University of California, Davis - Department of Economics

SPRING 2020 **ECN / ARE 200C: MICROECONOMIC THEORY** Professor Giacomo Bonanno

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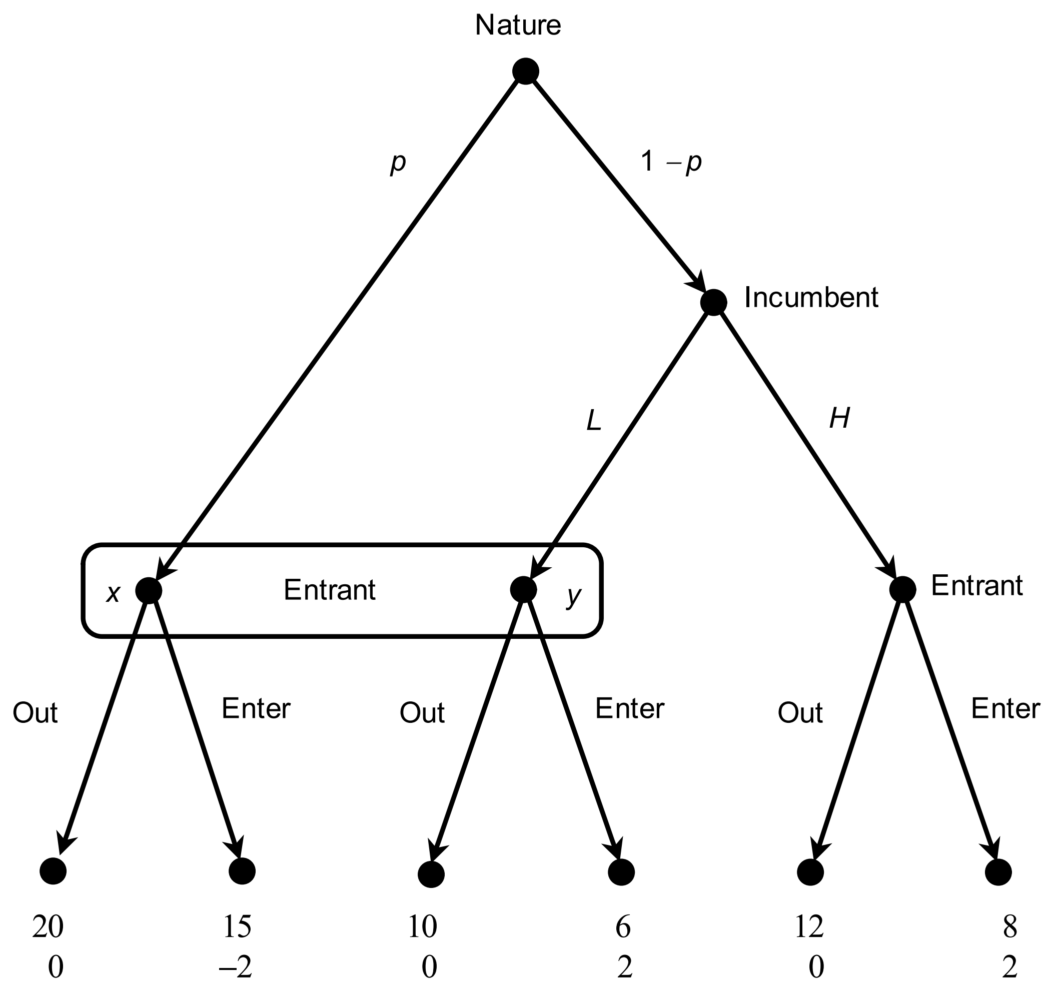
**MIDTERM EXAM** (total 100 points)

**Part 1** is a 40-point online Quiz in Canvas.

**Part 2.** [60 points] Consider the following game, where the top number is the Incumbent's payoff and the bottom number is the Entrant's payoff (all are von Neumann-Morgenstern payoffs). Assume that $0 < p < 1$ but $p \neq \frac{1}{2}$.




**(a)** [20 points] Write the corresponding strategic form.

**(b)** [20 points] Find all the subgame-perfect equilibria (including the mixed-strategy ones, if any).

**(c)** [20 points] For each subgame-perfect equilibrium determine if it is part of a weak sequential equilibrium .

**(a)**

ENTRANT

| INCUMBENT | Enter always | Out always | If H enter, if L out | If H out, if L enter |
|---|---|---|---|---|
| H | $(1-p)8+p15$ , $(1-p)2+p(-2)$ | $(1-p)12+p20$ , 0 | $(1-p)8+p20$ , $(1-p)2+p0$ | $(1-p)12+p15$ , $(1-p)0+p(-2)$ |
| L | $(1-p)6+p15$ , $(1-p)2+p(-2)$ | $(1-p)10+p20$ , 0 | $(1-p)10+p20$ , 0 | $(1-p)6+p15$ , $(1-p)2+p(-2)$ |

Simplifying:

ENTRANT

| INC | Enter always | Out always | If H enter, if L out | If H out, if L enter |
|---|---|---|---|---|
| H | $8+7p$, $2-4p$ | $12+8p$, 0 | $8+12p$, $2-2p$ | $12+3p$, $-2p$ |
| L | $6+9p$, $2-4p$ | $10+10p$, 0 | $10+10p$, 0 | $6+9p$, $2-4p$ |

**(b)** At a subgame-perfect equilibrium, the entrant enters when the incumbent Chooses $H$ (there is only one proper subgame which starts at the singleton node of the Entrant). Thus if we look for a subgame-perfect equilibrium we can delete the second and fourth columns. Thus we only need to look at the following matrix:

ENTRANT

| INCUMBENT | Enter always | If H enter, if L out |
|---|---|---|
| H | $8+7p$, $2–4p$ | $8+12p$, $2–2p$ |
| L | $6+9p$, $2–4p$ | $10+10p$, 0 |

Since $0 < p < 1$, the following inequalities hold:

$$8+7p > 6+9p, \qquad 10+10p > 8+12p, \qquad 2-2p > 2-4p$$

Thus the only candidate for a pure-strategy Nash equilibrium is (L, if H enter/if L out). We need to consider two cases.

**CASE 1**: if $0 > 2-4p$, that is, if $p > \frac{1}{2}$ (recall the assumption that $p \neq \frac{1}{2}$), then “If H enter, if L out” is a strictly dominant strategy for the Entrant and there is a unique pure-strategy Nash equilibrium given by (L, if H enter/if L out).

**CASE 2**: If $0 < 2-4p$, that is, $p < \frac{1}{2}$ then there is no pure-strategy Nash equilibrium. To find the mixed-strategy equilibrium, let $r$ be the probability with which the Incumbent chooses H. Then it must be that the Entrant is indifferent between her two strategies, that is $2-4p = (2-2p)r$ which yields $r = \dfrac{1-2p}{1-p}$. Similarly, letting $q$ be the probability of "Enter always", the Incumbent must be indifferent between his two pure strategies:
$(8+7p)q + (8+12p)(1-q) = (6+9p)q + (10+10p)(1-q)$, which yields $q = \frac{1}{2}$. The Nash equilibrium is thus given by $r = \frac{1-2p}{1-p}$ and $q = \frac{1}{2}$.

**(c)** In the pure-strategy equilibrium of Case 1, where $0 > 2-4p$, the Entrant's beliefs must be $\begin{pmatrix} x & y \\ p & 1-p \end{pmatrix}$, making "Out" the only rational choice. Thus the following is a weak sequential equilibrium :

$$\sigma = \left(\begin{array}{cc|cc|cc} L & H & Out & In & Out & In \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right),\ \mu = \begin{pmatrix} x & y \\ p & 1-p \end{pmatrix}$$

In the mixed-strategy equilibrium of Case 2, where $0 < 2-4p$, the prior probability of $x$ is $p$ and the prior probability of $y$ is $(1-p)\left(1-\dfrac{1-2p}{1-p}\right) = (1-p)\dfrac{p}{1-p} = p$. Thus the Entrant's beliefs must be $\begin{pmatrix} x & y \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$, making any mixture $\begin{pmatrix} Out & In \\ q & 1-q \end{pmatrix}$ rational for the Entrant at her information set $\{x,y\}$. Since $H$ gives the Incumbent a payoff of 8 (because "Enter" is the only rational choice of the Entrant at her singleton node), it can rational for the Incumbent to choose $\begin{pmatrix} L & H \\ \dfrac{p}{1-p} & \dfrac{1-2p}{1-p} \end{pmatrix}$ if and only if $L$ also gives a payoff of 8, which is the case if and only if $10q + 6(1-q) = 8$, that is, if $q = \frac{1}{2}$. Thus the following is a weak sequential equilibrium : $\sigma = \left(\begin{array}{cc|cc|cc} L & H & Out & In & Out & In \\ \frac{p}{1-p} & \frac{1-2p}{1-p} & \frac{1}{2} & \frac{1}{2} & 0 & 1 \end{array}\right),\ \mu = \begin{pmatrix} x & y \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

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**FINAL EXAM** ANSWER ALL QUESTIONS (total 100 points)

**1.** [40 points] There are four types of surgeons in the plastic surgery market. The following table gives, for each type of surgeon, the cost (to the surgeon) of a surgery and the value of the surgery to a patient:

| Type of surgeon | A | B | C | D |
|---|---|---|---|---|
| Cost of surgery to surgeon | \$17,000 | \$12,000 | \$6,000 | \$1,000 |
| Value of surgery to patient | \$30,000 | \$15,000 | \$10,000 | – \$10,000 |
| Proportion | 30% | 30% | 30% | 10% |

Type *D* surgeons are incompetent and do only harm to the patients (harm that will require an expense of \$10,000 to undo, hence the negative value of – \$10,000 to the patient). Each surgeon knows her own type, while a patient cannot tell the type of the surgeon she is hiring. Each surgeon is willing to perform a surgery if and only if she gets a fee that is at least as high as her own cost and each patient is willing to pay a fee of \$*p* for a surgery if and only if the expected value of the surgery she gets is at least as high as *p*. The information in the above table is common knowledge. Surgeons and patients are risk neutral.

**(a)** For every price $p > 0$, determine whether there is a market for plastic surgery and what type of surgeons are performing surgery.

**(b)** A new law comes into effect that introduces a new license for plastic surgeons. The new law is effective at eliminating the bad surgeons (type D), so that only Types A, B and C remain (this fact is known to everybody). The licensing increases the cost of each surgery by \$1,000 (for each type of doctor). Under the new law, for every price $p > 0$, determine whether there is a market for plastic surgery and what type of surgeons are performing surgery.

**2.** [40 points] A monopolist faces two consumers, one with demand function $D_H(P) = 12 - 2P$ and the other with demand function $D_L(P) = 12 - 3P$ . The monopolist cannot tell which consumer has the higher demand and which has the lower demand, although the monopolist knows the two demand functions. The monopolist decides to sell the good in bundles or packages. Denote a package as a pair $(Q,V)$ where $Q$ is the number of units of the product and $V$ is the price of the entire package (not the price per unit). He considers three options.

**Option 1**: sell only one package, targeted to the consumer with high demand.

**Option 2**: sell two identical packages, designed in such a way that each consumer will buy one package.

**Option 3**: sell two different packages, one targeted to the high-demand consumer and the other to the low-demand consumer.

The monopolist has the following cost function: $C(Q) = Q$

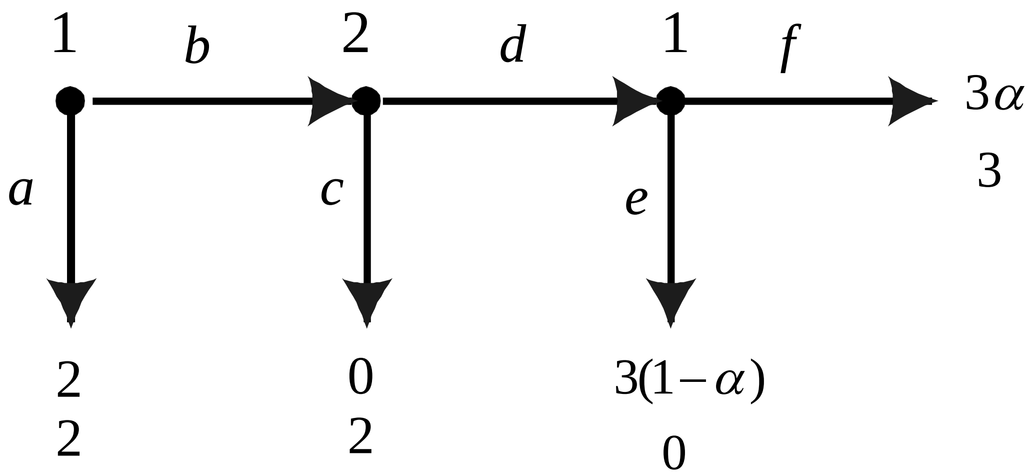
**(a)** [10 points] Determine the profit maximizing package for Option 1 and calculate the corresponding profits.

**(b)** [10 points] Determine the profit maximizing package for Option 2 and calculate the corresponding profits.

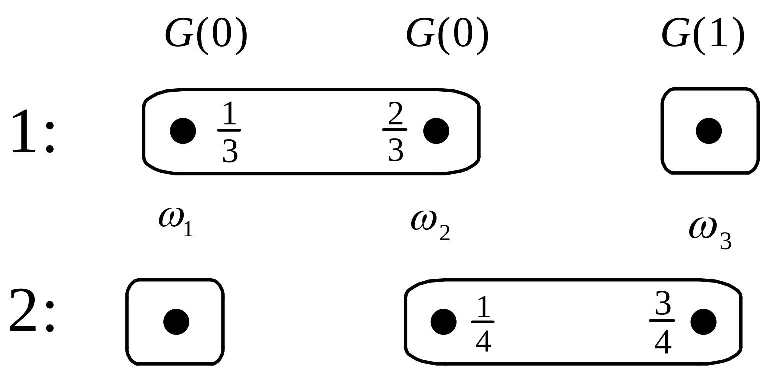
**(c)** [10 points] For Option 3 write the constraints that must be satisfied in order for each consumer to end up buying the package which is designed for her.

**(d)** [10 points] Determine the profit maximizing packages for Option 3.

**3.** [20 points] Consider the following perfect-information game with von Neumann-Morgenstern payoffs (the top number is Player 1's payoff, the bottom number is Player 2's payoff).




Suppose that there are only two possible values of α: 0 and 1. Let $G(0)$ be the above game with $\alpha = 0$ and $G(1)$ be the above game with $\alpha = 1$. Consider the following situation of incomplete information:




Apply the Harsanyi transformation to represent this situation of incomplete information as an extensive-form game.

**1.** **(a)** Consider first $p \geq$ **17,000**. Then all surgeons are willing to perform surgery. Thus the expected value to a buyer is

$$EV = \frac{3}{10}30,000 + \frac{3}{10}15,000 + \frac{3}{10}10,000 - \frac{1}{10}10,000 = 15,\ 500 < 17,000$$

Thus no such $p$ gives rise to an equilibrium.

Consider now **12,000 ≤** $p$ **< 17,000**. At any such price, Type A doctors drop out. Thus the expected value to a patient (updating probabilities according to Bayes' rule) is:

$$EV = \frac{3}{7}15,000 + \frac{3}{7}10,000 - \frac{1}{7}10,000 = \frac{65\,000}{7} < 12,000$$

Thus no such $p$ gives rise to an equilibrium.

Consider now **6,000 ≤** $p$ **< 12,000**. At any such price, Types A and B drop out. Thus the expected value to a patient (updating probabilities according to Bayes' rule) is:

$$EV = \frac{3}{4}10,000 - \frac{1}{4}10,000 = 5,000 < 6,000$$

Thus no such $p$ gives rise to an equilibrium.

Clearly $p < 6,000$ cannot yield an equilibrium because buyers know that, if at all, they would be dealing with a Type D doctor. Thus there is no value of $p$ that gives rise to an equilibrium.

**(b)** Because of the new law, the situation is now as follows:

| Type | $A$ | $B$ | $C$ |
|---|---|---|---|
| Worth To Surgeon | 18,000 | 13,000 | 7,000 |
| Worth To Patient | 30,000 | 15,000 | 10,000 |
| Proportion | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |

Consider first $p \geq$ **18,000**. The expected value to a buyer is

$$EV = \frac{1}{3}30,000 + \frac{1}{3}15,000 + \frac{1}{3}10,000 = 18,333.$$

**Thus, for every $p$ such that 18,000 ≤ $p$ ≤ 18,333 we have an equilibrium where all (licensed) types of surgeons are active in the market.**

Consider now **13,000 ≤ *p* < 18,000**. At any such price, Type A doctors drop out. Thus the expected value to a patient (updating probabilities according to Bayes' rule) is:

$$EV = \frac{1}{2}15,000 + \frac{1}{2}10,000 = 12,500 < 13,000$$

Thus no such *p* gives rise to an equilibrium.

Consider now $7,000 \le p < 13,000$. In this case only Type C doctors participate. Since surgery performed by a type C doctor is worth $10,000 to the patient, for an equilibrium we need $p \le 10,000$. **Hence any *p* with 7,000 ≤ p ≤ 10,000 gives rise to an equilibrium.**

Clearly, when $p < 7,000$ no doctor is willing to offer surgery and therefore there is no market.

**2.** Inverse demand is given by $P_H(Q) = \frac{12-Q}{2}$ and $P_L(Q) = \frac{12-Q}{3}$.

**(a)** Let $W_H(Q)$ be the willingness to pay of the *H* consumer for *Q* units of output. Then $W_H(Q) = \frac{1}{2}\left(12Q - \frac{Q^2}{2}\right) = 6Q - \frac{Q^2}{4}$. For Option 1 the monopolist would choose within the set $\{(Q,V) : V = W_H(Q)\}$. The corresponding profits are $\pi_1(Q) = 6Q - \frac{Q^2}{4} - Q$. This function is strictly concave. Solving $\frac{d\pi_1}{dQ} = 0$ we get $\boxed{Q_1^* = 10}$, $\boxed{V_1^* = 35}$ with corresponding profits $\boxed{\pi_1^* = 25}$.

**(b)** Let $W_L(Q)$ be the willingness to pay of the *L* consumer for *Q* units of output. Then $W_L(Q) = \frac{1}{3}\left(12Q - \frac{Q^2}{2}\right) = 4Q - \frac{Q^2}{6}$. For Option 2 the monopolist would choose within the set $\{(Q,V) : V = W_L(Q)\}$. The corresponding profits are $\pi_2(Q) = 2\left[4Q - \frac{Q^2}{6} - Q\right]$. This function is strictly concave. Solving $\frac{d\pi_2}{dQ} = 0$ we get $\boxed{Q_2^* = 9}$, $\boxed{V_2^* = \frac{45}{2}}$ with corresponding profits $\boxed{\pi_2^* = 27}$.

**(c)** Let $(Q_H, V_H)$ and $(Q_L, V_L)$ be the two packages. Then the incentive compatibility constraints are:

(1) $V_L \le W_L(Q_L)$ : the *L*-consumer is willing to buy "her" package

(2) $W_L(Q_L) - V_L \ge W_L(Q_H) - V_H$ : incentive compatibility constraint for the *L*-consumer (she does not prefer the *H* package to the *L* package)

(3) $V_H \le W_H(Q_H)$ : the *H*-consumer is willing to buy "her" package

(4) $W_H(Q_H) - V_H \ge W_H(Q_L) - V_L$ incentive compatibility constraint for the *H*-consumer (she does not prefer the *L* package to the H package);

Since $W_H(Q) > W_L(Q)$, (3) follows, as a strict inequality, from (1) and (4). Let ***S*** be the set of pairs $\left((Q_H, V_H), (Q_L, V_L)\right)$ that satisfy (1)-(4) and let $\left((Q_H^*, V_H^*), (Q_L^*, V_L^*)\right) \in \boldsymbol{S}$ be a pair that maximizes the firm's profits within **S**. Then (1) and (4) must be satisfied as equalities, that is, $V_L^* = W(Q_L^*) = \frac{1}{3}\left(12Q_L^* - \frac{(Q_L^*)^2}{2}\right)$ and

$$V_H^* = \mathrm{W_H}(Q_H^*) - \mathrm{W_H}(Q_L^*) + V_L^* \;=\; \frac{1}{2}\left(12Q_H^* - \frac{\left(Q_H^*\right)^2}{2} - 12Q_L^* + \frac{\left(Q_L^*\right)^2}{2}\right) + \frac{1}{3}\left(12Q_L^* - \frac{\left(Q_L^*\right)^2}{2}\right)$$

**(d)** Let $\boldsymbol{S}^*$ be the set of pairs $\left((Q_H,V_H),(Q_L,V_L)\right)$ that satisfy the last two equalities. Then if $\left((Q_H,V_H),(Q_L,V_L)\right)$ maximizes the firm's profits in **S** it must belong to $\boldsymbol{S}^*$. Within $\boldsymbol{S}^*$ the profit of the firm is given by

$$\pi = (V_H - Q_H) + (V_L - Q_L) = \frac{1}{2}\left(12Q_H - \frac{(Q_H)^2}{2} - 12Q_L + \frac{(Q_L)^2}{2}\right) + \frac{2}{3}\left(12Q_L - \frac{(Q_L)^2}{2}\right) - Q_H - Q_L$$

Thus the maximum of $\pi$ is found by solving

$$\frac{\partial \pi}{\partial Q_H} = \frac{10 - Q_H}{2} = 0 \quad \text{and} \quad \frac{\partial \pi}{\partial Q_L} = \frac{6 + 3Q_L - 4Q_L}{6} = 0$$

The solution is $\boxed{Q_H^* = 10}$ and $\boxed{Q_L^* = 6}$ with corresponding profits of $\boxed{\pi_3^* = 28}$.

**3.** The extensive game is as follows: