Consider the following 2-player, 2-period bargaining game of perfect information. Let $\delta_i$ be the discount factor of player $i$ ($i = 1, 2$). Player 1 begins by offering a share $x_0 \in [0, 1]$ of $\$1$ to player 2, who then accepts or rejects. If player 2 accepts, the payoffs are $1 - x_0$ to player 1 and $x_0$ to player 2. If player 2 rejects, he makes a counter-offer $x_1 \in [0, 1]$ which player 1 can either accept or reject. If player 1 accepts the counter-offer, the payoffs are $\delta_1 x_1$ to player 1 and $\delta_2 (1 - x_1)$ to player 2. If player 1 rejects, the payoffs are 0 to each player. Assume throughout that if a player is indifferent between accepting and not accepting an offer, then he will accept.

(a.1) Sketch the extensive game.
(a.2) Find the backward induction solution (not just the outcome but the entire strategies).
(a.3) Is the backward induction outcome Pareto efficient?
(a.4) At the backward induction solution, does being more patient (i.e. having a higher $\delta_2$) help or hurt player 2? Briefly explain the intuition.

Now suppose that before the bargaining begins, Player 2 can learn the art of patience by attending a self-hypnosis class. If he does not go to the class, his $\delta_2 = 0.3$. If he goes to the class he can increase his $\delta_2$, but the higher he wants his $\delta_2$ to be, the more effort he needs to expend at the class. The monetary cost of obtaining a given value of $\delta_2 \in [0.3, 1]$ is given by the following function: $c(\delta_2) = (\delta_2 - 0.3)^2$.

Whatever $\delta_2$ player 2 has achieved in the self-hypnosis class will be known to player 1 when she makes her initial offer $x_0$.

(b) Consider the modified bargaining game obtained from the one described before by adding an initial stage where player 2 decides whether or not to go to the self-hypnosis class and, if he decides to go, what value of $\delta_2$ to acquire. Solve the game by backward induction. Is it socially efficient for player 2 to go to the class to improve his bargaining skills?

Now forget about the self-hypnosis class and modify the game described initially by adding a third stage (first player 1 makes an offer, then player 2 makes a counter-offer -- if he rejects player 1’s offer -- and then player 1 makes a final offer, if she rejects 2’s offer; the final offer by Player 1 is either accepted or rejected by player 2; thus discounting takes place over three periods).

(c) Find the backward induction payoffs. Is the backward induction outcome Pareto efficient?

Now modify the 3-stage game of part (c) by allowing both players to go to the self-hypnosis class. Assume that if nobody attends the class, then $\delta_1 = \delta_2 = 0$; furthermore, the cost of training in patience for players 1 and 2 are, respectively, $c(\delta_1) = 0.5(\delta_1)^2$ and $c(\delta_2) = 0.5(\delta_2)^2$ for any $\delta_1, \delta_2 \in [0, 1]$. Suppose that the revised game is played sequentially and with perfect information. First player 1 decides on his $\delta_1$; then player 2, observing $\delta_1$, decides on his $\delta_2$; finally, the individuals play the 3-period game of part (c) with their chosen discount factors (which are common knowledge).

(d) Find the backward induction solution of this revised game.
(e) In the game of part (d), what would the value of $\delta_2$ be if only player 2 could go to the class, so that $\delta_1$ is fixed at $\delta_1 = 0$?