MIDTERM EXAM

ANSWER ALL QUESTIONS (total 100 points)

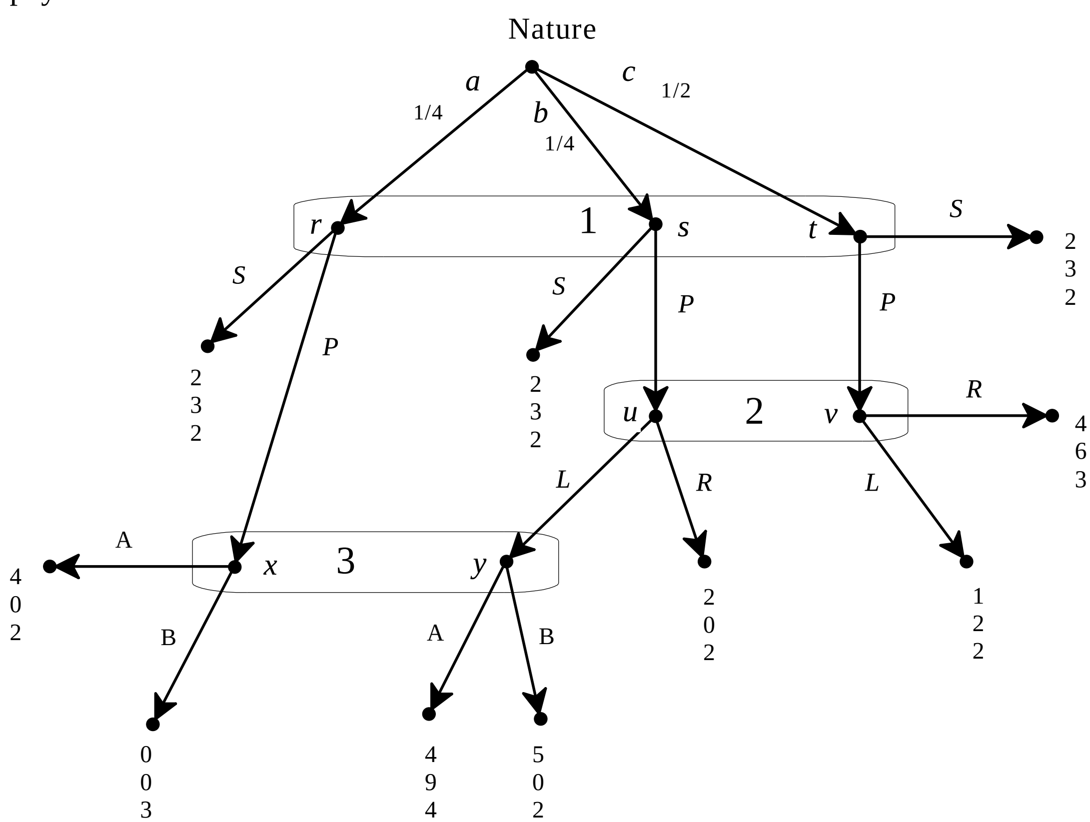
1. [20 points] Consider the strategic-form game with cardinal payoffs below, call it G, where, for every $i \in \{1,2\}$, $a_i - b_i - c_i + d_i \neq 0$. Assume that the parameters are such that G has a completely mixed Nash equilibrium σ (that is, every pure strategy in σ is played with positive probability) and there is no other completely mixed Nash equilibrium (although there may be other Nash equilibria where some pure strategies are played with zero probability).

		Player	2
		L	R
Player	T	a_1 , a_2	b_1 , b_2
1	B	c_1 , c_2	d_1 , d_2

(a) [10 points] Prove that Player 1 does not have a weakly dominant strategy.

(b) [10 points] Suppose we increase payoff entry a_1 in such a way that the sign of $a_1 - c_1$ does not change. Call the resulting game G'. Explain why G' must also have one and only one completely mixed Nash equilibrium.

2. [30 points]. Find all the pure-strategy weak sequential equilibria of the following game with cardinal payoffs.



3. [15 points] Consider Hotelling's model: two firms located on a street of length 1, zero production costs, N consumers (each with an infinite reservation price) uniformly distributed along the street facing a linear transportation cost of αd where α is a positive constant and d is distance. We saw in class that if the firms are "not too close" then there exists a Nash equilibrium in prices, while if the firms are "too close" then there is no Nash equilibrium. Restricting attention to symmetric locations (so that $x_1 = 1 - x_2$) find the ***smallest*** distance between the two firms which is such that there exists a Nash equilibrium in prices.

4. [35 points] Consider a homogeneous-product industry facing the following inverse demand function: Q = 83 − P. Initially there are three firms in this industry. The cost function of firm $i \in \{1,2,3\}$ is denoted by $C_i(q_i)$. We will consider a number of different alternatives.

(a) **(a.1)** [9 points] Suppose that all the firms have the same cost function given by $C(q) = 3q$. Find the Cournot-Nash equilibrium and the corresponding profits.
(a.2) [6 points] Firms 2 and 3 decide to merge, thus turning the industry into a duopoly. Find the Cournot-Nash equilibrium of the industry after the merger.
(a.3) [2 points] Was the merger profitable?

(b) Now suppose that in the initial situation where there were three firms, the cost functions were as follows: $C_1(q_1) = 3q_1$, $C_2(q_2) = 3q_2$ and $C_3(q_3) = \left(\frac{q_3}{3}\right)^2$. You don't need to compute the Cournot-Nash equilibrium of the three-firm industry. Assume that Firms 2 and 3 merge and the merged firm is free to use either only one or both of the production facilities of the two firms.
(b.1) [6 points] Write the cost function and the profit function of the merged firm.
(b.2) [6 points] Suppose that in the after-merger industry firm 1 produces 20 units and the merged firm produces 40 units. What are the profits of the two firms?
(b.3) [6 points] Is the situation described under (b.2) a Cournot-Nash equilibrium?

1. **(a)** There are two ways of proving this.

Method 1. Suppose that T weakly dominates B; then either $a_1 > c_1$ and $b_1 \geq d_1$ (Case 1) or $a_1 \geq c_1$ and $b_1 > d_1$ (Case 2). Let q (with $0 < q < 1$) be the probability with which Player 2 plays L at the completely mixed-strategy equilibrium. Then in both Cases 1 and 2 $a_1 q + b_1(1-q) > c_1 q + d_1(1-q)$, so that T is strictly better than B (and thus Player 1 is not indifferent between T and B). The proof for the case where B weakly dominates T is similar (reverse the inequalities).

Method 2. As shown below, $\text{sgn}(a_1 - c_1) = \text{sgn}(d_1 - b_1)$ so that if $a_1 > c_1$ then $d_1 > b_1$ and if $a_1 < c_1$ then $d_1 < b_1$.

(b) Let $q = \sigma_2(L)$ be the probability with which Player 2 plays L at the completely mixed equilibrium. Then Player 1 must be indifferent between playing T and playing B, that is, it must be that $qa_1 + (1-q)b_1 = qc_1 + (1-q)d_1$. Solving for q we get $\sigma_2(L) = \dfrac{d_1 - b_1}{(a_1 - c_1) + (d_1 - b_1)}$. Since $0 < \sigma_2(L) < 1$, $\dfrac{1}{\sigma_2(L)} > 1$, that is, $\dfrac{(a_1 - c_1) + (d_1 - b_1)}{d_1 - b_1} = \dfrac{a_1 - c_1}{d_1 - b_1} + 1 > 1$. Hence it must be that $\dfrac{a_1 - c_1}{d_1 - b_1} > 0$, implying that $\text{sgn}(a_1 - c_1) = \text{sgn}(d_1 - b_1)$.

2. First let's see if there is a weak sequential equilibrium where Player 1's strategy is P. Then Player 2's beliefs must be $\begin{pmatrix} u & v \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$. Suppose that Player 2's strategy is R. Then Player 3's beliefs must be $\begin{pmatrix} x & y \\ 1 & 0 \end{pmatrix}$, making B the only rational choice. Then Player 2's expected payoffs are: $L \to \frac{1}{3}0 + \frac{2}{3}2 = \frac{4}{3}$ and $R \to \frac{1}{3}0 + \frac{2}{3}6 = 4$. Hence R is indeed rational. Thus we only need to check the rationality of playing P for Player 1: $S \to 2$, $P \to \frac{1}{4}0 + \frac{1}{4}2 + \frac{1}{2}4 = 2.5$. Hence P is indeed rational. Thus we have found one weak sequential equilibrium: $\left((P,R,B), \left(\begin{array}{ccc|cc|cc} r & s & t & u & v & x & y \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{3} & \frac{2}{3} & 1 & 0 \end{array} \right) \right)$.

Now, let's see if there is another weak sequential equilibrium where Player 1's strategy is P. As before, Player 2's beliefs must be $\begin{pmatrix} u & v \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$. Suppose that Player 2's strategy is L. Then Player 3's beliefs must be $\begin{pmatrix} x & y \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$, so that $A \to \frac{1}{2}2 + \frac{1}{2}4 = 3$ and $B \to \frac{1}{2}3 + \frac{1}{2}2 = 2.5$; hence the rational choice is A. Then Then Player 2's expected payoffs are: $L \to \frac{1}{3}9 + \frac{2}{3}2 = \frac{13}{3}$ and $R \to \frac{1}{3}0 + \frac{2}{3}6 = 4$. Hence L is indeed rational. Thus we only need to check the rationality of playing P for Player 1: $S \to 2$, $P \to \frac{1}{4}4 + \frac{1}{4}4 + \frac{1}{2}1 = 2.5$.

Hence P is indeed rational. Thus we have found one weak sequential equilibrium: $\left((P,L,A),\begin{pmatrix} r & s & t & u & v & x & y \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{3} & \frac{2}{3} & \frac{1}{2} & \frac{1}{2}\end{pmatrix}\right)$.

Next we see if there are weak sequential equilibria where Player 1's strategy is *S*. Since Player 1's expected payoff with *S* is 2, he must not get more than 2 with *P*. Previous calculations show that with *P* player 1 gets more than 2 if the strategies of Players 2 and 3 are (*R,B*) or (*L,A*). Thus we need to check (*R,A*) and (*L,B*). With (*R,A*), $P \to \frac{1}{4}4+\frac{1}{4}2+\frac{1}{2}4=3.5$ so it doesn't work. With (*L,B*), $P \to \frac{1}{4}0+\frac{1}{4}5+\frac{1}{2}1=1.25$; thus (*S,L,B*) could be part of a weak sequential equilibrium. We need to augment it with beliefs that rationalize *L* and *B*. Since the information sets of Players 2 and 3 are not reached, any beliefs are allowed there by Bayes' rule. We need $\begin{pmatrix} x & y \\ p & 1-p\end{pmatrix}$ to be such that $2p+4(1-p)\le 3p+2(1-p)$, that is, $p\ge\frac{2}{3}$, and we need $\begin{pmatrix} u & v \\ q & 1-q\end{pmatrix}$ to be such that $0q+2(1-q)\le 0q+6(1-q)$, that is, $q=1$. Hence the following are weak sequential equilibria for every $p\in\left[\frac{2}{3},1\right]$: $\left((S,L,B),\begin{pmatrix} r & s & t & u & v & x & y \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 1 & 0 & p & 1-p\end{pmatrix}\right)$.

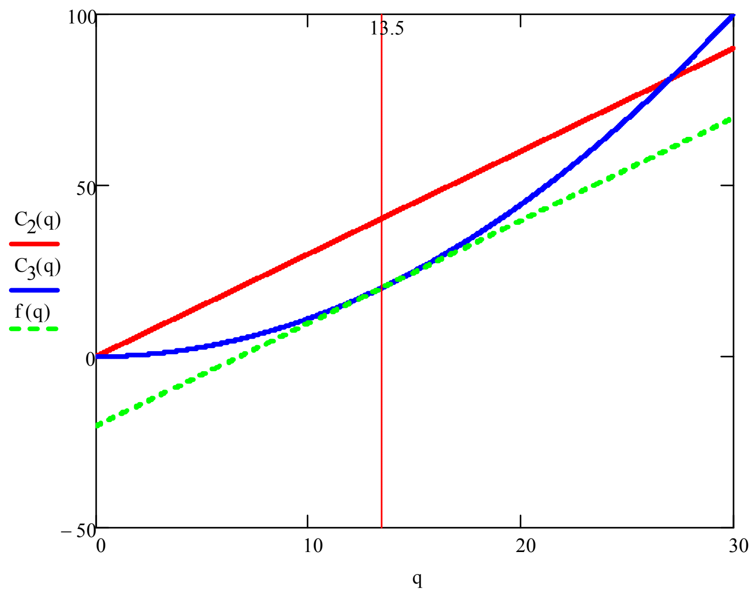
In conclusion the following are pure-strategy weak sequential equilibria:

- $\left((P,R,B),\begin{pmatrix} r & s & t & u & v & x & y \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{3} & \frac{2}{3} & 1 & 0\end{pmatrix}\right)$
- $\left((P,L,A),\begin{pmatrix} r & s & t & u & v & x & y \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & \frac{1}{3} & \frac{2}{3} & \frac{1}{2} & \frac{1}{2}\end{pmatrix}\right)$
- $\left((S,L,B),\begin{pmatrix} r & s & t & u & v & x & y \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} & 1 & 0 & p & 1-p\end{pmatrix}\right)$ for every $p\in\left[\frac{2}{3},1\right]$.

3. Let x be the location of firm 1 (so that the location of firm 2 is $1-x$). Then the indifferent consumer is located at $z=\frac{1}{2}+\frac{p_2-p_1}{2\alpha}$. Demand for firm 1 is $D_1=zN$ and demand for firm 2 is $D_2=(1-z)N$. The profit functions are $\Pi_1=p_1D_1$ and $\Pi_2=p_2D_2$. Solving $\frac{\partial\Pi_1}{\partial p_1}=0$ and $\frac{\partial\Pi_2}{\partial p_2}=0$ we get $p_1^*=p_2^*=\alpha$ with corresponding profits $\Pi_1^*=\Pi_2^*=\frac{\alpha}{2}N$. For this to be a Nash equilibrium it must be the case that, given that firm 2 charges a price equal to α, firm 1 cannot increase its profits by charging a price at which it captures the whole market (and the symmetric condition for firm 2). The price for firm 1 that captures the entire market is a price slightly less than the price that makes the consumer on top of firm 2 indifferent between the two firms. This price is found by solving $p_1+\alpha(1-x-x)=\alpha$ which gives $p_1=2\alpha x$. A price slightly less than this gives firm 1 the entire market with profits slightly less than $2\alpha xN$. Thus we need $\frac{\alpha}{2}N\ge 2\alpha xN$, that is, $x\le\frac{1}{4}$. Hence there is a Nash equilibrium in prices as long as the distance between the two firms is at least $\frac{3}{4}-\frac{1}{4}=\frac{1}{2}$.

4. **(a.1)** The profit function of firm i is $\Pi_i = q_i(83 - q_1 - q_2 - q_3) - 3q_i$. The Cournot-Nash equilibrium is given by $q_1 = q_2 = q_3 = 20$. Each firm makes a profit of 400. **(a.2)** The profit function of the merged firm is $\Pi_m = q_m(83 - q_1 - q_m) - 3q_m$. The Cournot-Nash equilibrium is given by $q_1 = q_m = \frac{80}{3} = 26.67$. The profit of each firm in the industry is 711.11. Since 711.11 < 2(400), the merger was not profitable.

(b.1) The merged firm is allowed to divide its total output freely between the two facilities. Let q_2 denote the total output of the merged firm, x the amount produced in the facility with cost function $C(x) = 3x$ and $q_2 - x$ the amount produced in the facility with cost function $C(y) = \left(\frac{y}{3}\right)^2$. Then, if the firm wants to produce q_2 units, it will choose x to $\underset{x}{Max}\left[3x + \left(\frac{q_2 - x}{3}\right)^2\right]$. The FOC is $3 - \frac{2}{9}(q_2 - x) = 0$ whose solution is $x = q_2 - \frac{27}{2} = q_2 - 13.5$. Hence the cost function of the merged firm is $C_2(q_2) = \begin{cases} \left(\frac{q_2}{3}\right)^2 & \text{if } q_2 \le 13.5 \\ \left(\frac{13.5}{3}\right)^2 + 3(q_2 - 13.5) & \text{if } q_2 > 13.5 \end{cases}$ and the profit function is $\Pi_2(q_1, q_2) = q_2(83 - q_1 - q_2) - C_2(q_2)$. At output level 13.5 marginal cost is equalized between the two cost functions $C(x) = 3x$ and $C(y) = \left(\frac{y}{3}\right)^2$, as the following figure shows:



(b.2) The profit function of firm 1 is $\Pi_1(q_1, q_2) = q_1(83 - q_1 - q_2) - 3q_1$. $\Pi_1(20, 40) = 400$. The profit of the merged firm is: $\Pi_2(20, 40) = 820.25$.

(b.3) While $q_1 = 20$ is a best reply to $q_2 = 40$ for Firm 1, $q_2 = 40$ is not a best reply to $q_1 = 20$ for the merged firm. In fact, given $q_1 = 20$, in the range where $q_2 > 13.5$ the profit function of the merged firm is given by the expression $q_2(83 - 20 - q_2) - \left(\frac{13.5}{3}\right)^2 - 3(q_2 - 13.5)$ which is maximized at $q_2 = 30$. Hence (20,40) is not a Nash equilibrium.

FINAL EXAM
ANSWER ALL QUESTIONS (total 100 points)

1. [40 points] Consider the following one-sided incomplete information situation. Two players must decide whether to make a costly investment. They must make their choices simultaneously. For each player there are two possible states of the world: good and bad. If a player invests in a bad state for her, she will lose \$$c$ million for sure. If a player invests in a good state for her, she will make a net profit of \$1 million, but only if the other player also invests (there are strategic links between their investments), otherwise she will lose \$$c$ million. (Thus if only one of the players invests, that player will lose \$$c$ regardless of the state.) If either player decides not to invest, her profits are zero. It is common knowledge between the players that the state is good for Player 2. It is also common knowledge that Player 1 knows whether the state is good or bad for her, while Player 2 does not have this information. Player 2 believes that Player 1 is facing a good state with probability p and this belief is also common knowledge. To summarize, if a player's state is good, her von Neumann-Morgenstern payoffs are given by the following matrix (the row represents a player's action and the column represents her opponent's action):

Good State	If the other player invests	If the other player does Not invest
Invest	1	$-c$
Not Invest	0	0

where c > 0. On the other hand, if a player's state is bad, her von Neumann-Morgenstern payoffs are given by the following matrix.

Bad state	If the other player invests	If the other player does Not invest
Invest	$-c$	$-c$
Not Invest	0	0

(a) [10 points] Represent this situation using a set of states, information partitions and probability distributions.

(b) [10 points] Apply the Harsanyi transformation to represent the situation as an extensive-form game.

(c) [10 points] Write down the strategic form corresponding to the extensive-form game of part (b). Let Player 1 be the row player.

(d) [10 points] Find the pure strategy Bayesian Nash equilibria of this game with incomplete information.

2. [60 points] Consider an individual whose von Neumann-Morgenstern utility-of-wealth function is

$$U(m) = \begin{cases} \sqrt{m} & \text{if she exerts no effort} \\ \sqrt{m} - c & \text{if she exerts effort} \end{cases} \quad \text{with } c > 0.$$

The individual has an initial wealth of w_0 and faces a potential loss of ℓ with $0 < \ell < w_0$. The probability of her incurring a loss is p_e if she exerts effort and p_n if she chooses no effort, with $0 < p_e < p_n < 1$.

(a) [2 points] Suppose that insurance is not available. For what values of *c* will she choose to exert effort? [Assume that, if indifferent, she decides **not** to exert effort.]

(b) [4 points] In a diagram where on the horizontal axis you measure wealth in the bad state (W_1) and on the vertical axis wealth in the good state (W_2) sketch the indifference curves that go through the no-insurance point (*NI*) (one corresponding to effort and the other to no effort).

(c) [4 points] For the case where $w_0 = 2{,}500$, $\ell = 1{,}600$, $p_e = \frac{1}{20}$, $p_n = \frac{1}{10}$ calculate the slopes of the two curves of part (b) at the *NI* point.

From now on assume that *c* belongs to the range found in part (a), that the insurance industry is a monopoly and that the monopolist knows $U(m), w_0, \ell, c, p_e$ and p_n. Furthermore, assume that, if indifferent between insuring and not insuring, the individual chooses to insure.

(d) [8 points] Suppose first that effort is observable and verifiable and can thus be specified in the contract. **(d.1)** Among the contracts that require the individual to exert effort, which contract maximizes the monopolist's profit? **(d.2)** Among the contracts that require the individual *not* to exert effort, which contract maximizes the monopolist's profit? **(d.3)** Calculate the maximum expected profit for the monopolist for the case where $w_0 = 2{,}500$, $\ell = 1{,}600$, $p_e = \frac{1}{20}$, $p_n = \frac{1}{10}$, $c = \frac{15}{16}$. **(d.4)** For the case where $w_0 = 2{,}500$, $\ell = 1{,}600$, , $p_e = \frac{1}{4}$, $p_n = \frac{1}{2}$, $c = \frac{75}{16}$ calculate the maximum expected profit for the monopolist.

From now on suppose that effort is **not** observable and thus cannot be made part of the contract. Describe an insurance contract as a pair (h, d) where *h* is the premium and *d* is the deductible. Whatever decision the individual makes concerning insurance, she will then choose whether or not to exert effort (of course, this decision is made prior to the time when the state, good or bad, is realized). Let $EU_e(h,d)$ be the individual's expected utility if she purchases contract (h,d) and exerts effort and $EU_n(h,d)$ be her expected utility if she purchases contract (h,d) and exerts no effort and let $\Delta(h,d) = EU_e(h,d) - EU_n(h,d)$.

(e) [8 points] **(e.1)** Are there contracts (h,d) such that $\Delta(h,d) < 0$? **(e.2)** Let (h_A, d_A) be the contract that makes the individual indifferent between (1) not insuring and (2) purchasing contract (h_A, d_A) and exerting no effort and is such that $\Delta(h,d) = 0$. Show where contract (h_A, d_A) lies in the (W_1, W_2)-diagram of part (b) [draw a new diagram]. **(e.3)** For the case where $w_0 = 2{,}500$, $\ell = 1{,}600$, $p_e = \frac{1}{20}$, $p_n = \frac{1}{10}$, $c = \frac{15}{16}$ write the equations whose solution gives contract (h_A, d_A).

(f) [8 points] In the (W_1, W_2)-diagram show the set of contracts that the individual considers just as good as no insurance.

(g) [8 points] Of all the contracts that belong to the set of part (f), which is the profit-maximizing one?

(h) [8 points] For the case where $w_0 = 2{,}500$, $\ell = 1{,}600$, $p_e = \frac{1}{20}$, $p_n = \frac{1}{10}$, $c = \frac{15}{16}$ where the contract (h_A, d_A) of part (e.2) is given by $\left(\frac{1599}{256}, \frac{48675}{32}\right)$, find the profit-maximizing contract.

(i) [10 points] Suppose now that *e* and *n* are not chosen by the individual but they are innate characteristics of individuals: those who have the *e* gene are drawn by nature to exert effort and those who have the *n* gene are compelled by nature to avoid effort. The monopolist cannot tell individuals apart. Let *N* be the total number of individuals, of whom N_e have the *e* gene and N_n have the *n* gene (thus $N_e + N_n = N$). Each individual knows her own type. Describe how the monopolist maximizes its profits in this case and use a diagram to illustrate.

University of California, Davis -- Department of Economics

SPRING 2016 **ECN/ARE 200C: MICROECONOMIC THEORY** Giacomo Bonanno

Final Exam $\boxed{\text{ANSWERS}}$

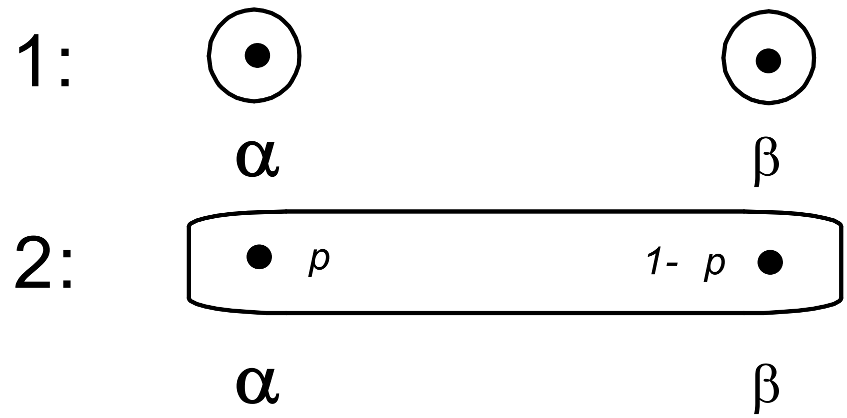
1. **(a)** I means "invest", N means "not invest". State α is good for both players and state β is bad for Player 1 and good for Player 2.

State α:

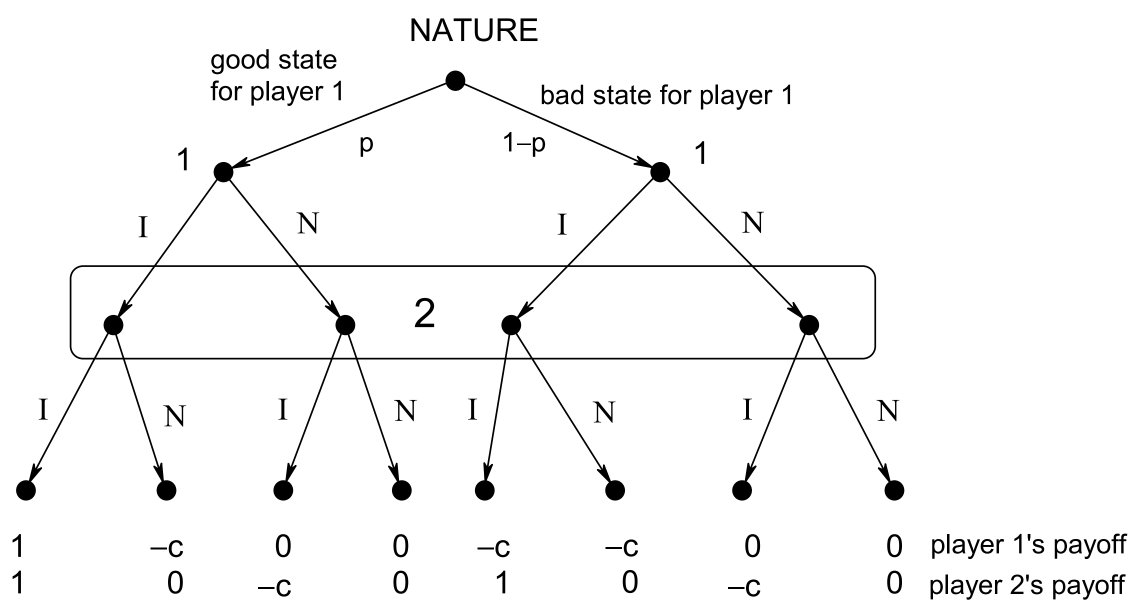
player 1 \ player 2	I	N
I	1 , 1	−c , 0
N	0 , −c	0 , 0

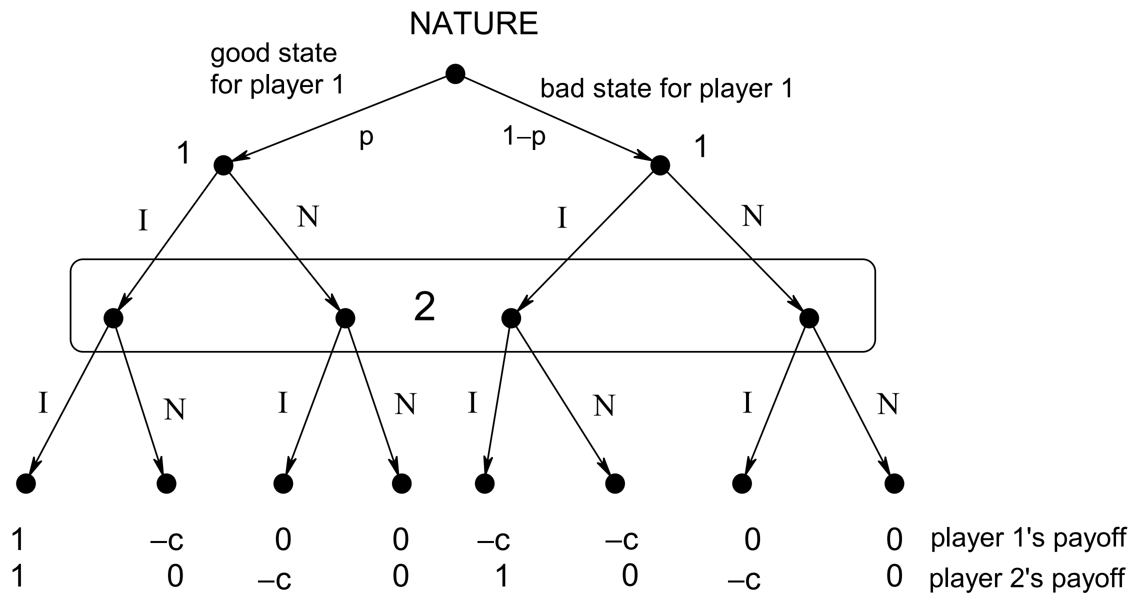
State β:

player 1 \ player 2	I	N
I	−c , 1	−c , 0
N	0 , −c	0 , 0



(b) The extensive game is as follows:





I = invest
N = not invest

(c) The corresponding normal form is:

		Player 2 Invest	Not invest
Player 1	Invest always	$p-c(1-p)$, 1	$-c$,0
	Never invest	0 , $-c$	0 , 0
	If good invest. if bad do not	p , $p-c(1-p)$	$-cp$, 0
	If good no inv. if bad invest	$-c(1-p)$, $1-p-cp$	$-c(1-p)$, 0

(d) (Never invest, Not invest) is always a Bayesian equilibrium, that is, for all the parameter values. This is the unique pure-strategy Bayesian equilibrium if $p < \frac{c}{1+c}$ (that is, if $p-c(1-p) < 0$). If $p \geq \frac{c}{1+c}$ (that is, if $p-c(1-p) \geq 0$) then there is another pure-strategy Bayesian equilibrium, namely (If state is good invest, if state is bad do not invest; Invest).

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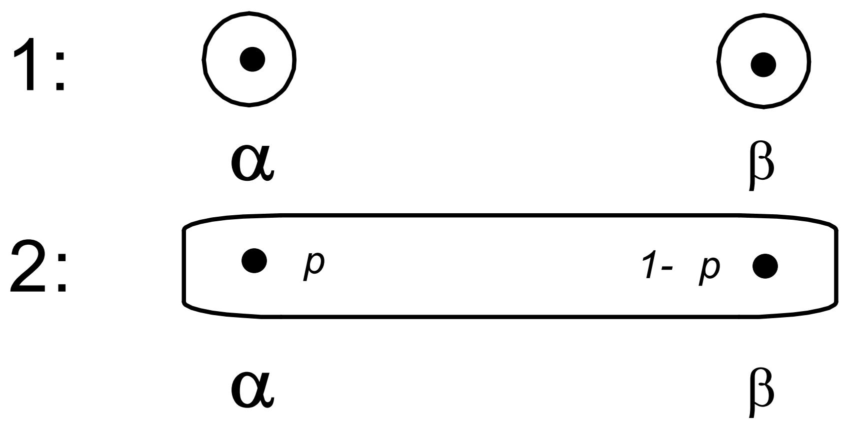
Final Exam $\boxed{\text{ANSWERS}}$

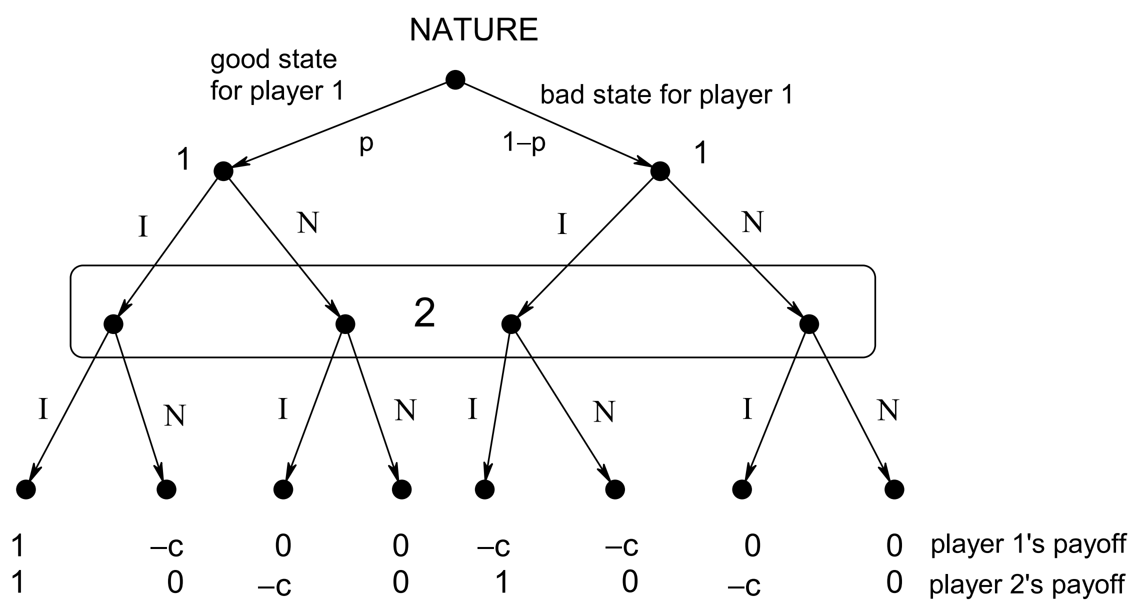
1. **(a)** I means "invest", N means "not invest". State α is good for both players and state β is bad for Player 1 and good for Player 2.

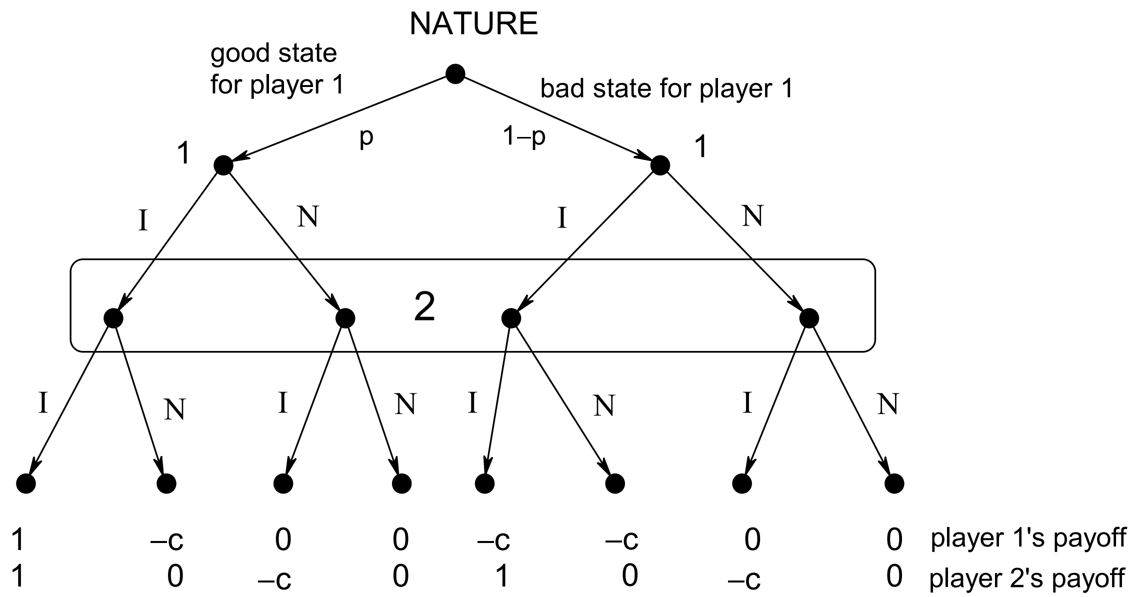
		player 2	
		I	N
player 1	I	1 , 1	−c , 0
	N	0 , −c	0 , 0

		player 2	
		I	N
player 1	I	−c , 1	−c , 0
	N	0 , −c	0 , 0



(b) The extensive game is as follows:





I = invest
N = not invest

(c) The corresponding normal form is:

Player 2

Player 1		Invest	Not invest
	Invest always	$p-c(1-p)$, 1	$-c$,0
	Never invest	0 , $-c$	0 , 0
	If good invest. if bad do not	p , $p-c(1-p)$	$-cp$, 0
	If good no inv. if bad invest	$-c(1-p)$, $1-p-cp$	$-c(1-p)$, 0

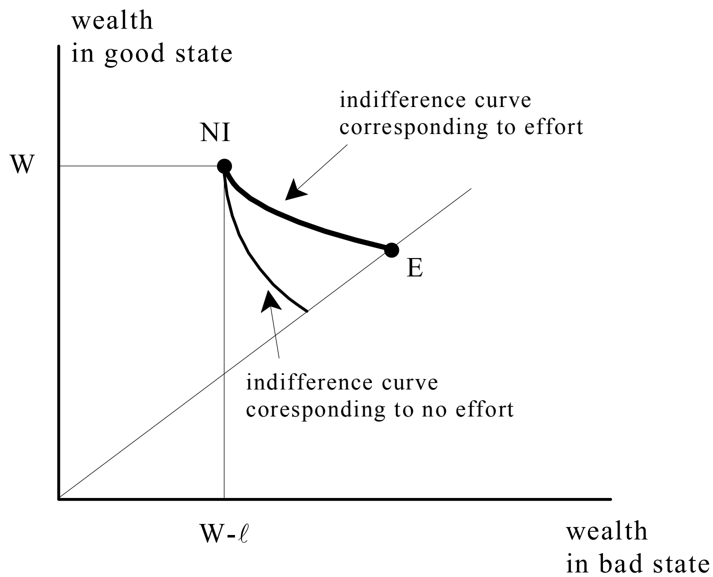
(d) (Never invest, Not invest) is always a Bayesian equilibrium, that is, for all the parameter values. This is the unique pure-strategy Bayesian equilibrium if $p < \frac{c}{1+c}$ (that is, if $\mathrm{p-c(1-p)} < 0$). If $p \geq \frac{c}{1+c}$ (that is, if $\mathrm{p-c(1-p)} \geq 0$) then there is another pure-strategy Bayesian equilibrium, namely (If state is good invest, if state is bad do not invest; Invest).

(e) There was a problem with this question, so everybody will get 8 points for free.

2.

(a) Let $EU_n(NI) = p_n\sqrt{w_0 - \ell} + (1-p_n)\sqrt{w_0}$ be her expected utility if she has no insurance and chooses no effort and let $EU_e(NI) = p_e\sqrt{w_0 - \ell} + (1-p_e)\sqrt{w_0} - c$ be her expected utility if she has no insurance and chooses to exert effort. Then it must be that $c < \left(p_e\sqrt{w_0 - \ell} + (1-p_e)\sqrt{w_0}\right) - \left(p_n\sqrt{w_0 - \ell} + (1-p_n)\sqrt{w_0}\right)$, that is, $\boxed{c < (p_n - p_e)\left(\sqrt{w_0} - \sqrt{w_0 - \ell}\right)}$.

(b)

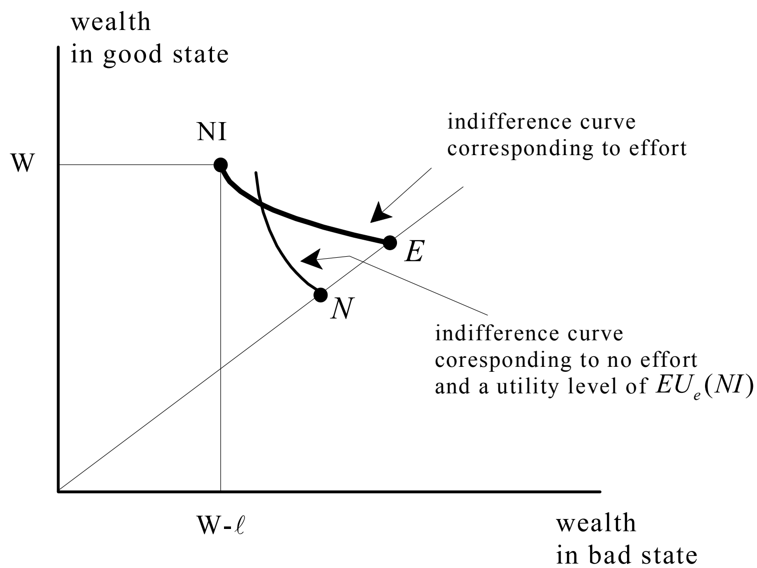


(c) The slope of the e-indifference curve at NI is $-\frac{p_e}{1-p_e}\left(\frac{U'(w_0-\ell)}{U'(w_0)}\right) = -\frac{1}{19}\left(\frac{\frac{1}{2\sqrt{900}}}{\frac{1}{2\sqrt{2,500}}}\right) = -\frac{5}{57} = -0.0877$

and the slope of the n-indifference curve is $-\frac{p_n}{1-p_n}\left(\frac{U'(w_0-\ell)}{U'(w_0)}\right) = -\frac{1}{9}\left(\frac{\frac{1}{2\sqrt{900}}}{\frac{1}{2\sqrt{2,500}}}\right) = -\frac{5}{27} = -0.1852$.

(d) (d.1) The full-insurance contract shown as point E in the above diagram (and the diagram below).

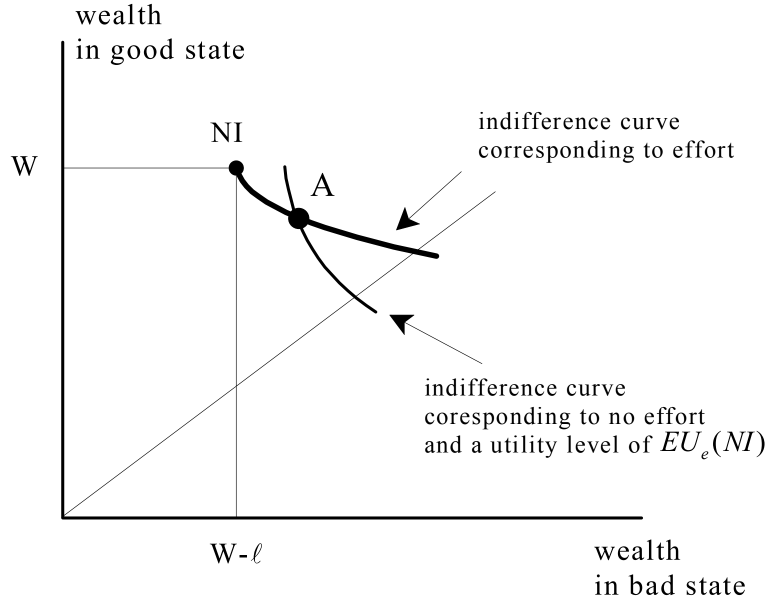
(d.2) The full-insurance contract shown as point N in the diagram below: it is the full-insurance contract on the no-effort indifference curve corresponding to a utility level equal to $EU_e(NI)$ (given the assumption about the range of values of c, if the individual does not to insure then she chooses to exert effort and thus her reservation utility is $EU_e(NI)$). If the monopolist were to offer the contract given by the intersection of the indifference curve corresponding to no effort that goes through the NI point and the 45^0 line, the consumer would reject it (because he can get higher utility by choosing NI end effort) and thus the monopolist's profits would be zero.



(d.3) In this case $EU_n(NI) = 48$ and $EU_e(NI) = 48.0625$. Thus the contract of part (d.1) (point *E* in the diagram) is given by the solution to $\sqrt{2,500-h} - c = 48.0625$ which is $h = 99$. The monopolist's expected profit from this contract is $99 - \frac{1}{20}(1,600) = 19$. The contract of part (d.2) (point *N* in the diagram) is given by the solution to $\sqrt{2,500-h} = 48.0625$ which is $h = 189.996$. Thus the monopolist's expected profit from this contract is $189.996 - \frac{1}{10}(1,600) = 29.996$. Hence in this case the monopolist would offer a contract that requires the individual **not** to exert effort and pay a premium of \$189.996 for full insurance.

(d.4) In this case $EU_n(NI) = 40$ and $EU_e(NI) = 40.3125$. Thus the contract of part (d.1) (corresponding to point *E* in the above diagram) is given by the solution to $\sqrt{2,500-h} - c = 40.3125$ which is $h = 475$. Thus the monopolist's expected profit from this contract is $475 - \frac{1}{4}(1,600) = 75$. The contract of part (d.2) (corresponding to point *N* in the above diagram) is given by the solution to $\sqrt{2,500-h} = 40.3125$ which is $h = 874.9$. Thus the monopolist's expected profit from this contract is $874.9 - \frac{1}{2}(1,600) = 74.9$. Thus in this case the monopolist would offer a contract that requires the individual to exert effort (and pay a premium of \$475 for full insurance).
[There is a trade-off between requiring effort and no effort: no effort is good for the monopolist because the probability of having to cover the loss is lower, but at the same time the individual faces a lower risk and therefore is willing to pay less for full insurance.]

(e) **(e.1)** Yes: all the full-insurance contracts, that is, all contracts of the form $(h,0)$. When fully insured the individual has no incentive to exert effort. **(e.2)** We are looking for a contract (h_A, d_A) such that $\Delta(h_A, d_A) = 0$ [that is, $EU_e(h_A, d_A) = EU_n(NI)$]. [Note the condition $\Delta(h_A, d_A) = 0$ was not mentioned in the question, so if you chose any other contract(s) on the steeper indifference curve through point A in the figure below, then your answer is correct.] We can think of *NI* as the contract $(0, \ell)$. By hypothesis $EU_e(NI) > EU_n(NI)$ and thus $\Delta(0, \ell) > 0$. On the other hand at contract $(h_E, 0)$, corresponding to point $E = (w_0 - h_E, w_0 - h_E)$ at the intersection of the 45° line and the *n*-indifference curve, $\Delta(h_E, 0) < 0$. Thus there must be a contract (h_A, d_A), corresponding to point $A = (w_0 - h_A - d_A, w_0 - h_A)$ on the n-indifference curve, such that $\Delta(h_A, d_A) = 0$:



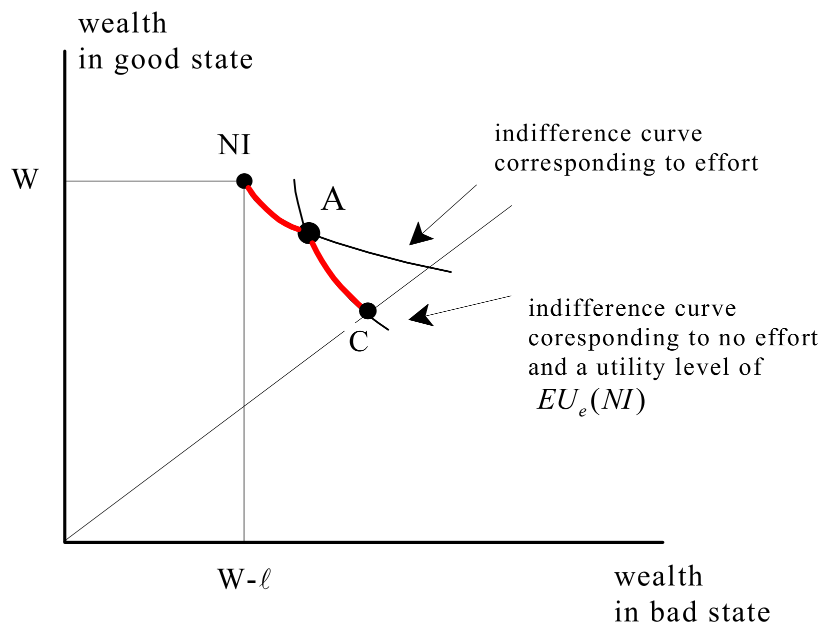
(e.3) $\frac{1}{20}\sqrt{2,500-h-d} + \frac{19}{20}\sqrt{2,500-h} - \frac{15}{16} = \frac{1}{10}\sqrt{2,500-h-d} + \frac{9}{10}\sqrt{2,500-h}$

(this equation says that accepting contract A and exerting effort gives the same utility as not insuring and not exerting effort)

$$\frac{1}{20}\sqrt{2,500-h-d} + \frac{19}{20}\sqrt{2,500-h} - \frac{15}{16} = \frac{1}{20}\sqrt{900} + \frac{19}{20}\sqrt{2,500} - \frac{15}{16}$$

(this equation says that contract A lies on the reservation-indifference-curve-with –effort).

(f) It is the union of the portion of the e-indifference curve through NI up to point A and the portion of the n-indifference curve through A from there on, shown as the thick kinked curve in the figure below:



(g) The monopolist would only consider either offering contract A or contract C and will chose the one of the two that yields higher profits.

(h) The profit from contract $\left(\frac{1599}{256}, \frac{48675}{32}\right)$ is given by $\frac{1,599}{256} - \frac{1}{20}\left(1,600 - \frac{48,675}{32}\right) = \frac{589}{256} = 2.301$ (using the assumption that, if indifferent, the individual chooses to insure). Contract C is given by the solution to $\sqrt{2,500-h} = EU_e(NI)$ (where $EU_e(NI) = 48.0625$), which is $h = \frac{48,639}{256} = 189.996$, with corresponding profits of $\frac{48,639}{256} - \frac{1}{10}1,600 = \frac{7,679}{256} = 29.996$. Thus the monopolist would offer contract C.

(i) This is the standard two-type situation. The L types are those with the e gene.