## HOMEWORK 2 (for due date see the web page)

An industry consists of two firms selling a perfectly homogenous product. Firm i = 1,2 chooses quantity  $q_i \ge 0$  which it produces at cost  $cq_i$  (with  $c \ge 0$ ; thus the two firms have the same cost function). The firms make these quantity choices simultaneously, and all output is then sold at the price  $P = a - b(q_1 + q_2)$  (a > c, b > 0).

(a) Find the Cournot-Nash equilibrium.

Now let us depart from the standard Cournot model by assuming that Firm 1 is owned by investor A, who owns the share  $\alpha_1 \ge \frac{1}{2}$  of Firm 1 and the share  $(1-\alpha_2)$  (with  $\alpha_2 \ge \frac{1}{2}$ ) of Firm 2, and Firm 2 is owned by investor B, who owns the complementary shares. Firm 1's objective is to maximize the income of investor A (which consists of the fraction  $\alpha_1$  of the profits of Firm 1 plus the fraction  $(1-\alpha_2)$  of the profits of Firm 2) and Firm 2's objective is to maximize the income of investor B.

[As you work through the problem, check your algebra by verifying that your formulas coincide with the solution found in part (a) when you substitute  $\alpha_1 = \alpha_2 = 1$ .]

- (b) Calculate Firm 1's best response function. How does it depend on  $\alpha_1$  and  $\alpha_2$ ?
- (c) Find the Nash equilibrium quantities.
- (d) For the special case where  $\alpha_1 = \alpha_2 = \alpha > 0$  how do equilibrium quantities vary with the value of  $\alpha$ ?
- (e) Still assuming that  $\alpha_1 = \alpha_2 = \alpha$ , is there a value of  $\alpha$  such that total industry output at the Nash equilibrium coincides with monopoly output?