Consider the following two-player game where the payoffs are von Neumann-Morgenstern payoffs. Nature moves first and chooses Player 1’s type, which is either $A$ with probability $p$ (with $0 < p < 1$) or $B$ with probability $(1-p)$. Player 1 observes Nature’s move (that is, learns her type) and chooses an action $a_1 \in \{U,D\}$. Player 2 sees Player 1’s choice but not Nature’s choice and chooses an action $a_2 \in \{L,R\}$. When Player 1’s type is $A$ the payoffs are (as usual, in each pair the number on the left is Player 1’s payoff and the number on the right is Player 2’s payoff) $L$ $R$
\begin{array}{ll}
U & 4, 4 \\
D & 1, 1
\end{array}$
and when Player 1’s type is $B$ the payoffs are
\begin{array}{ll}
L & R \\
U & 2, 0 \\
D & 0, 1
\end{array}$.

Call an equilibrium (1) separating if Player 1’s choice as type $A$ is different from her choice as type $B$, (2) pooling if Player 1 makes the same choice, whatever her type.

(a) Draw the extensive-form game.

(b) Is there a pure-strategy separating weak sequential equilibrium? If there is then give the details, if there isn’t then prove it.

(c) For what values of $p$ does the game have a pure-strategy pooling weak sequential equilibrium? For every value of $p$ either prove that a pooling weak sequential equilibrium does not exist or find them all.

(d) For what values of $p$ does the game have a weak sequential equilibrium in which type $B$ plays $U$ with probability 1 and type $A$ chooses each action with positive probability?