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MIDTERM EXAM (total 100 points). Answer all questions.

1. [50 points] There are two candidates in a presidential election: L and R. Before the election, each candidate has to commit to a political position (or platform) out of ten possible positions, that can be ranked from left to right on the political spectrum, with position 1 being the extreme left position and 10 the extreme right position. The voters are uniformly distributed across the ten positions, in the sense that, for each position, 10% of the voters identify with that position:

10%	10%	10%	10%	10%	10%	10%	10%	10%	10%
1	2	3	4	5	6	7	8	9	10

Each voter will vote for the candidate whose announced position is closest to the position with which the voter identifies (nobody is going to abstain from voting). If a group of voters are indifferent between two candidates, then half of them will vote for one and half of them will vote for the other.

The two candidates choose their positions simultaneously and independently. Denote an outcome as a pair (ℓ, r) where ℓ is the percentage of votes that candidate L gets and r is the percentage of votes that candidate R gets. Each candidate only cares about the percentage of votes that she gets and prefers a higher percentage to a lower percentage, that is,

- $(\ell_1, r_1) \succ_L (\ell_2, r_2)$ if and only if $\ell_1 > \ell_2$ and $(\ell_1, r_1) \sim_L (\ell_2, r_2)$ if and only if $\ell_1 = \ell_2$.
- $(\ell_1, r_1) \succ_R (\ell_2, r_2)$ if and only if $r_1 > r_2$ and $(\ell_1, r_1) \sim_R (\ell_2, r_2)$ if and only if $r_1 = r_2$.

(a) [10 points] Fill in the first two rows (corresponding to choices 1 and 2 for candidate L) of the strategic-form game writing only the utility of candidate L. What can you conclude from these payoffs?

(b) [9 points] Explain why, for a candidate, it is not true that choosing position 2 is dominated by choosing position 3.

(c) [9 points] Show that, if the opponent does not choose position 1, then for the other candidate choosing position 2 is strictly dominated by choosing position 3.

(d) [10 points] Determine the output of the iterated deletion of strictly dominated strategies (IDSDS).

(e) [12 points] Find all the pure-strategy Nash equilibria of this game.

2. [36 points] There are three firms in a homogeneous-product industry. Firm 1 is a large, dominant firm, while firms 2 and 3 are small firms. The industry inverse demand function is given by $P = 57 - 3Q$ (where P is the price and Q is industry output). All firms have the same cost function given by $C(q) = 3q$. The industry is organized as follows. Firm 1 moves first and commits to an output level. Firms 2 and 3 observe firm 1's output level and then simultaneously and independently choose their own output levels (thus they behave as Cournot competitors for the residual demand).

(a) [30 points] Find the subgame-perfect equilibrium of this game (express it properly in terms of strategies).

(b) [6 points] Calculate the equilibrium output levels and profits of all the firms and the equilibrium price.

3. [14 points] Find all the Nash equilibria of the following game, where the payoffs are von Neumann-Morgenstern payoffs.

		Player 2			
		D	E	F	G
Player 1	A	4 , 2	1 , 1	2 , 0	3 , 4
	B	1 , 4	2 , 2	5 , 1	4 , 3
	C	3 , 1	0 , 2	1 , 8	2 , 1

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SPRING 2024 **ECN/ARE 200C: MICROECONOMIC THEORY** Giacomo Bonanno

Midterm Exam ANSWERS

1. We can take $U_L(\ell, r) = \ell$ as utility function of candidate L and $U_R(\ell, r) = r$ as utility function of candidate R.

(a) The numbers below are percentages and also utilities:

	1	2	3	4	5	6	7	8	9	10
1	50	10	15	20	25	30	35	40	45	50
2	90	50	20	25	30	35	40	45	50	55

Clearly, choosing position 2 strictly dominates choosing position 1.

(b) $U_L(2,1) = 90$ while $U_L(3,1) = 85$, that is, when candidate R is at position 1 then position 2 gives candidate L 90% of the votes while position 3 gives only 85% of the votes.

(c) See the following table:

	2	3	4	5	6	7	8	9	10
2	50	20	25	30	35	40	45	50	55
3	80	50	30	35	40	45	50	55	60

(d) First deleted 1 and 10 for both candidates, then delete 2 and 8 then 3 and 7 then 4 and 8. The output is {(5,5), (5,6), (6,5), (6,6)}.

(e) The pure-strategy Nash equilibria are: (5,5), (5,6), (6,5) and (6,6). They all give the same payoff to both players, namely 50 (each gest 50% of the votes)

2. Let q_i be the output of firm *i*. In the subgame where firm 1 has chosen q_1, the profit functions of firms 2 and 3 are given by $\Pi_2 = q_2\left[57 - 3\left(q_1 + q_2 + q_3\right)\right] - 3q_2$ and $\Pi_3 = q_3\left[57 - 3\left(q_1 + q_2 + q_3\right)\right] - 3q_3$. The Cournot-Nash equilibrium of the subgame is given by the solution to $\frac{\partial \Pi_2}{\partial q_2} = 0$ and $\frac{\partial \Pi_3}{\partial q_3} = 0$ which is $q_2^* = q_3^* = \begin{cases} 6 - \frac{q_1}{3} & \text{if } q_1 < 18 \\ 0 & \text{if } q_1 \geq 18 \end{cases}$. Thus the profit function of firm 1 is given by $\Pi_1 = \begin{cases} q_1\left[57 - 3\left(q_1 + 2\left(6 - \frac{q_1}{3}\right)\right)\right] - 3q_1 & \text{if } q_1 < 18 \\ q_1\left[57 - 3q_1\right] - 3q_1 & \text{if } q_1 \geq 18 \end{cases}$. The bottom function is decreasing in q_1 (in the range $q_1 \geq 18$) so that its maximum in the range $q_1 \geq 18$ is attained at $q_1 = 18$ and is equal to 0; the maximum of the top function is given by the solution to $\frac{d\Pi_1}{dq_1} = 0$, which is $q_1 = 9$, and its value is 81. Thus the subgame-perfect equilibrium output levels are

$q_1 = 9,\ q_2 = q_3 = 3$ and the corresponding profits are $\Pi_1 = 81,\ \Pi_2 = \Pi_2 = 27.$ The equilibrium price is 12. The subgame-perfect equilibrium is

$$\left(q_1 = 9,\ R_2(q_1) = \begin{cases} 6 - \frac{q_1}{3} & \text{if } q_1 < 18 \\ 0 & \text{if } q_1 \geq 18_1 \end{cases},\ R_3(q_1) = \begin{cases} 6 - \frac{q_1}{3} & \text{if } q_1 < 18 \\ 0 & \text{if } q_1 \geq 18 \end{cases} \right)$$

3. First apply the IDSDS procedure. Step1: delete C (strictly dominated by A). Step 2: delete E and F (strictly dominated by both D and G). The output is:

		Player 2: D	Player 2: G
Player 1	A	4 , 2	3 , 4
	B	1 , 4	4 , 3

Let p be the probability of A and q the probability of D. Then it must be that $4q + 3(1-q) = q + 4(1-q)$, so that $q = \frac{1}{4}$. Furthermore, it must be that $2p + 4(1-p) = 4p + 3(1-p)$ so that $p = \frac{1}{3}$. Thus there is a unique mixed-strategy Nash equilibrium given by: $\left(\begin{array}{ccc|cccc} A & B & C & D & E & F & G \\ \frac{1}{3} & \frac{2}{3} & 0 & \frac{1}{4} & 0 & 0 & \frac{3}{4} \end{array} \right)$.

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FINAL EXAM

ANSWER ALL QUESTIONS (total 100 points)

1. [38 points] The insurance industry is a monopoly and it faces two types of potential customers: *L* and *H*. All the potential customers have the same von Neumann-Morgenstern utility-of-money function $U(m) = \sqrt{m}$, the same initial wealth $W_0 = 3{,}600$ and the same potential loss $\ell = 1{,}100$; however, they differ in the probability of incurring a loss: for type *L* it is $p_L = 10\%$ and for type *H* it is $p_H = 20\%$. Each potential customer knows her own type. There are 500 type-*L* individuals and 500 type-*H* individuals in the population. Type *H* individuals have the option of spending \$C to align themselves with the *L* types, in the sense of reducing their probability of loss to 10%. All of the above is known to the monopolist. The monopolist is risk neutral. Assume the following:

- If indifferent between insuring and not insuring, every individual will choose to insure.
- Every type-*H* individual, if indifferent between investing \$*C* and not investing, will choose **not** to invest.
- The decision of a type-*H* individual to invest or not invest is **not** observed by the monopolist (although the monopolist knows that the H-types have the option to invest at a cost of \$C).

(a) If the monopolist were able to identify the *L* types and offered a contract only to them,

(a.1) [4 points] what contract would it offer?

(a.2) [4 points] what would its profits be from insuring the *L* types?

(b) [10 points] If the monopolist were to offer the contract of Part (a) also to the H types, would they purchase it?

(c) [5 points] Write an inequality involving *C* (the investment cost) which, if satisfied, implies that type-*H* individuals choose **not** to invest if they don't insure. [No need to solve the inequality.]

Assume that the monopolist is unable to identify the types, so that any contract(s) that it offers can be bought by anybody who wishes to buy it.

(d) [15 points] Suppose that *C* = 10 and that the monopolist offers a contract with premium *h* = 115 and deductible *d* = 200. What will its profits be?

2. [42 points] There are two types of workers, L and H. The productivity of type i is π_i with $\pi_H > \pi_L > 0$. Each worker knows her own type and productivity, while potential employers cannot tell types apart and only know that the fraction μ_H (with $0 < \mu_H < 1$) of the population consists of type H individuals (and the fraction $1 - \mu_H$ are of type L). Each worker chooses an amount of education $y \geq 0$ before applying for a job. Education has no effect on productivity and is costly: the cost of acquiring y units of education is $c_L(y) = y$ for the L types and $c_H(y) = y^2$ for the H types. Employers offer the following wage schedule: $w(y) = \begin{cases} \pi_L & \text{if } y < \hat{y} \\ \pi_H & \text{if } y \geq \hat{y} \end{cases}$. The payoff of a worker of type i who has purchased y units of education and is paid wage w is $w - c_i(y)$. Define a signaling equilibrium as a situation where all the workers make education choices that lead them to being paid an amount equal to their productivity. [Note: when you set up the relevant inequalities, you can use strict inequalities.]

(a) **(a.1)** [4 points] What restrictions on the parameters π_L and π_H are necessary and sufficient for the existence of a signaling equilibrium?
(a.2) [4 points] When a signaling equilibrium does exist, find all the values of $\hat{y}$ that are compatible with a signaling equilibrium.
(a.3) [4 points] Give three values of $\hat{y}$, π_L and π_H that give rise to a signaling equilibrium.

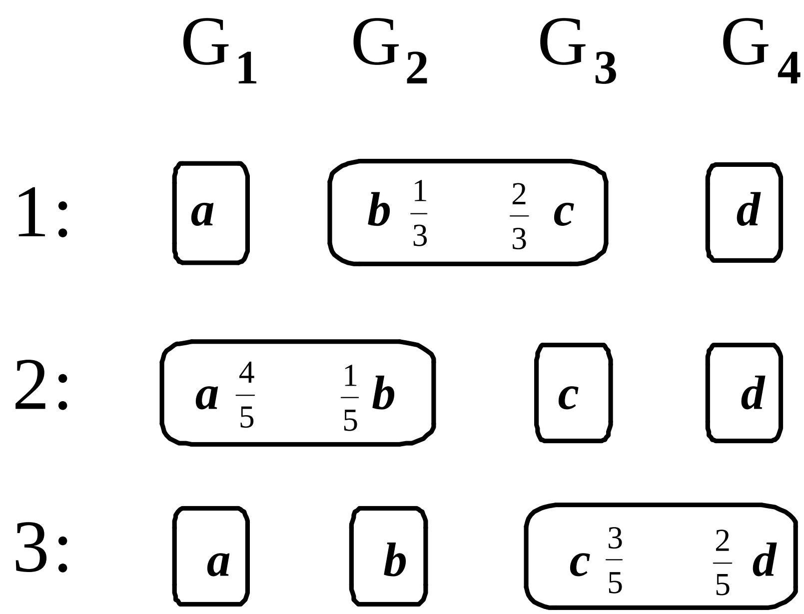
(b) [9 points] Suppose that $\pi_L = 2$ and $\pi_H = \frac{9}{4}$ and that the economy is initially at a signaling equilibrium with $\hat{y} = 0.4$. The government now steps in, closes down all schools, forces every potential worker to choose $y = 0$ and forces employers to hire all workers at the same wage equal to the average productivity of the population. For what values of μ_H (recall that μ_H is the fraction of the population that consists of type H individuals) are the workers better off than in the earlier signaling equilibrium?

For the remainder of this question, suppose that, in addition to obtaining education, workers can also acquire some other certification of an activity that, just like education, does not affect productivity and is costly to acquire. The cost of acquiring $x \geq 0$ units of this additional certification is the same for both types and is equal to $c(x) = x$. The amount of this additional certification is observable by employers (as is the level of education).

Employers now offer the following wages: $w(x, y) = \begin{cases} \pi_L & \text{if } x < \hat{x} \text{ or } y < \hat{y} \\ \pi_H & \text{if } x \geq \hat{x} \text{ and } y \geq \hat{y} \end{cases}$. Each worker chooses x and y at the same time.

(c) **(c.1)** [7 points] For arbitrary values of π_L and π_H (with $\pi_H > \pi_L > 0$), find a necessary and sufficient condition on $\hat{x}$ for the existence of a signaling equilibrium.
(c.2) [7 points] Suppose that $\pi_L = 3, \pi_H = 4, \hat{x} = 0.5, \hat{y} = 0.6$. What values of x and y will the two types choose?
(c.3) [7 points] Suppose that $\pi_L = 2, \pi_H = 7, \hat{x} = 3, \hat{y} = 2.5$. What values of x and y will the two types choose?

3. [20 points] Consider the situation of incomplete information shown below, where G_1, G_2, G_3 and G_4 are three-player, simultaneous, strategic-form games with different payoffs. In each of those games, each player has two strategies (call them *L* and *R*).



Draw the extensive-form game-frame (that is, the extensive form without payoffs) that results from applying the Harsanyi transformation to the above situation of incomplete information. Make Player 1 move first, then Player 2 and then Player 3.

Final Exam ANSWERS

1. **(a.1)** A full-insurance contract with premium $h_L = 119$ (given by the solution to the equation $\sqrt{3,600 - h} = \underbrace{\frac{1}{10}\sqrt{3,600 - 1,100} + \frac{9}{10}\sqrt{3,600}}_{=59}$). **(a.2)** $500(119 - \frac{1}{10}1100) = 4,500$.

(b) Yes: it would guarantee them a utility of $\sqrt{3,600 - 119} = 59$ (without investment) while no insurance without investment would yield a utility of $\frac{1}{5}\sqrt{3,600 - 1,100} + \frac{4}{5}\sqrt{3,600} = 58$ and no insurance with investment would yield a utility of

$\frac{1}{10}\sqrt{3,600 - 1,100 - C} + \frac{9}{10}\sqrt{3,600 - C} < \frac{1}{10}\sqrt{3,600 - 1,100} + \frac{9}{10}\sqrt{3,600} = 59$

(c) $\underbrace{\frac{1}{5}\sqrt{3,600 - 1,100} + \frac{4}{5}\sqrt{3,600}}_{=58} \geq \frac{1}{10}\sqrt{3,600 - 1,100 - C} + \frac{9}{10}\sqrt{3,600 - C}$ (which simplifies to $C \geq 116.64$).

(d) The expected utility from the contract (115, 200) for a type L individual is

$$\frac{1}{10}\sqrt{3,600 - 115 - 200} + \frac{9}{10}\sqrt{3,600 - 115} = 58.86$$

which is less than the expected utility of no insurance (= 59) and thus type-L individuals will not insure.

Since C = 10 does not satisfy the inequality of part (c), the H types would choose to invest if not insured, guaranteeing themselves an expected utility of $\frac{1}{10}\sqrt{3,600 - 1,100 - 10} + \frac{9}{10}\sqrt{3,600 - 10} = 58.91$ (while no insurance with no investment would yield a utility of 58). The contract would give them the following utility:

$$\begin{cases} \frac{1}{5}\sqrt{3,600 - 115 - 200} + \frac{4}{5}\sqrt{3,600 - 115} = 58.69 & \text{if they don't invest} \\ \frac{1}{10}\sqrt{3,600 - 115 - 200 - 10} + \frac{9}{10}\sqrt{3,600 - 115 - 10} = 58.78 & \text{if they invest} \end{cases}$$

Thus they are better off not insuring (and investing). Hence the monopolist would end up with no customers and thus a profit of 0.

2.

(a) **(a.1)** At a signaling equilibrium type L workers choose y = 0 and type H workers choose $y = \hat{y}$. This is rational only if (1) $\pi_L > \pi_H - \hat{y}$ (that is, $\hat{y} > \pi_H - \pi_L$) and (2) $\pi_H - \hat{y}^2 > \pi_L$ (that is, $\hat{y} < \sqrt{\pi_H - \pi_L}$). These two inequalities can be simultaneously satisfied if and only if $\pi_H - \pi_L < \sqrt{\pi_H - \pi_L}$ which is true if and only if $\pi_H - \pi_L < 1$.

(a.2) Assuming that $\pi_H - \pi_L < 1$, every value of $\hat{y}$ such that $\pi_H - \pi_L < \hat{y} < \sqrt{\pi_H - \pi_L}$ gives rise to a signaling equilibrium. **(a.3)** There are, of course, many. For example, $\pi_H = \frac{1}{2}$, $\pi_L = \frac{1}{4}$, $\hat{y} = \frac{3}{8}$.

(b) The average productivity is $\bar{\pi} = \mu_H \pi_H + (1 - \mu_H)\pi_L$. Clearly $\pi_L < \bar{\pi} < \pi_H$. Type-$L$ workers are better off after government intervention since they make the same choice of education as before (namely y = 0) but are paid more ($\bar{\pi}$ instead of π_L). Type-H workers are better off if and only if

$\underbrace{\mu_H \pi_H + (1-\mu_H)\pi_L}_{\bar{\pi}} > \pi_H - \hat{y}^2$, that is, if $\mu_H > 1 - \dfrac{\hat{y}^2}{\pi_H - \pi_L}$; replacing the values $\pi_L = 2,\ \pi_H = \dfrac{9}{4}, \hat{y} = 0.4$ we get $\mu_H > 1 - \dfrac{(0.4)^2}{0.25} = 0.36$ (=36%).

(c)(c.1) At a signaling equilibrium type *L* workers choose *y* = 0 and *x* = 0 and type *H* workers choose $y = \hat{y}$ and $x = \hat{x}$. This is rational if only if (1) $\pi_L > \pi_H - \hat{y} - \hat{x}$ (that is, $\hat{y} > \pi_H - \pi_L - \hat{x}$) and (2) $\pi_H - \hat{y}^2 - \hat{x} > \pi_L$ (that is, $\hat{y} < \sqrt{\pi_H - \pi_L - \hat{x}}$). These two inequalities can be simultaneously satisfied, if and only if $\pi_H - \pi_L - \hat{x} < 1$, that is, if and only if $\hat{x} > \pi_H - \pi_L - 1$.

(c.2) If $\pi_L = 3, \pi_H = 4, \hat{x} = 0.5, \hat{y} = 0.6$ then $\pi_H - \pi_L - \hat{x} = 0.5$ and $\sqrt{\pi_H - \pi_L - \hat{x}} = 0.707$. Thus the inequalities of part (c.1) are satisfied and we have a signaling equilibrium where the *L*-types choose *x* = *y* = 0 and the *H*-types choose *x* = 0.5 and *y* = 0.6.

(c.3) If $\pi_L = 2, \pi_H = 7, \hat{x} = 3, \hat{y} = 2.5$ then $\pi_H - \pi_L - \hat{x} = 2$ and $\sqrt{\pi_H - \pi_L - \hat{x}} = 1.414$ so that the inequalities of part (c.1) are not satisfied. Both types choose *x* = *y* = 0.

3. There is a common prior given by: $\begin{pmatrix} a & b & c & d \\ \frac{12}{25} & \frac{3}{25} & \frac{6}{25} & \frac{4}{25} \end{pmatrix}$. Thus the Harsanyi transformation can indeed be applied to yield the following extensive form: