins} \\ \text{if } (P,P) \text{ or } (R,R) \text{ or } (S,S) & \text{then } D = \text{draw} \end{cases}$$

Assume the following standard preferences:

Player 1's preferences:  $W_1 \succ D \succ W_2$  Player 2's preferences:  $W_2 \succ D \succ W_1$ .

Consider the following variant of this game: instead of using only one hand, each player uses both hands; then, when both players have displayed both hands, each player chooses which hand to remove and this is done simultaneously. The final outcome is determined by the remaining two hands as detailed above. Although this is not a simultaneous game, since there are two stages, we can think of it as a simultaneous strategic-form game by imagining each player formulating, in advance, a strategy defined as a complete contingent plan of action. Define a strategy as follows: there is an initial choice  $(\ell, r) \in \{P, R, S\} \times \{P, R, S\}$  of what gesture to display with the left hand  $(\ell)$  and what gesture to display with the right hand (r), and then a second choice from the set  $\{L, R\}$  where L means "remove left hand" and R means "remove right hand", as a function of the configuration of the two hands of the other player.

- (a) How many such strategies does each player have?
- (b) Are there any Nash equilibria?
- (c) (c.1) Describe a set of strategies for Player 1 that are all equivalent to each other.(c.2) What is the cardinality of a largest set of equivalent strategies for Player 1?
- (d) Is it the case that, for Player 1, the strategy, call it  $\hat{s}_1$ , of choosing (*P*,*P*) in the first stage and then removing the left hand, whatever hand configuration Player 2 displays, is weakly dominated by a strategy involving the choice of (*S*,*S*) in the first stage?
- (e) Are there any strategies of Player 1 that are weakly dominated? If so, give an example of a weakly dominated strategy and explain what other strategy weakly dominates it.

**2.** A seller has *m* identical units ( $m \ge 1$ ) of a commodity for sale. There are *n* bidders (n > m), each wishing to purchase at most one unit. All the bidders have the same valuation for one unit of the good, given by V > 0. The seller asks each bidder *i* to submit a sealed bid  $b_i$ , which can be any non-negative real number. The bids are submitted simultaneously. The seller then arranges the bids in decreasing order (from highest to lowest) and then sells the *m* units to the first *m* bidders on that list and charges each of them the (m+1)<sup>th</sup> bid. If there are ties, then the seller favors the bidder(s) with the lower index (e.g. bidder 5 has a lower index than bidder 8). For example, suppose that m = 5, n = 7 and the bids are as follows:  $b_1 = \$3$ ,  $b_2 = \$6$ ,  $b_3 = \$7$ ,  $b_4 = \$3$ ,  $b_5 = \$5$ ,  $b_6 = \$2$ ,  $b_7 = \$3$ . Then the seller arranges the bids as follows:  $b_3 = 7$ ,  $b_2 = 6$ ,  $b_5 = 5$ ,  $b_1 = 3$ ,  $b_4 = 3$ ,  $b_7 = 3$ ,  $b_6 = 2$  and sells 1 unit to each of

bidders 3,2,5,1 and 4 for \$3 (the sixth bid on that list).

- (a) What is this auction called if m = 1?
- (b) For the case where m > 1, show that each player has a weakly dominant strategy.
- (c) Show that the strategy of part (b) is not strictly dominant.
- (d) What is the seller's revenue at the dominant-strategy equilibrium?