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MIDTERM EXAM (total 100 points)

1. [40 points] Consider the following interaction between a police officer (Player 1) and a motorist (Player 2). At the start of the interaction, the police officer observes the motorist speeding. The officer has three choices: leave the motorist alone (*L*), pull the motorist over to give her a ticket (*T*) or pull her over to arrest her (*A*). If the officer leaves the motorist alone then the game ends and both players get a payoff of 0. If the officer pulls her over, then the motorist decides whether to stay put (*s*) or drive away (*d*). When the motorist makes this decision, she only knows that the officer has pulled her over, but cannot tell whether the officer has decided to give her a ticket or to arrest her. If the motorist drives away, then she is equally likely to get caught (*C*) and to escape (*E*). The von Neumann-Morgenstern payoffs are as follows:

- If the officer pulls the motorist over to give her a ticket and she stays put, then the officer gets 3 and the motorist gets -5.
- Arresting the motorist means extra paperwork for the police officer. Therefore, if the officer pulls the motorist over to arrest her and she stays put, then the officer gets a payoff of just 2, while the motorist gets -10.
- If the officer pulls the motorist over and the motorist drives away and is caught, then the officer gets a payoff of 5, while the motorist gets -15.
- If the officer pulls the motorist over and the motorist drives away and escapes, then the motorist gets 0; the officer gets -10 if he was pulling the motorist over to ticket her, and -11 if he was pulling her over to arrest her (since he will still have to do the extra paperwork).

(a) **(a.1)** [8 points] Represent this situation as an extensive-form game with the officer moving first.

(a.2) [14 points] Find all the subgame-perfect equilibria, including the mixed-strategy ones.

(b) **(b.1)** [8 points] Now represent the situation in a slightly different way: the officer is still moving first, but makes his decision in two steps: first he decides between leaving the motorist alone (*L*) and not leaving the motorist alone ($\neg$L); in the latter case he subsequently makes the further decision whether to go for a ticket or for an arrest. The remaining part of the game is the same as above.

(b.2) [10 points] Find all the subgame-perfect equilibria, including the mixed-strategy ones.

2. [45 points] Mr. Wasser owns the only firm that produces bottled water in an isolated town in California. The demand for bottled water is

$P = 80 - Q$ (P = price, Q = industry output). Production of bottled water is characterized by a constant marginal cost equal to 4 and zero fixed cost.

(a) Mr Wasser wants to retire and appoint a manager to run the firm. He has a choice between offering the manager as compensation either 10% of the profits of the firm (Contract 1) or 10% of the revenue of the firm (Contract 2). The manager will then make production decisions to maximize his own income.

(a.1) [8 points] Which contract does the manager prefer?

(a.2) [2 points] Which contract will Mr Wasser offer to the manager? [Mr Wasser's aim is to maximize his own income, which is equal to the firm's profit minus the compensation paid to the manager.]

Now consider a different scenario. Just before his planned retirement Mr Wasser learns that Ms Shui is about to open a competing bottled-water-producing firm in town (with the same costs as the existing firm). Mr Wasser has the first mover advantage. First he decides whether to appoint a manager with Contract 1 or with Contract 2. This decision is made public. Afterwards the appointed manager makes a production decision to which he will be committed and which, again, is made public ("commitment" means that the chosen output cannot be changed). After observing all of this, Ms Shui chooses her output with the aim to maximize the profit of her own income.

(b) [8 points] Sketch the perfect-information game described above.

(c) [27 points] Find all the backward induction solutions. Note: do not just give the outcome but the actual solution.

3. [15 points] Consider the following simultaneous game. There are 100 players in a room. Each player is given an envelope with his/her name on it and can either put nothing in the envelope of put $100 of his/her own money. A referee then collects all the envelopes.

- If at least 60 players put $100 in their envelopes, then the referee returns the envelope to each **of those** players with an extra $50 in it (so each of those players finds $150 in his/her envelope), while any player who did not put anything in his/her envelope receives nothing.
- If fewer than 60 players put $100 in their envelopes then the referee keeps all the envelopes and leaves the room with a big smile, never to be seen again.

Each player is selfish and greedy, that is, is interested only in his/her own wealth and prefers more money to less.

(a) [8 points] Find all the pure-strategy Nash equilibria of this game.

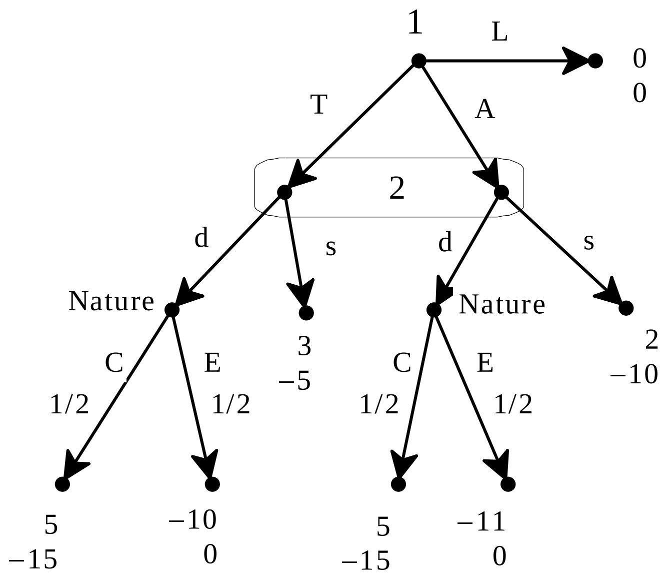
(b) [7 points] Explain why the other pure-strategy profiles are not Nash equilibria.

University of California, Davis -- Department of Economics

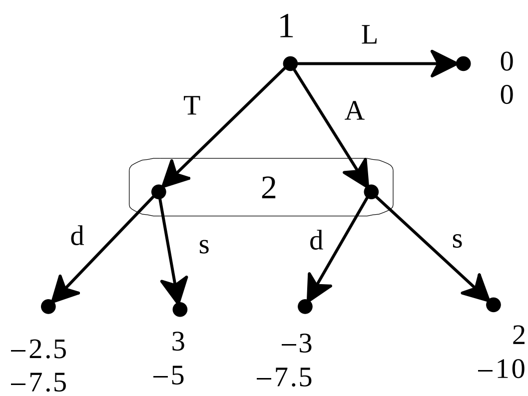
SPRING 2022 **ECN/ARE 200C: MICROECONOMIC THEORY** Giacomo Bonanno

Midterm Exam ANSWER

1. **(a.1)** The game is as follows (an alternative representation is one where there are no moves of Nature and the outcomes associated with Td and Ad are lotteries):



(a.2) First replace Nature's move with expected payoffs:



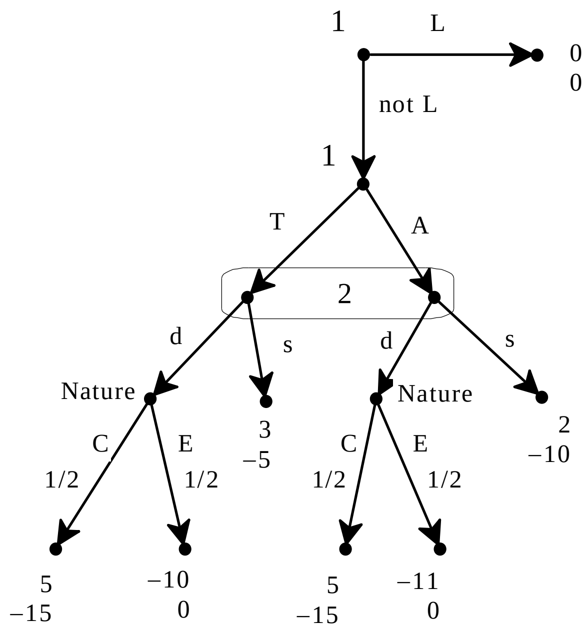
Since the game has no proper subgames, the set of subgame-perfect equilibria coincides with the set of Nash equilibria. The strategic form is as follows:

		Player 2: *d*		*s*	
Player 1	*L*	0	0	0	0
	A	−3	−7.5	2	−10
	T	−2.5	−7.5	3	−5

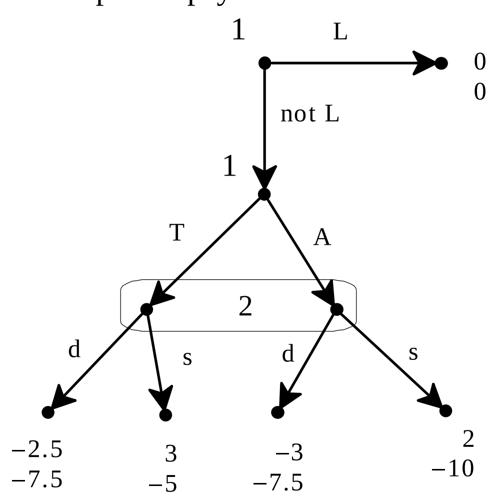
Note that *A* is strictly dominated by *T* and thus cannot be played with positive probability at a Nash equilibrium. There are two pure-strategy Nash equilibria: (*T*,*s*) with expected payoffs (3, −5) and (*L*,*d*) with expected payoffs (0, 0). There is also an infinite number of mixed-strategy equilibria:

$$\begin{pmatrix} L & A & T & | & d & s \\ 1 & 0 & 0 & | & p & 1-p \end{pmatrix}$$ for every $p \geq \frac{6}{11} = 0.545$, with expected payoffs (0,0).

(b.1) The game is as follows:



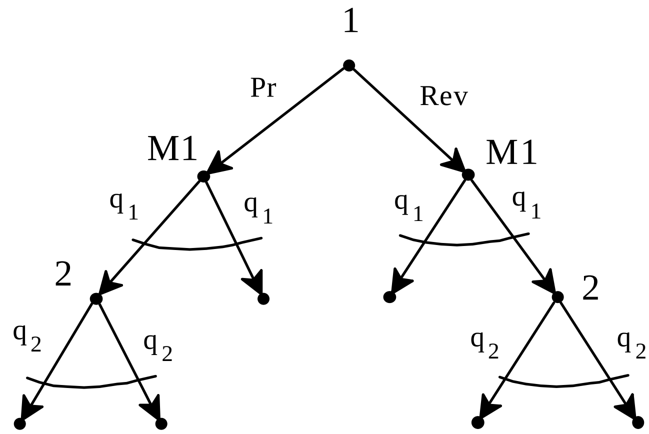
(b.2) First replace Nature's move with expected payoffs:



The proper subgame has a unique Nash equilibrium: (*T*,*s*) (because *T* strictly dominates *A* and against *T* the unique best response is *s*). Thus the unique subgame-perfect equilibrium is $\left((\neg L, T), s\right)$.

2. **(a)** The revenue function is $R(Q) = Q(80 - Q)$ and the profit function is $\Pi(Q) = Q(80 - Q) - 4Q$. If the manager is given a percentage of profits then he will choose *Q* to maximize profits, that is, he will choose *Q* so that $\frac{d\Pi}{dQ} = 0$. The solution is *Q* = 38 with corresponding total profits of $\Pi(38) = 1,444$, so that the manager's income is $\frac{1}{10}1444 = 144.4$ and Mr Wasser's income is $\frac{9}{10}(1,444) = \boxed{1,299.6}$ If the manager is given a percentage of revenue then he will choose *Q* to maximize revenue, that is, he will choose *Q* so that $\frac{dR}{dQ} = 0$. The solution is *Q* = 40 with corresponding revenue of $R(40) = 1,600$, so that the manager's income is 160 and Mr Wasser's income is $\frac{9}{10}(1,600) - 4(40) = \boxed{1,280}$. Hence the manager prefers the revenue contract while Mr Wasser prefers the profit contract.

(b) The extensive form is as follows. Player 1 is Mr Wasser, M1 is Wasser‘s manager and Player 2 is Ms Shui. Pr means “appoint a manager with profit-sharing contract”, Rev means “appoint a manager with revenue-sharing contract”, q_1 is the output of Firm 1 (which is Mr. Wasser’s firm) and q_2 the output of Firm 2 (Ms. Shui’s firm).



(c) The profit function of Firm 2 is $\Pi_2(q_1, q_2) = q_2[80 - (q_1 + q_2)] - 4q_2$. Ms Shui will choose q_2 to solve $\frac{\partial \Pi_2}{\partial q_2} = 0$ yielding the best reply function $\boxed{BP(q_1) = 38 - \frac{q_1}{2}}$. The profit function of Firm 1 is $\Pi_1(q_1, q_2) = q_1[80 - (q_1 + q_2)] - 4q_2$ and the revenue function is $R_1(q_1, q_2) = q_1[80 - (q_1 + q_2)]$. If M1 is on a profit contract then he will choose q_1 to solve $\frac{\partial \Pi_1(q_1, BP(q_1))}{\partial q_1} = 0$. The solution is $q_1 = 38$ with corresponding profits of 722 so that Mr Wasser’s income is $\frac{9}{10}722 = \boxed{649.8}$. If M1 is on a revenue contract then he will choose q_1 to solve $\frac{\partial R_1(q_1, BP(q_1))}{\partial q_1} = 0$. The solution is $q_1 = 42$ with corresponding revenue of 882 so that Mr Wasser’s income $\frac{9}{10}882 - 4(42) = \boxed{625.8}$. Thus the backward-induction solution is:

- Mr Wasser’s strategy: choose profit-sharing contract.
- M1’s strategy: choose $q_1 = 38$ if on a profit contract and choose $q_1 = 42$ if on a revenue contract.
- Ms Shui’s strategy: if M1 chooses q_1, whether he is on a profit contract or a revenue contract, choose $q_2 = 38 - \frac{q_1}{2}$.

3. **(a)** There are two pure-strategy Nash equilibria: one where everybody leaves his/her the envelope empty and one where everybody puts $100 in his/her envelope.

(b) Let n be the number of players who put $100 in their envelopes. Both $n = 0$ and $n = 100$ are Nash equilibria. We need to show that no n, with $0 < n < 100$, is a Nash equilibrium. Pick an arbitrary such n. If $n < 60$, then any player who contributed $100 loses that money while if he switched to leaving the envelope empty he would be better off. If $n \geq 60$, then any player who is not contributing anything would be better off by switching to contributing $100: she would make a profit of $50 (note that this would be true even if $n = 59$, so there are two reasons why $n = 59$ is not a Nash equilibrium).

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FINAL EXAM ANSWER ALL QUESTIONS (total 100 points)

1. [40 points] The insurance industry is a monopoly. There are two types of potential customers, R and L. There are n_R individuals of type R and n_L individuals of type L. Both types are identical in all respects, except their preferences. They all have the same initial wealth of \$160,000, they face the same potential loss of \$70,000 with a probability of 10%. Group R individuals have the utility-of-money function $\sqrt{m}$, while group L individuals have the utility-of-money function $100 \ln(m)$. Let w_1 denote wealth in the bad state (when the loss occurs) and w_2 wealth in the good state. In all the diagrams, measure w_1 on the horizontal axis.

(a) **(a.1)** [2 points] For each group calculate the slope at the no-insurance point of the indifference curve that goes through the no-insurance point.

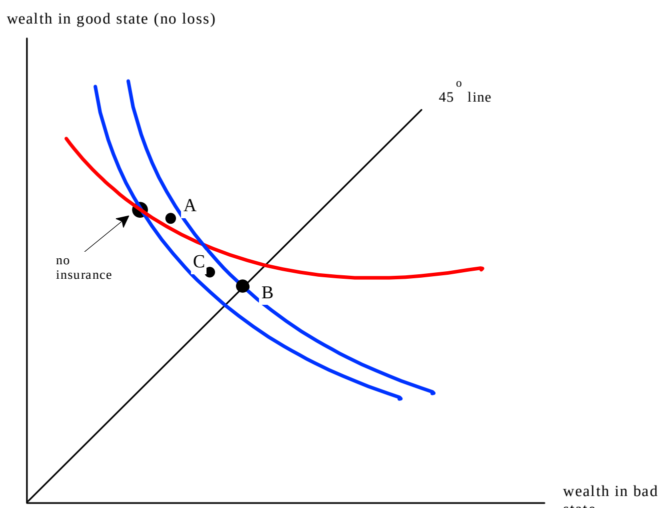
(a.2) [4 points] Sketch the two indifference curves (no need to compute them: a rough sketch is sufficient, as long as you clearly label each indifference curve with the corresponding type of consumer and are clear about the shape).

(b) **(b.1)** [2 points] Write the equation that needs to be solved in order to find the maximum premium that a type R individual is willing to pay for full insurance.

(b.2) [2 points] Write the equation that needs to be solved in order to find the maximum premium that a type L individual is willing to pay for full insurance.

(b.3) [3 points] Which of the two premia is larger? (You don't need to compute them in order to answer the question.)

There is asymmetric information: while each individual knows his/her own type, the firm cannot tell to which group applicants belong; however, the firm knows the information given above (initial wealth, utility functions, etc.). Let $A = (w_1^A, w_2^A)$, $B = (w_1^B, w_2^B)$ and $C = (w_1^C, w_2^C)$ be the contracts illustrated in the figure below.



(c) [6 points] Write down an expression for the firm's profits if it offers only contract B.

(d) [5 points] Write down an expression for the firm's profits if it offers contracts B and C.

(e) [6 points] Write down an expression for the firm's profits if it offers contracts A, B and C.

(f) [10 points] Suppose that the two-contract profit-maximizing choice for the firm is the pair where one contract is the full insurance contract with premium \$8,200. Write a system of equations whose solution gives the premium and deductible of the other contract.

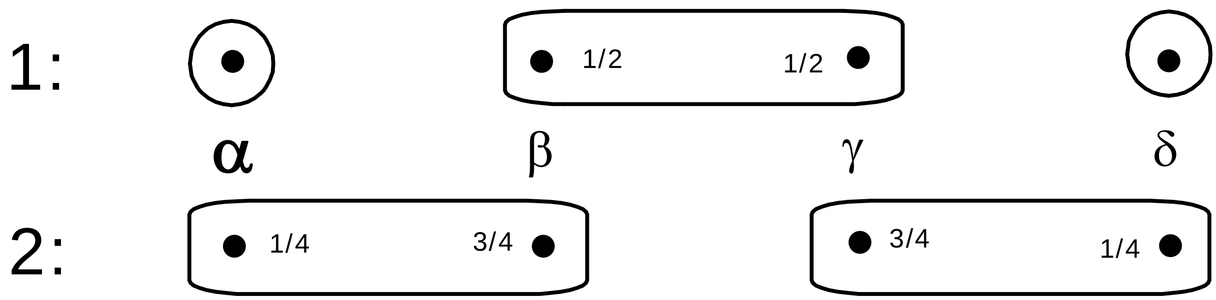
2. [60 points] Consider the following two-player situation of incomplete information (where the four strategic-form games are to be understood as simultaneous games and the payoffs are von Neumann-Morgenstern payoffs).

1 \ 2	L	R
T	4,4	0,0
B	2,0	2,2

1 \ 2	L	R
T	0,4	4,0
B	2,0	0,2

1 \ 2	L	R
T	0,4	4,0
B	2,0	0,2

1 \ 2	L	R
T	4,4	0,0
B	2,0	2,2



(a) [10 points] Find the common prior.

(b) [10 points] Apply the Harsanyi transformation to represent this situation of incomplete information as a game with imperfect information, making Player 1 move before Player 2.

(c) [16 points] Prove that there are no pure-strategy weak sequential equilibria.

(d) [4 points] Explain why there are no pure-strategy Nash equilibria.

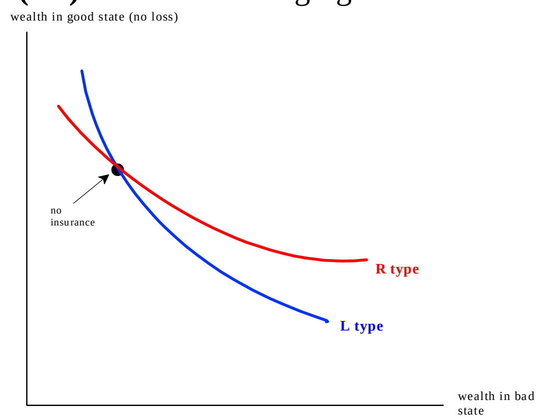
(e) [10 points] Under what circumstances would it be rational for Player 1 to choose a completely mixed behavioral strategy at his non-trivial (i.e. non-singleton) information set?

(f) [10 points] Assuming that Player 2 chooses the same behavioral strategy at all of her information sets and using the insights obtained from part (e), find a weak sequential equilibrium.

Final Exam ANSWERS

1. **(a.1)** The slope of the indifference curve at point $(90,000, 160,000)$ is given by $-\dfrac{0.1\ U'(90,000)}{0.9\ U'(160,000)}$.

For group R this is equal to $-\frac{4}{27} = -0.148$ and for group L this is equal to $-\frac{16}{81} = -0.198$. Thus the L-indifference curve is steeper. **(a.2)** See the following figure.



(b.1) $\sqrt{160,000 - h} = 0.1\sqrt{90,000} + 0.9\sqrt{160,000}$ $(= 390)$ (The solution is 7,900.)

(b.2) $\ln(160,000 - h) = 0.1\ \ln(90,000) + 0.9\ \ln(160,000)$ (The solution is 8,946.)

(b.3) The one for group L since they have a steeper indifference curve.

(c) Only type L will buy. Since $w_1^B = w_2^B = w_B$ the firm's profits will be

$$\pi(B) = n_L \left[\underbrace{(160,000 - w_B)}_{premium} - 0.1(70,000) \right] = n_L (153,000 - w_B).$$

(d) Only type L will buy and they will all buy contract B. Thus the same as in (c).

(e) Type L will buy contract B and type R will buy contract A. Thus profits will be

$$\pi(ABC) = n_L (153,000 - w_B) + n_R \left[\underbrace{160,000 - w_2^A}_{premium} - 0.1(70,000) + 0.1 \left(\underbrace{w_2^A - w_1^A}_{deductible} \right) \right]$$

$$= n_L (153,000 - w_B) + n_R (153,000 - 0.9\, w_2^A - 0.1 w_1^A)$$

(f) For the R type, expected utility of no insurance (NI) is 390 (computed above). Call the second contract $E = (h_E, d_E)$. The first equation says that the R type is indifferent between NI and E:

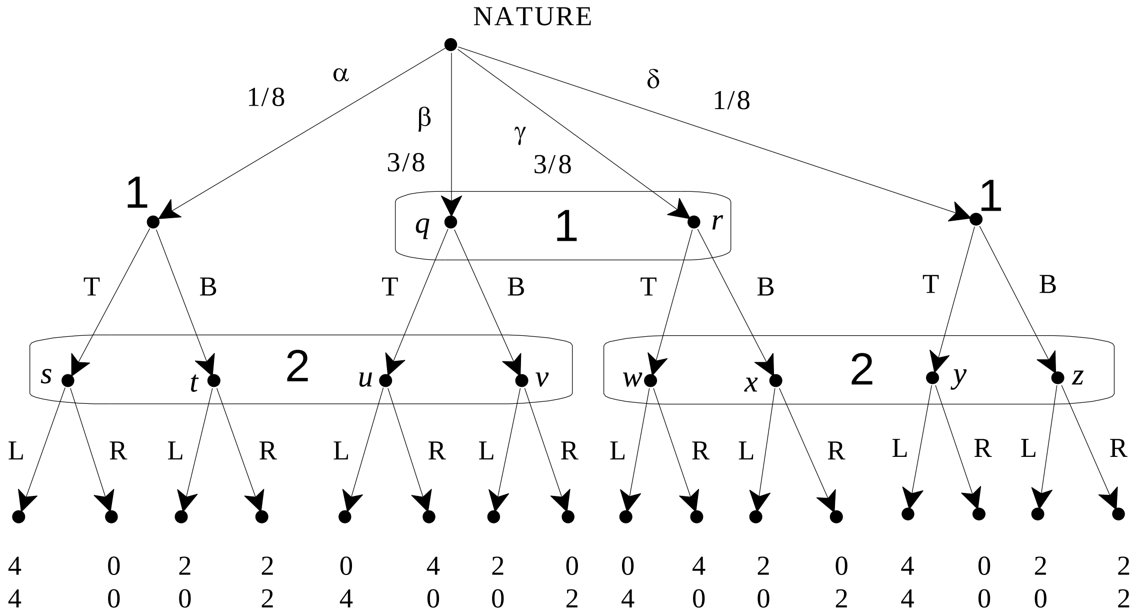
$$\tfrac{1}{10}\sqrt{160,000 - h_E - d_E} + \tfrac{9}{10}\sqrt{160,000 - h_E} = 390$$

For the L type the full-insurance contract give a utility of $100\ln(160,000 - 8,200) = 1193.032$. The second equation says that the L type is indifferent between this contract and E:

$$\tfrac{1}{10}100\ln(160,000 - h_E - d_E) + \tfrac{9}{10}100\ln(160,000 - h_E) = 1193.032.$$

2. **(a)** The common prior is $\begin{pmatrix} \alpha & \beta & \gamma & \delta \\ \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{pmatrix}$

(b) The game is as follows:



(c) First of all, the beliefs of Player 1 must be $\begin{pmatrix} q & r \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

- Suppose that the strategy of Player 2 is *LL*. Then by sequential rationality Player 1 must choose *T* at both singleton nodes; at his information set in the middle, $T \to \frac{1}{2}0 + \frac{1}{2}0 = 0$ while $B \to \frac{1}{2}2 + \frac{1}{2}2 = 2$. Thus Player 1's strategy must be *TBT*. Hence Player 2's beliefs must be $\left(\begin{array}{cccc|cccc} s & t & u & v & w & x & y & z \\ \frac{1}{4} & 0 & 0 & \frac{3}{4} & 0 & \frac{3}{4} & \frac{1}{4} & 0 \end{array}\right)$. Then at her information set on the left, $L \to \frac{1}{4}4 + \frac{3}{4}0 = 1$ and $R \to \frac{1}{4}0 + \frac{3}{4}2 = 1.5$, so that *L* is **not** rational there. Thus there is no pure-strategy weak sequential equilibrium where Player 2's strategy is *LL*.
- Suppose that the strategy of Player 2 is *LR*. Then by sequential rationality Player 1's strategy must be *TTB*. Hence Player 2's beliefs must be $\left(\begin{array}{cccc|cccc} s & t & u & v & w & x & y & z \\ \frac{1}{4} & 0 & \frac{3}{4} & 0 & \frac{3}{4} & 0 & 0 & \frac{1}{4} \end{array}\right)$. Then at her information set on the right Player 2 should play *L* (*L* gives 3, while *R* gives 0.5). Hence there is no pure-strategy weak sequential equilibrium where Player 2's strategy is *LR*.
- Suppose that the strategy of Player 2 is *RL*. Then Player 1's strategy must be *BTT*. Hence Player 2's beliefs must be $\left(\begin{array}{cccc|cccc} s & t & u & v & w & x & y & z \\ 0 & \frac{1}{4} & \frac{3}{4} & 0 & \frac{3}{4} & 0 & \frac{1}{4} & 0 \end{array}\right)$. Then at her left information set Player 2 should play *L* (*L* gives 3, while *R* gives 0.5). Hence there is no pure-strategy weak sequential equilibrium where Player 2's strategy is *RL*.

- Suppose that the strategy of Player 2 is *RR*. Then Player 1's strategy must be *BTB*. Hence Player 2's beliefs must be $\left(\begin{array}{cccc|cccc} s & t & u & v & w & x & y & z \\ 0 & \frac{1}{4} & \frac{3}{4} & 0 & \frac{3}{4} & 0 & 0 & \frac{1}{4} \end{array}\right)$. Then at her left information set Player 2 should play *L* (*L* gives 3, while *R* gives 0.5). Hence there is no pure-strategy weak sequential equilibrium where Player 2's strategy is *RR*.

(d) Because this is a game where every information set is reached with positive probability, no matter what strategies the players choose. In these games the set of Nash equilibria coincides with the set of weak sequential equilibria.

(e) As before, the beliefs of Player 1 must be $\begin{pmatrix} q & r \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. We saw above that if Player 2 chooses a pure strategy then at Player 1's information set in the middle either *T* is strictly better than *B* or *vice versa.* Thus Player 2 must be playing mixed behavioral strategies. Let *p* the probability of *L* at Player 2's information set on the left and *q* the probability of *L* at Player 2's information set on the right. Then at Player 1's information set in the middle *T* gives an expected payoff of $\frac{1}{2}4(1-p)+\frac{1}{2}4(1-q)=4-2p-2q$ and B gives an expected payoff of $\frac{1}{2}2p+\frac{1}{2}2q = p+q$. Thus it is rational for Player 1 to choose a completely mixed behavioral strategy at that information set if and only if $4-2p-2q = p+q$, that is, if and only if $p+q=\frac{4}{3}$. If $p = q$ (which is what is assumed in part f) then this means that $p=q=\frac{2}{3}$.

(f) By the calculations in part (e), the behavioral strategy of Player 2 at both information sets is $\begin{pmatrix} L & R \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}$. Then at Player 1's node on the left $T \to 4\frac{2}{3}=\frac{8}{3}$ and $B \to 2$. Hence *T* is the rational choice. Similarly, *T* is the rational choice at Player 1's node on the right. Thus Player 1's behavioral strategy must be $\left(\begin{array}{cc|cc|cc} T & B & T & B & T & B \\ 1 & 0 & r & 1-r & 1 & 0 \end{array}\right)$ for some $0 < r < 1$. It follows that Player 2's beliefs must be $\left(\begin{array}{cccc|cccc} s & t & u & v & w & x & y & z \\ \frac{1}{4} & 0 & \frac{3}{4}r & \frac{3}{4}(1-r) & \frac{3}{4}r & \frac{3}{4}(1-r) & \frac{1}{4} & 0 \end{array}\right)$, so that $L \to \frac{1}{4}4+\frac{3}{4}r4=1+3r$ and $R \to \frac{3}{4}(1-r)2=\frac{3}{2}(1-r)$. Hence, it is rational for Player 2 to choose a mixed behavioral strategy if and only if $1+3r=\frac{3}{2}(1-r)$, that is, if and only if $r=\frac{1}{9}$. Hence the following is a weak sequential equilibrium:

$$\sigma=\left(\begin{array}{cc|cc|cc|cc|cc} T & B & T & B & T & B & L & R & L & R \\ 1 & 0 & \frac{1}{9} & \frac{8}{9} & 1 & 0 & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} \end{array}\right), \quad \mu=\left(\begin{array}{cc|cccc|cccc} q & r & s & t & u & v & w & x & y & z \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{12} & \frac{2}{3} & \frac{1}{12} & \frac{2}{3} & \frac{1}{4} & 0 \end{array}\right).$$