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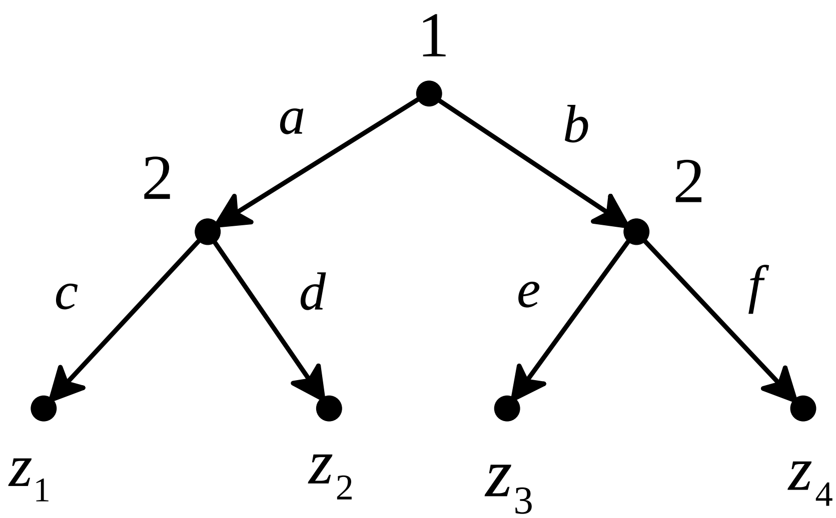
MIDTERM EXAM (total 100 points). Answer all questions.

1. [13 points] Consider the following extensive-form game-frame with perfect information:



The players have complete and transitive preferences over the set of outcomes $\{z_1, z_2, z_3, z_4\}$ and the preferences are such that the corresponding game has exactly three backward-induction solutions, namely $(a,(c,e))$, $(b,(c,e))$ and $(b,(c,f))$.

(a) [4 points] What can you infer about the preferences of Player 1?

(b) [4 points] What can you infer about the preferences of Player 2?

(c) [5 points] Write all the complete and transitive preference relations of **Player 1** that are compatible with the above information.

2. [27 points] Each member of a class of $n \geq 3$ students is simultaneously deciding whether or not to contribute a fixed amount of money, F, toward the provision of a class party. The party will be held if and only if **at least *k* students, but not all, contribute** (thus $2 \leq k < n$), and the quality of the party does not depend on the amount contributed. Any contributions are non-refundable: contributors do not get their money back even if the party is not held. For each student, the value of the party, V, is greater than their potential contribution ($V > F$), and each student only cares about their own benefit from the party and their own expenditures. As a result, each student ranks the possible outcomes as follows, from best to worst: (i) any outcome in which the party is held and the student does **not** contribute; (ii) any outcome in which the party is held and the student contributes; (iii) any outcome in which the party is **not** held and the student does **not** contribute; (iv) any outcome in which the party is **not** held and the student contributes.

(a) [8 points] For the case where $k = 2$ and $n = 3$, write the strategic-form game using utility values from the set $\{2,4,6,8\}$ (thus utilities are even numbers).

(b) [4 points] For the game of part (a) find **all** the pure-strategy Nash equilibria.

(c) [4 points] For the game of part (a) are the Nash equilibria Pareto efficient?

(d) [7 points] Now for the general case, that is, for any k and n with $2 \le k < n$, find **all** the pure-strategy Nash equilibria of the corresponding game and fully justify your answer (in particular, show that there are no other Nash equilibria).

(e) [4 points] If in the game of part (a), instead of even numbers, we had chosen odd numbers for utilities, how would the set of Nash equilibria have been affected?

3. [60 points] This question asks you to compare the equilibria of various games with the same payoffs and different information structures.

GAME 1. This is a game with perfect information. Player 1 moves first choosing between U and D. Player 2 sees Player 1's move and chooses between L and R. The von Neumann Morgenstern payoffs are as follows:

	L	R
U	$3,2$	$1,1$
D	$5,0$	$2,2$

(a) [6 points] Draw the extensive-form game and write the corresponding strategic-form game.

(b) [2 points] Find all the pure-strategy Nash equilibria.

(c) [3 points] Find the output of the iterated deletion of strictly dominated strategies (IDSDS).

(d) [4 points] Find the output of the iterated deletion of weakly dominated strategies (IDWDS).

(e) [2 points] Find all the backward-induction solutions.

GAME 2. In this game again Player 1 moves first choosing between U and D. If Player 1 chooses U then Nature chooses either a with probability $1-\varepsilon$ or b with probability ε, where $0 < \varepsilon < 1$. If Player 1 chooses D then Nature chooses either a win probability ε or b with probability $1-\varepsilon$. Player 2 moves after Nature, observing Nature's choice but without observing Player 1's choice. Player 2 chooses between L and R. The payoffs are the same as above; in particular, they depend only on the choices of Player 1 and 2 and not on Nature's choices.

(f) [16 points] Draw the extensive-form game and write the corresponding strategic form game.

(g) [6 points] Find all the pure-strategy Nash equilibria of this game (your answer should be conditional on the value of ε).

(h) [3 points] What happens to the set of pure-strategy Nash equilibria as $\varepsilon \to 0$?

(i) [4 points] Are there values of ε for which the following is a mixed-strategy Nash equilibrium: the players play each of their pure strategies with equal probability? Explain your answer.

(j) [14 points] Are there values of ε, p and q, with $p, q \in (0,1)$, for which the following is a mixed-strategy Nash equilibrium: $\begin{pmatrix} U & D & | & LL & LR \\ p & 1-p & | & q & 1-q \end{pmatrix}$?