University of California, Davis -- Department of Economics

SPRING 2025 **ECN / ARE 200C: MICROECONOMIC THEORY** Giacomo Bonanno

Midterm Exam $\boxed{\text{ANSWERS}}$

1. **(a)** Since $(a,(c,e))$ and $(b,(c,e))$ are both backward-induction solutions, it must be that Player 1 is indifferent between z_1 and z_3. Since $(b,(c,f))$ is a backward-induction solution but $(a,(c,f))$ is not, it must be that Player 1 prefers z_4 to z_1. Thus there are only two inferences that we can make about Player 1's preferences: (1) $z_1 \sim_1 z_3$ and (2) $z_4 \succ_1 z_1$ (and thus also $z_4 \succ_1 z_3$)

(b) Since both *e* and *f* are part of a backward-induction solution, it must be that Player 2 is indifferent between z_3 and z_4. Since *c* is part of a backward-induction solution but *d* is not, it must be that Player 2 prefers z_1 to z_2; if Player 2 were indifferent between z_1 and z_2, at least one of the following would be a backward-induction solution: $(b,(d,f))$ (if, for Player 1, $z_4 \succsim_1 z_2$) or $(a,(d,f))$ (if, for Player 1, $z_2 \succsim_1 z_4$). Thus there are only two inferences that we can make about Player 2's preferences: (1) $z_3 \sim_2 z_4$ and (2) $z_1 \succ_2 z_2$.

(c) There are five possibilities, depending on where z_2 appears in the ranking:

$$\begin{pmatrix} best & z_2 \\ & z_4 \\ worst & z_1, z_3 \end{pmatrix}, \begin{pmatrix} best & z_2, z_4 \\ worst & z_1, z_3 \end{pmatrix}, \begin{pmatrix} best & z_4 \\ & z_2 \\ worst & z_1, z_3 \end{pmatrix}, \begin{pmatrix} best & z_4 \\ worst & z_1, z_2, z_3 \end{pmatrix}, \begin{pmatrix} best & z_4 \\ & z_1, z_3 \\ worst & z_2 \end{pmatrix}.$$

2. **(a)** The game is as follows (C means "contribute" and N means "Not contribute")

	Player 2: C			Player 2: N		
Player 1: C	2	2	2	6	8	6
Player 1: N	8	6	6	4	4	2

Player 3: C

	Player 2: C			Player 2: N		
Player 1: C	6	6	8	2	4	4
Player 1: N	4	2	4	4	4	4

Player 3: N

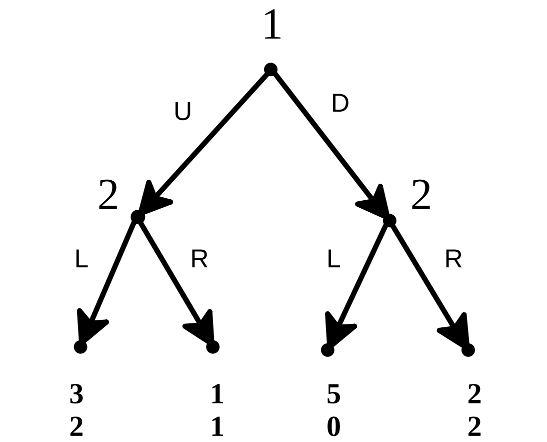
(b) There are four Nash equilibria, which are highlighted above: one where nobody contributes and three where exactly 2 people contribute.

(c) The equilibrium (N,N,N) is Pareto inefficient (it is Pareto dominated by any of the other equilibria), while the remaining equilibria are Pareto efficient.

(d) One equilibrium is where nobody contributes. All the other equilibria are those where exactly *k* people contribute. There is no equilibrium where more than *k* people contribute, because one of them can increase his payoff by switching to *N* (the party is still held and he saves money). There is no equilibrium where at least one, but less than *k*, people contribute, because any of the contributors can increase his payoff by switching to *N* (the party is not held in any case and he saves money).

(e) Of course we would have had the same set of Nash equilibria!

3. **(a)** The extensive game and corresponding strategic game are as follows:



Player 2

Player 1		LL	LR	RL	RR
	U	3, 2	3, 2	1, 1	1, 1
	D	5, 0	2, 2	5, 0	2, 2

(b) The pure-strategy Nash equilibria are: (U,LR) and (D,RR).

(c) For Player 2, RL is strictly dominated by LR. After deleting RL, there are no more strictly dominated strategies. Thus the output of IDSDS is

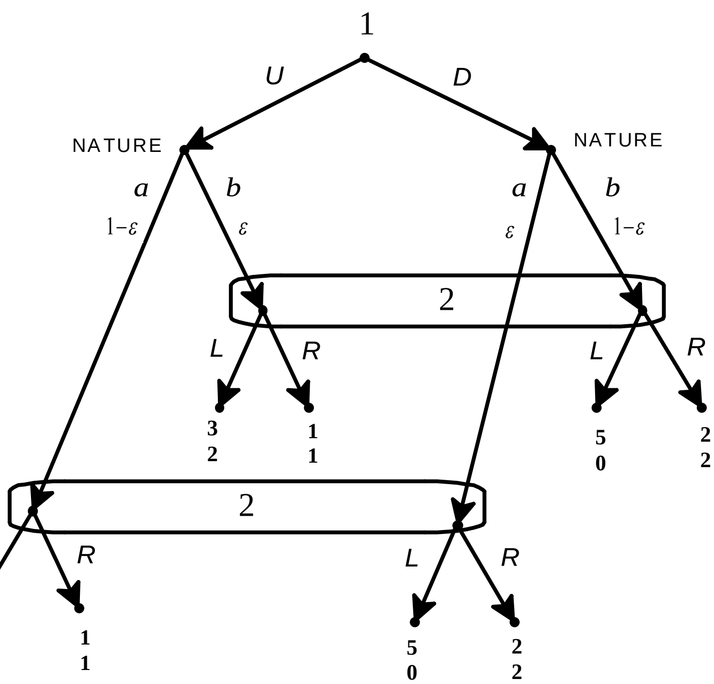
Player 2

Player 1		LL	LR	RR
	U	3, 2	3, 2	1, 1
	D	5, 0	2, 2	2, 2

(d) In the first round delete LL, RL and RR for Player 2 (all weakly dominated by LR) and in the second round delete D for Player 1. Thus the output of IDWDS is (U,LR).

(e) The backward-induction solution is (U,LR).

(f) The extensive-form game and corresponding strategic-form game are as follows (the first element of the strategy of Player 2 refers to her top information set in the figure):



		Player 2			
		LL	LR	RL	RR
Player 1	U	3, 2	1+2ε, 1+ε	3–2ε, 2–ε	1, 1
	D	5, 0	5–3ε, 2ε	2+3ε, 2–2ε	2, 2

(g) (D,RR) is the only pure-strategy Nash equilibrium for every value of ε. There is no other pure-strategy Nash equilibrium where Player 1 plays D, no matter what the value of ε (for every $0<\varepsilon<1$, $2>2\varepsilon$ and $2>2-2\varepsilon$). There is no pure-strategy Nash equilibrium where Player 1 plays *U*, because Player 2's best reply to *U* is *LL* and *U* is not a best reply to *LL*.

(h) There is a discontinuity at $\varepsilon=0$: along the sequence as long as $\varepsilon>0$ there is only one Nash equilibrium, namely (D,RR), but at $\varepsilon=0$ a second one appears, namely (U,RL).

(i) No. If Player 2 plays each of her pure strategies with probability ¼ then with *U* Player 1 gets a payoff of $\frac{3+1+2\varepsilon+3-2\varepsilon+1}{4}=2$ and with *D* he gets a payoff of $\frac{5+5-3\varepsilon+2+3\varepsilon+2}{4}=3.5$; thus *D* is the unique best reply, so that the mixed-strategy $\begin{pmatrix} U & D \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ is not a best reply.

(j) First of all, Player 1 must be indifferent between U and D, which requires $3q+(1+2\varepsilon)(1-q)=5q+(5-3\varepsilon)(1-q)$, that is, $\boxed{q=\frac{5\varepsilon-4}{5\varepsilon-2}}$ which is positive 1 if and only if $\boxed{\varepsilon>\frac{4}{5}}$.

Similarly, Player 2 must be indifferent between LL and LR, which requires $2p=(1+\varepsilon)p+2\varepsilon(1-p)$, that is, $\boxed{p=\frac{2\varepsilon}{1+\varepsilon}}$ (which is always strictly between 0 and 1, since $0<\varepsilon<1$). However, we must also check that Player 2 cannot to better with RL or RR. If she plays RR her payoff is $p+2(1-p)=2-p$ and this is not greater than 2*p* if and only if $\boxed{p\geq\frac{2}{3}}$. Note that $\frac{2\varepsilon}{1+\varepsilon}\geq\frac{2}{3}$ if and only if $\varepsilon\geq\frac{1}{2}$. If she plays RL her payoff is $(2-\varepsilon)p+(2-2\varepsilon)(1-p)=\varepsilon p$ which is always not greater than 2*p*. Thus the answer is: if $\boxed{\varepsilon>\frac{4}{5}}$ then $\left(\begin{array}{cc|cc} U & D & LL & LR \\ \frac{2\varepsilon}{1+\varepsilon} & \frac{1-\varepsilon}{1+\varepsilon} & \frac{5\varepsilon-4}{5\varepsilon-2} & \frac{2}{5\varepsilon-2} \end{array}\right)$ is a Nash equilibrium.