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HOMEWORK 4 (for due date see the web page)

An Incumbent Monopolist (IM) and a Potential Entrant (PE) play the following game. First the IM decides whether to be **passive** or **committed**. Commitment costs $**C** and the cost is non recoverable (sunk cost). Then the PE observes the action taken by the IM and decides whether to **enter** or **stay out**. If she stays out, her payoff is $\boldsymbol{k}$ (independent of whether the IM chose to be passive or committed), whereas the IM's payoff is $\boldsymbol{M}$ if passive and $(\boldsymbol{M-C})$ if committed. Assume that $M > C > 0$. If the PE decides to enter, then the two firms play a simultaneous Cournot game where the demand function is given by $D_{pass}(P) = a - bP$ if the IM chose to be passive and $D_{comm}(P) = c - dP$ if the IM chose to be committed. Production costs are zero for both firms. The situation, however, is complicated by the fact that the PE's opportunity cost of entry k is known only to her and not to the IM. Let K be the set of possible values of k. Suppose that the value of k is chosen (e.g. by a referee) according to a given probability distribution over K. The probability distribution is known to both players. We can think of this choice as a move by *Nature* or *Chance*, which occurs at the beginning of the game. When the PE makes her choices, she is told what the value of k is, while the IM is never told what value of k was realized. Both players are risk neutral.

(a) For the simple case where (i) $K = \{k_1, k_2\}$ and (ii) in the Cournot game each firm has only two possible choices of output (denote them by q_1 and q_2), represent this situation as an extensive-form game with imperfect information. **You do not need to write the payoffs**: just show the structure of the game.

(b) Explain why in the game of part (a) above, the post-entry Cournot games are not subgames.

(c) Suppose that $K = \{1, 4, 6, 12\}$, the probability distribution over K is the uniform distribution, $a = 15$, $b = 5$, $c = 6$, $d = 2$. Determine whether a rational incumbent chooses commitment (identify rationality with Nash equilibrium and assume that the PE's choice of output in the post-entry interaction does not depend on the opportunity cost of entry). [Hint: first solve the Cournot "subgames" as if they truly were subgames; note that your answer must be conditional on the values of the parameters M and C.]

Now let us change the game. First the IM decides whether to be passive or committed (as before, commitment costs $$C$ and is irreversible). Then Nature selects the opportunity cost of entry $k \in K$ (that is, the profit that the potential entrant could make in the best alternative investment) according to the cumulative distribution function F [thus, for every number x, $F(x)$ is the probability that the opportunity cost of entry k is less than or equal to x]. The value of k is then revealed to **both** IM and PE (and becomes common knowledge between them). Then the PE decides whether or not to enter and if she enters then there is a simultaneous duopoly game (which we do not specify: it could be a Cournot game or a price-setting game). Let D_I and D_E be the incumbent's and entrant's profits, respectively, at the Nash equilibrium of the duopoly game following entry with a passive incumbent, and H_I and H_E be their respective profits at the Nash equilibrium of the duopoly game following entry with a committed incumbent (H_I **includes** the commitment cost C). Assume that if she is indifferent between entering and not entering, the PE will choose to enter.

(d) Draw the extensive form of this game for the case where $K = \{k_1, k_2\}$ (replace each duopoly game with the corresponding equilibrium payoffs; do write all the payoffs).

(e) Assume that $K = [A, B]$ (the closed interval between A and B, $0 < A < B$) and $A < H_E < D_E < B$. Under what conditions is there commitment at every subgame-perfect equilibrium? Under what conditions are the subgame-perfect equilibria characterized by the fact that the incumbent is passive?

(f) Suppose that $K = \{1, 2, 4, 7\}$, Prob$\{1\}$ = Prob$\{2\}$ = Prob$\{4\}$ = 1/5, Prob$\{7\}$ = 2/5, $M = 8$, $C = 2$, $D_I = D_E = \frac{7}{2} = 3.5$, $H_I = H_E = \frac{3}{2} = 1.5$. Would a rational incumbent choose commitment? (Identify rationality with behavior consistent with a subgame-perfect equilibrium.)
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