1. [56 points] Two agents, 1 and 2, can contribute to the provision of a public good. The good will be provided if and only if the sum of their contributions exceeds some critical level $X > 0$. In this case, each agent obtains a benefit $B > 0$. However, contributing is costly and contributions are non-refundable. The cost of contributing $x \geq 0$ is $c(x) = x$. The payoff of a player who contributes $x$ is $B - x$ if the public good is provided and $-x$ if the public good is not provided. Assume that $2B > X$ so that providing the good is efficient.

Suppose first that the agents simultaneously choose contribution levels $x_1$ and $x_2$, after which the good is provided if and only if $x_1 + x_2 \geq X$.

(a) [4 points] Find the pure strategy Nash equilibria for the case where $B > X$.

(b) [4 points] Find the pure strategy Nash equilibria for the case where $B = X$.

(c) (c.1) [4 points] Find the pure strategy Nash equilibria for the case where $B < X$.
   (c.2) [4 points] Explain why there is at least one Nash equilibrium where both players contribute a positive amount.

Suppose now that the agents contribute sequentially, with agent 1 contributing in period 1 and agent 2 in period 2. Assume that contributions are observable and that the good is provided as soon as the total contribution level is at least $X$ (that is, either in the first period, the second period, or never). Suppose that there is no discounting, so that if an agent contributes $x$, her payoff is $B - x$, whether the public good is provided in period 1 or in period 2, and $-x$ if it isn’t provided.

(d) Suppose that $B > X$.
   (d.1) [4 points] Find the best reply function of Player 2.
   (d.2) [4 points] Find the backward-induction solution(s).
   (d.3) [4 points] Is the public good provided in the first period, the second period, or never?

(e) Suppose that $B = X$.
   (e.1) [4 points] Find the best reply function of Player 2.
   (e.2) [4 points] Find the backward-induction solution(s).
   (e.3) [4 points] Is the public good provided in the first period, the second period, or never?
   (e.4) [4 points] Find a Nash equilibrium that is not a backward-induction solution.

(f) Suppose that $B < X$ (recall that, by assumption, $2B > X$).
   (f.1) [4 points] Find the best reply function of Player 2.
   (f.2) [4 points] Find the backward-induction solution(s).
   (f.3) [4 points] Is the public good provided in the first period, the second period, or never?
2. [32 points] It has been documented in the psychology literature that the probability that an individual offers assistance to someone in need is lower when he is in a group than when he alone (see Latané, B. and Nida, S., Ten years of research on group size and helping, Psychological Bulletin, 89, 1981, pp. 308-324). Let us show this phenomenon in a simple game. There are \( N \geq 2 \) potential helpers (call them players) of a person in need. Each player decides whether to Help or Ignore. Decisions are made simultaneously and independently. Each player cares about whether the victim is helped, whether she helps the victim and whether anyone else helps the victim. The von Neumann-Morgenstern payoffs are denoted as follows:

- \( a \) := payoff to a player if she helps and no one else helps,
- \( b \) := payoff to a player if she does not help and someone else helps,
- \( c \) := payoff to a player if she helps and someone else helps,
- \( d \) := payoff to a player if no one helps.

All the players are identical and the game and payoffs are common knowledge.

**a)** [4 points] Assume that each player is concerned with the welfare of the victim (wants the victim to be helped) but, at the same time, help involves effort, so that each player would rather free ride on someone else’s help. Express these two assumptions in terms of restrictions on the parameters \( a, b, c, d \).

**b)** [4 points] Find all the pure-strategy Nash equilibria of this game.

**c)** [4 points] Find the symmetric mixed-strategy equilibrium (symmetric means that all the players adopt the same strategy).

In what follows consider the special case where \( a = 10, b = 16, c = 8, d = 2 \).

**d.1)** [4 points] Show that, for each player, at the mixed-strategy equilibrium the probability that she helps is decreasing in \( N \).

**d.2)** [4 points] Calculate the probability, call it \( h \), that the victim is helped at the mixed-strategy equilibrium.

**d.3)** [4 points] Show that \( h \) is decreasing in \( N \).

**d.4)** [4 points] Verify this by computing the value of \( h \) when \( N = 2 \) and when \( N = 8 \).

**d.5)** [4 points] Calculate the limit of \( h \) as \( N \) goes to infinity.

3. [12 points] Consider the following two-player game-frame. Rudy is a fugitive and it is common knowledge between him and the Chief of Police that (1) there are only two locations, A and B, where he could be hiding and (2) the police will be able to pinpoint the exact location of Rudy’s cellphone, but only if it is on, that is, if it communicates with the cell towers. Rudy makes three consecutive decisions: first whether to hide at location A or at location B, second whether to turn off his cell phone or leave it on and third - in case he leaves his cellphone on - whether to keep it with himself or give it to an accomplice to take it to the other location and leave it there (turned on). The next day, the police chief -- not having observed any of the above decisions by Rudy -- after consulting the tracking device and checking whether or not there is a signal from the cellphone and, if there is, where it is coming from, decides whether to storm location A or location B.

**a)** [6 points] Draw an extensive-form game-frame to represent this situation.

**b)** [3 points] How many strategies does the police chief have?

**c)** [3 points] How many strategies does Rudy have?