University of California, Davis -- Department of Economics

ECN/ARE **200C** : **MICRO THEORY**

SPRING 2025 Professor Giacomo Bonanno

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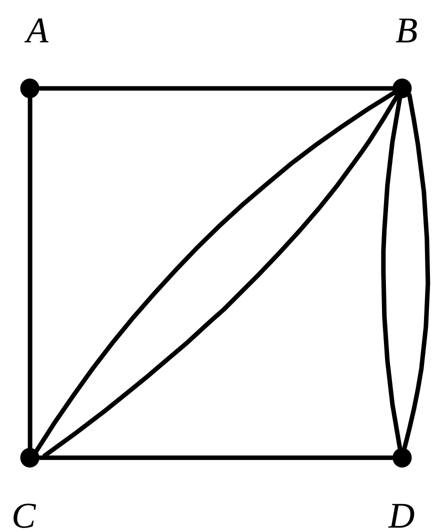
HOMEWORK 3 **(for due date see the web page)**

1. Consider the following perfect-information game. The figure below shows four locations (*A*,*B*,*C*,*D*) and each line represents a bridge connecting two locations. Player 2 is located at point *A* and her objective is to reach point *D*. Player 1, whose objective is to prevent Player 2 from reaching point *D*, moves first and blows up one bridge. Then Player 2 crosses one bridge from point *A* to a contiguous location and Player 1 observes this. Then Player 1 blows up another bridge. Then Player 2 crosses a bridge again to move from his current location to a contiguous location (possibly reversing his previous crossing), and so on. The game terminates if Player 2 reaches point *D* (Player 2's most preferred outcome) or if it becomes impossible for Player 2 to reach point *D* (Player 1's most preferred outcome). Each player has a strict ranking of the two possible outcomes.



(a) Explain why it is the case that either Player 1 has a strategy that for sure will prevent Player 2 from reaching point *D* or Player 2 has a strategy that for sure will allow her to reach point *D*.

(b) Describe in words the strategy of part (a).
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2. Consider a population of N people labeled $i = 1, 2, ..., N$. Each chooses between two actions, IN, which corresponds to adopting a new technology, and OUT, which corresponds to staying with the *status quo* technology. If n **other** players choose IN, the payoff to player i is

$$S(i,n) = a - nb \quad \text{if } i \text{ chooses } OUT$$

$$B(i,n) = c + dn - e \quad \text{if } i \text{ chooses } IN$$

(again: n is the number of players, *not including player i*, who choose IN).

Assume that $b, d > 0$, $a > c - e$ and $c + d - e > a - b$. This implies that the new technology and the old technology are both characterized by "network externalities", in the sense that the benefits of using a given technology are greater when the number of other people using it is larger.

You are asked to consider two different versions of this game. **In both versions assume that N = 3.**

Version 1: Players make their choices of technology simultaneously. With respect to Version 1:

(a) Does any of the players have a dominant strategy?

(b) Find all of the pure strategy Nash equilibria of this game.

Version 2: Assume that the players make their decisions sequentially: first Player 1 then Player 2 and then Player 3, with each player observing earlier choices (if any). With respect to Version 2:

(c) **(c.1)** Draw the extensive form of this game, including payoffs.

(c.2) How many strategies do the players have?

(d) Find the backward-induction solution(s) for all possible values of a, b, c, d and e subject to $b, d > 0$, $a > c - e$ and $c + d - e > a - b$.