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HOMEWORK 7 (for due date see the web page)

Consider a market for loans to finance investment projects. All investment projects require an outlay of $\$X$. There are 2 types of projects: good and bad. A bad project has probability p_B of yielding a revenue of $\$R$ and a probability $(1-p_B)$ of yielding zero revenue. A good project yields $\$R$ with probability p_G and zero with probability $(1-p_G)$, with $0 \le p_B < p_G \le 1$. Assume that $\boldsymbol{R > X}$. Each entrepreneur has one project, and the fraction of entrepreneurs with **bad** projects is λ , with $0 < \lambda < 1$.

Entrepreneurs go to banks to borrow the cash to make the initial outlay (assume for now that they must borrow the entire amount). A **loan contract** specifies the total amount w that is supposed to be repaid to the bank, where $w \le R$. Entrepreneurs and banks are *risk neutral*. Each entrepreneur knows the type of his project, but the banks do not. *In the event that a project yields zero revenue, the entrepreneur defaults on her loan contract, and the bank receives nothing.* Let r be the risk-free rate of interest (the rate that banks must pay to their depositors in order to obtain funds to be used for loans). There are many competing banks and in equilibrium **each bank will earn zero expected profit**.

(a) **(a.1)** For what values of the parameters is it the case that efficiency requires that *all and only* the good projects be financed?

(a.2) For these values of the parameters, what would happen in the absence of asymmetric information, i.e., if banks could distinguish good and bad entrepreneurs? What would the expected utility of each entrepreneur be?

From now on assume that the parameter values satisfy the restriction of part (a).

(b) Now assume asymmetric information: banks cannot distinguish good and bad entrepreneurs. Find the value of w and the parameter restrictions that yield a pooling equilibrium, that is, an equilibrium where all types of entrepreneurs apply for a loan. Calculate the equilibrium expected utility of each entrepreneur of type G and of type B at the equilibrium.
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Now suppose that an entrepreneur can offer to contribute **$c** toward the initial outlay of $X, where $c \in [0, X]$. If she contributes $c, she only needs to borrow $($X - c$) from a bank. Now a loan contract is a pair $C = (c, w)$ specifying the amount w that is supposed to be repaid if the entrepreneur contributes c [and thus borrows $(X - c)$]. Also assume that the entrepreneur is liquidity constrained, so her effective cost of contributing c is $(1+\rho)c$, where $\rho > r$ (that is, the entrepreneur has to borrow that money at an interest rate ρ that exceeds the risk-free rate r). As before, in the event that a project yields zero revenue, the entrepreneur defaults on her loan contract, and the bank receives nothing; however, the loan for the amount c is fully guaranteed by the entrepreneur's illiquid assets (his house) and therefore the repayment of $(1+\rho)c$ cannot be avoided.

(c) Let $U_\theta(c, w)$ denote the expected payoff of an entrepreneur of type $\theta \in \{G,B\}$ from the contract $C = (c, w)$. Find $U_G(c, w)$ and $U_B(c, w)$.

(d) Suppose that all the banks use the following strategy: offer a loan of $($X - c$) with repayment $$w$ to all those entrepreneurs who offer to contribute $$c$ with $c \geq c_0$ and refuse a loan to all those entrepreneurs who offer to contribute $$c$ with $c < c_0$. Find conditions on the values of c_0 and w that would lead to all and only the G-type entrepreneurs applying for (and obtaining) a loan and each bank making zero expected profits. (That is, find values of c_0 and w that yield a separating equilibrium.) Assume that each entrepreneur will apply for a loan if and only if her expected utility is positive.

(e) From now on assume that $X = 50$, $R = 100$, $p_G = 1$, $p_B = \frac{1}{10}$, $r = 0$, $\lambda = \frac{1}{3}$, $\rho = \frac{1}{10}$.

(e.1) Give the answers to (b)-(d) for these values of the parameters.

(e.2) Compare the payoffs of the two types in the equilibrium without asymmetric information of part (a), in the pooling equilibrium of part (b) and in the separating equilibrium of part (d) with the lowest value of c_0. Rank them in terms of efficiency. [Hint: calculate average expected utility.]