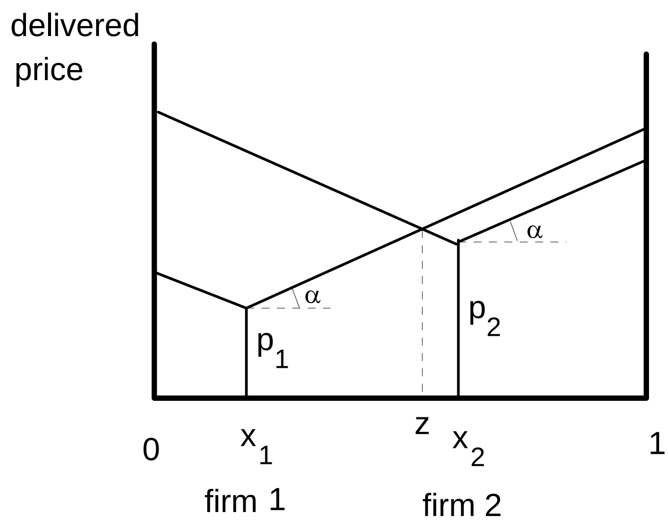
HOTELLING'S MODEL

Cournot's model assumes that the products of all the firms in the industry are identical, that is, all consumers view them as perfect substitutes. It is a very useful model in that it enables us to prove in a simple way such claims as: "the larger the number of firms in an industry the stronger the competition among them", "consumers benefit from the entry of new firms in an industry", "perfect competition can be thought of as an approximation to what happens in industries where the number of firms is very large", "one of the factors that determine the number of firms in an industry is the size of the fixed cost", etc.

Bertrand in his review (1883) of Cournot's book criticized the assumption that firms choose output levels, maintaining that in reality the main decision firms have to make is what price to charge for their products. He showed that if output competition is replaced by price competition, while maintaining the assumption of product homogeneity, then the only equilibrium is one where price is equal to marginal cost.

Another criticism of Cournot's model concerns the assumption of product homogeneity. Very few products can be considered identical. Sugar, milk, cement, might be good examples of products that do not differ across firms. Yet the local convenience store might charge a bit more for a carton of milk than the nearest supermarket and some consumers might be willing to pay the small premium associated with convenience and proximity. As a matter of fact, it is ***very*** difficult to think of examples of truly homogeneous products. Furthermore, firms seem to spend a lot of money and effort in the attempt to differentiate their products from those of their competitors. Think, for example, of the frequent-flyer programs introduced by airlines. A firm can either add something "real" to the product in order to differentiate it from its competitors' products or it can try to add - usually through advertising - a "perception" that the product is different, even though it is not.

The first model of product differentiation is due to Hotelling (1929). Imagine a town with a Main Street of length 1. There are N consumers living on this street and they are uniformly distributed along the street, that is, on a segment of the street of length x there are xN consumers (thus, if N = 900 and we take a segment of length 1/3 then on this segment lives 1/3 of the population, i.e. 300 consumers). Two firms offer the same product (e.g. milk). Firm 1 is located at point x_1 and firm 2 is located at point x_2 (let firm 1 be to the left of firm 2, so that $0 \le x_1 \le x_2 \le 1$). Firm 1 charges (mill) price p_1, while firm 2 charges (mill) price p_2. Consumers have a transportation cost of α per unit of distance. In the following diagram, the vertical lines represent mill prices, while the oblique lines represent "delivered" prices, i.e. mill price plus transportation cost.



The consumer located at z is indifferent between the two firms. Those located to the left of z prefer firm 1, while those located to the right of z prefer firm 2. Hence firm 1 will serve the market segment [0,z], while firm 2 will serve the market segment [z,1]. Obviously, the location of the indifferent consumer depends on both prices. Given p_1 and p_2, the indifferent consumer is given by the solution to:

$$p_1 + \alpha (z - x_1) = p_2 + \alpha (x_2 - z).$$

Thus

$$z = \frac{p_2 - p_1}{2\alpha} + \frac{x_2 + x_1}{2}.$$

Hence the demand functions are given by:

$$D_1(p_1,p_2) = z\,N = \left(\frac{p_2 - p_1}{2\alpha} + \frac{x_2 + x_1}{2}\right) N$$

and

$$D_2(p_1,p_2) = (1 - z)\,N = \left(1 - \frac{p_2 - p_1}{2\alpha} - \frac{x_2 + x_1}{2}\right) N.$$

To see firms' incentive to differentiate their products, first consider the case where both firms are located at $\frac{1}{2}$, i.e. $x_1 = x_2 = \frac{1}{2}$. Assume that both firms have the same cost function:

$$TC = c\,q$$

(with $c > 0$) and that $\boldsymbol{\alpha = 1}$. Then the unique Nash equilibrium in prices (Bertrand-Nash equilibrium) is given by $p_1 = p_2 = c$. Hence both firms make zero profits in equilibrium. Consider now the case where $x_1 = 0$ and $x_2 = 1$ (that is, the two firms are located at the extremes). The profit function of firm 1 is (recall that $\alpha = 1$):

$$\pi_1(p_1,p_2) = p_1\left(\frac{p_2 - p_1}{2} + \frac{1}{2}\right) N - c\left(\frac{p_2 - p_1}{2} + \frac{1}{2}\right) N$$

while the profit function of firm 2 is given by:

$$\pi_2(p_1,p_2) = p_2\left(\frac{1}{2} - \frac{p_2 - p_1}{2}\right) N - c\left(\frac{1}{2} - \frac{p_2 - p_1}{2}\right) N$$

As usual, the Nash equilibrium is given by the solution to the system of equations: $\frac{\partial \pi_1}{\partial p_1} = 0$ and $\frac{\partial \pi_2}{\partial p_2} = 0$. The solution is:

$$p_1 = p_2 = c + 1$$

$$\pi_1 = \pi_2 = \frac{1}{2}N.$$