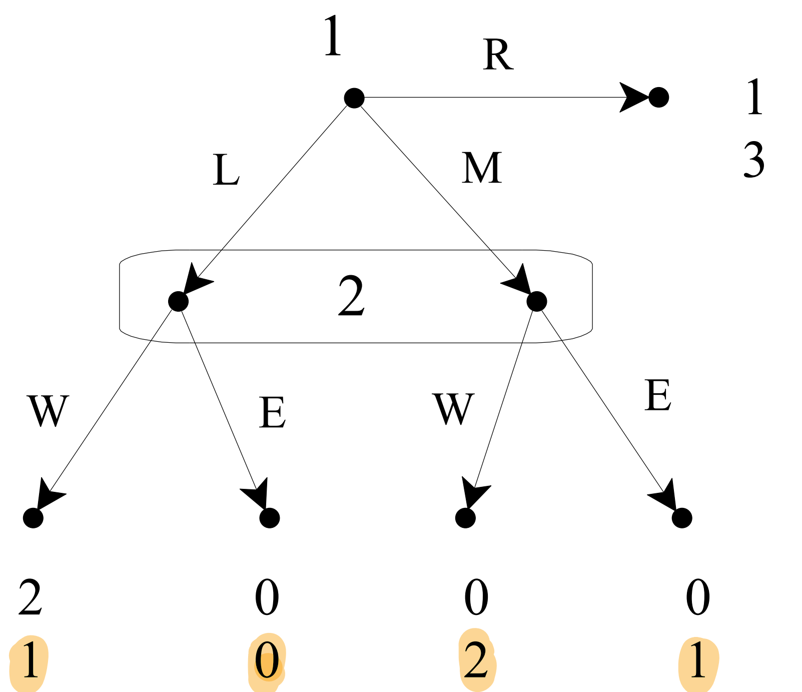
WEAK SEQUENTIAL EQUILIBRIUM



		Player 2	
		W	***E***
Player 1	***L***	2, 1	0, 0
	M	0, 2	0, 1
	R	1, 3	1, 3

Two NE: (L, W), (R, E)

both trivially SPE

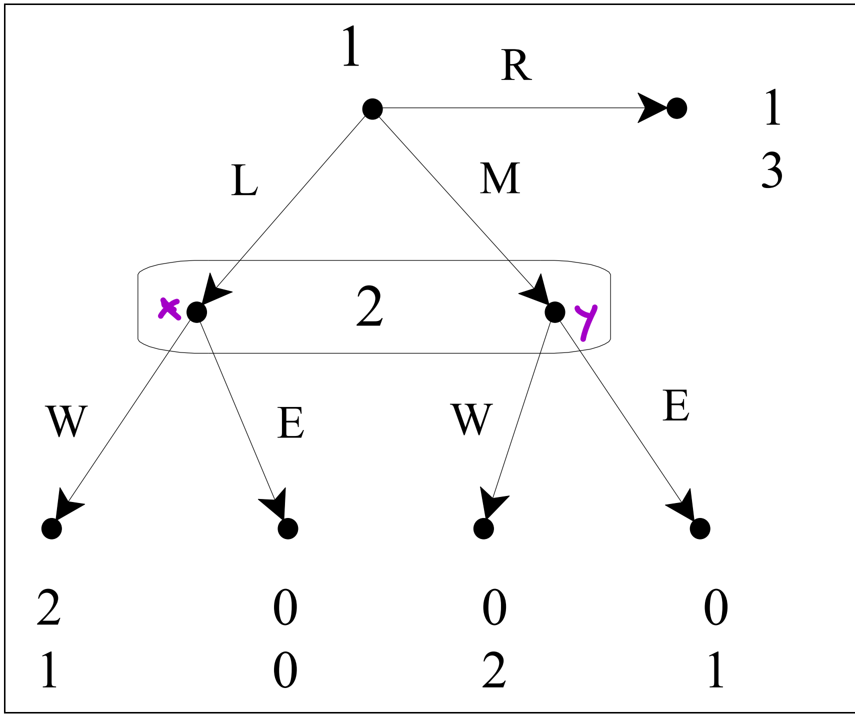
want to eliminate

W strictly dominates E at 2's information set

E is a non-credible threat

Definition. An ***assessment*** is a pair (σ, μ) where σ is a profile of behavioral strategies and μ is a "system of beliefs" defined as a collection of probability distributions, one for each information set over the set of nodes in that information set.

$\mu = \begin{pmatrix} x & y \\ p & 1-p \end{pmatrix} \quad 0 \le p \le 1$



Nash and Subgame-Perfect equilibria: (R,E) and (L,W)

$\left(\overbrace{(R,E)}^{\sigma}, \underbrace{\begin{pmatrix} x & y \\ p & 1-p \end{pmatrix}}_{\mu} \right)$

$\left((L,W), \begin{pmatrix} x & y \\ p & 1-p \end{pmatrix} \right)$

SEQUENTIAL RATIONALITY. At each information set, the action(s) taken by the player who has to move must be optimal given the player's beliefs at that information set and given the subsequent strategies (of his and of the other players).

$E \rightarrow p\,0 + (1-p)\,1 = 1-p$

$W \rightarrow p \cdot 1 + (1-p)\,2 = 2-p$

W better than E for any $p \in [0,1]$ so E is not sequentially rational

W is better than E for every $p \in [0,1]$ so W is sequentially rational

Definition. An ***assessment*** is a pair (σ, μ) where σ is a profile of behavioral strategies and μ is a "system of beliefs" defined as a collection of probability distributions, one for each information set over the set of nodes in that information set.

$$\sigma = \left(\begin{array}{ccc|cc:cc|cc} A & B & C & D & E & F & G & H & I \\ \frac{1}{6} & \frac{4}{6} & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & 1 & 0 & 1 & 0 \end{array}\right) \qquad \mu = \left(\begin{array}{cc|ccc} s & t & u & v & x \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{4} & \frac{2}{4} & \frac{1}{4} \end{array}\right)$$

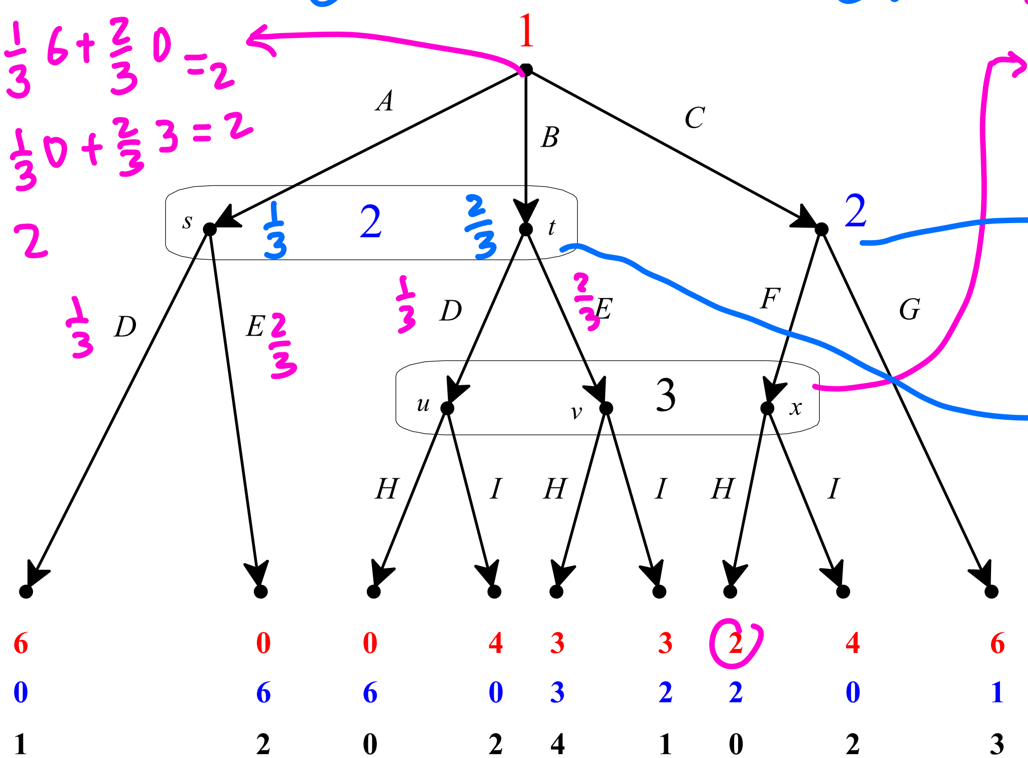
$H \to \frac{1}{4}0 + \frac{1}{2}4 + \frac{1}{4}0 = 2$

$I \to \frac{1}{4}2 + \frac{1}{2}1 + \frac{1}{4}2 = 1$

$A \to \frac{1}{3}6 + \frac{2}{3}0 = 2$

$B \to \frac{1}{3}0 + \frac{2}{3}3 = 2$

$C \to 2$



$G \to 1$

$F \to 2$ (F followed by H)

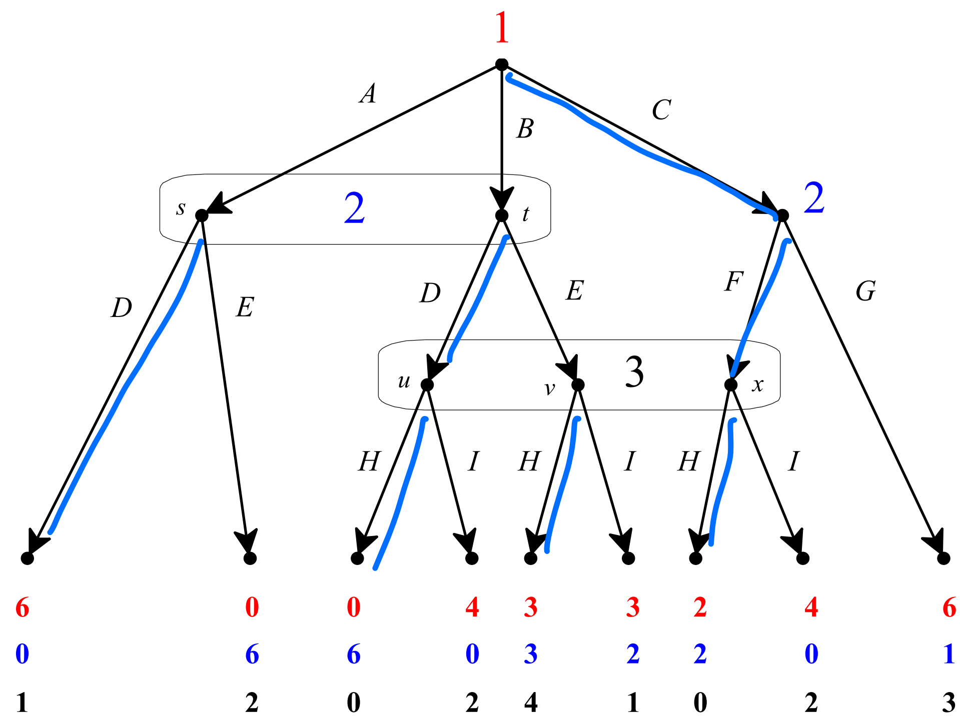
$D \to \frac{1}{3}0 + \frac{2}{3}6 = 4$

$E \to \frac{1}{3}6 + \frac{2}{3}3 = 2 + 2 = 4$

$P(s \mid \{A,B\} \mid \{s,t\}) = \dfrac{\frac{1}{6}}{\frac{1}{6} + \frac{4}{6}} = \frac{1}{5}$

SEQUENTIAL RATIONALITY. At each information set, the action(s) taken by the player who has to move must be optimal given the player's beliefs at that information set and given the subsequent strategies (of his and of the other players).

Definition. An information set is **reachable** by the strategy profile σ if at least one node in it is reached with positive probability when the game is played according to σ.



$\sigma = (C, D, F, H)$ makes

$\{s,t\}$ not reachable

$\{u,v,x\}$ is reachable

$\sigma' = \left(\begin{pmatrix} A & B & C \\ \varepsilon & 0 & 1-\varepsilon \end{pmatrix}, \ldots \right)$

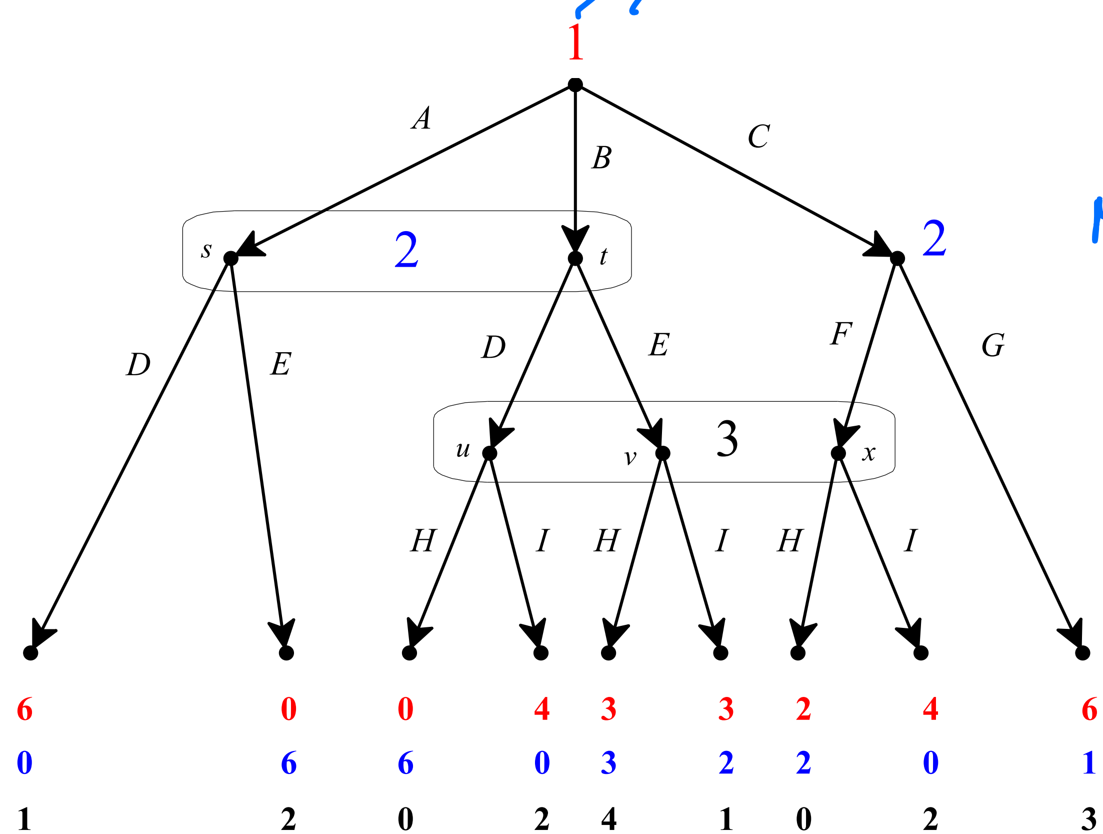
$\{s,t\}$ is reachable

CONSISTENCY WITH BAYES' RULE: For information sets that are reachable, beliefs should be obtained using Bayes' rule.

$\frac{1}{5}$ $\frac{4}{5}$ only Bayes consistent beliefs for Player 2 given σ

$$\sigma = \begin{pmatrix} A & B & C & D & E & F & G & H & I \\ \frac{1}{6} & \frac{4}{6} & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & 1 & 0 & 1 & 0 \end{pmatrix} \quad \mu = \begin{pmatrix} s & t & u & v & x \\ \cancel{\frac{1}{3}} & \cancel{\frac{2}{3}} & \frac{1}{4} & \frac{2}{4} & \frac{1}{4} \end{pmatrix}$$

is **not** Bayes consistent because ... $\{s,t\}$ is reachable (with probability $\frac{1}{6} + \frac{4}{6} = \frac{5}{6}$)



$$\mu(s) = P(s \mid \{s,t\}) = \frac{P(s)}{P(\{s,t\})} = \frac{\frac{1}{6}}{\frac{5}{6}} = \frac{1}{5}$$

trivially Bayes' consistent

Definition. An assessment (σ,μ) is a **Weak Sequential Equilibrium** if it satisfies the two requirements: sequential rationality and consistency with Bayes' rule (at reachable information sets).

$$\sigma=\left(\begin{array}{ccc|cc|cc|cc} A & B & C & D & E & F & G & H & I \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right) \quad \mu=\left(\begin{array}{cc|ccc} s & t & u & v & x \\ \frac{1}{3} & \frac{2}{3} & 0 & \frac{1}{4} & \frac{3}{4} \end{array}\right)$$ is a weak sequential equilibrium.

trivially Bayes' consistent

$D \to \frac{1}{3}0+\frac{2}{3}0=0$

$E \to \frac{1}{3}6+\frac{2}{3}2=4$

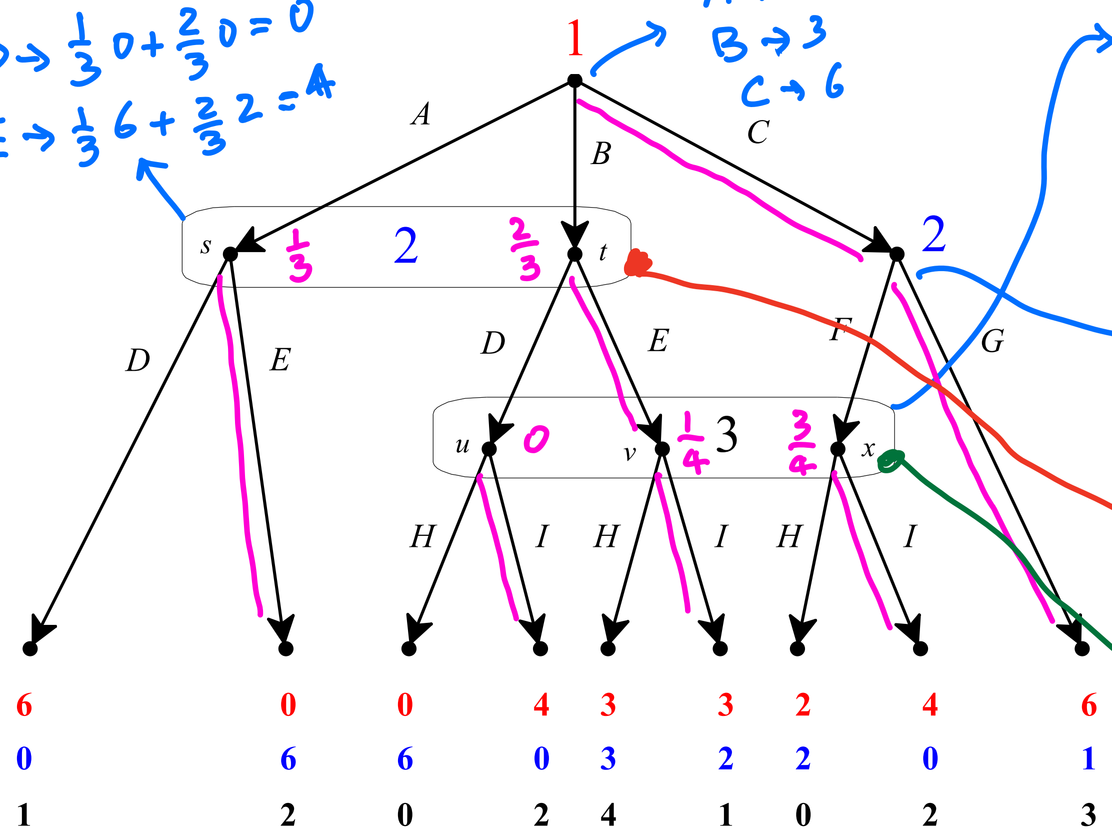
$A \to 0$

$B \to 3$

$C \to 6$

$H \to \frac{1}{4}4+\frac{3}{4}0=1$

$I \to \frac{1}{4}1+\frac{3}{4}2=\frac{7}{4}$



$G \to 1, \quad F \to 0$

sequential rationality ✓

$P(\{s,t\}|\sigma)=0$ Bayes' rule not applicable

Bayes' rule not applicable

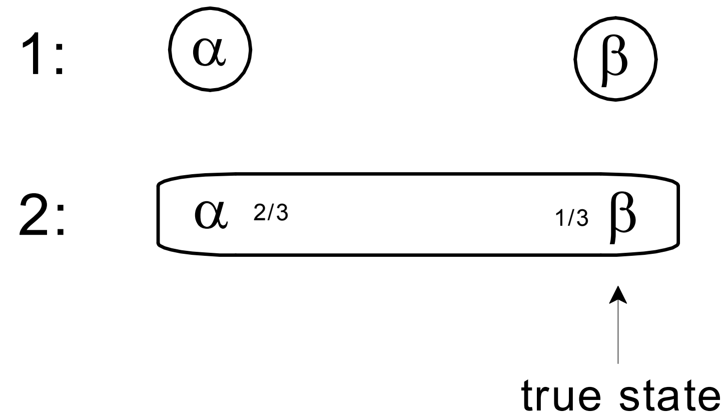
Theorem. If (σ,μ) is a weak sequential equilibrium then σ is a Nash equilibrium.

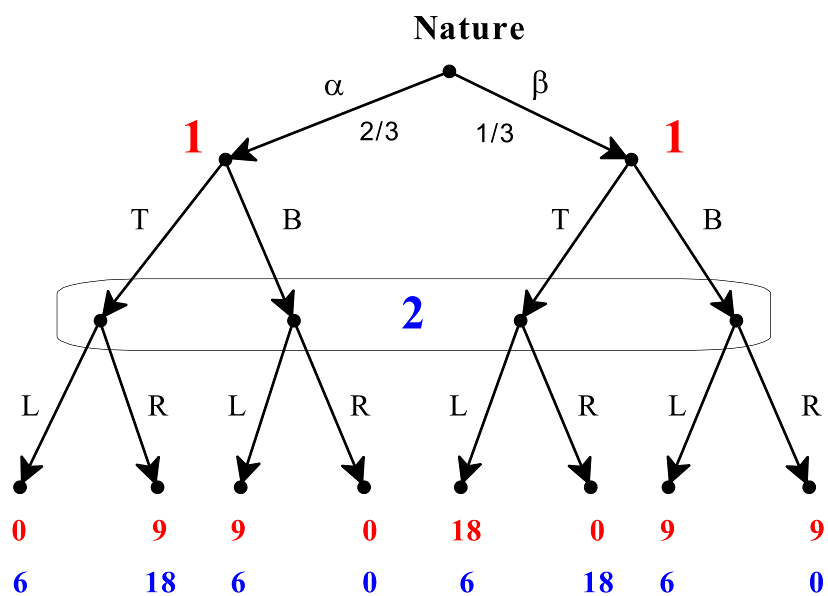
G_1

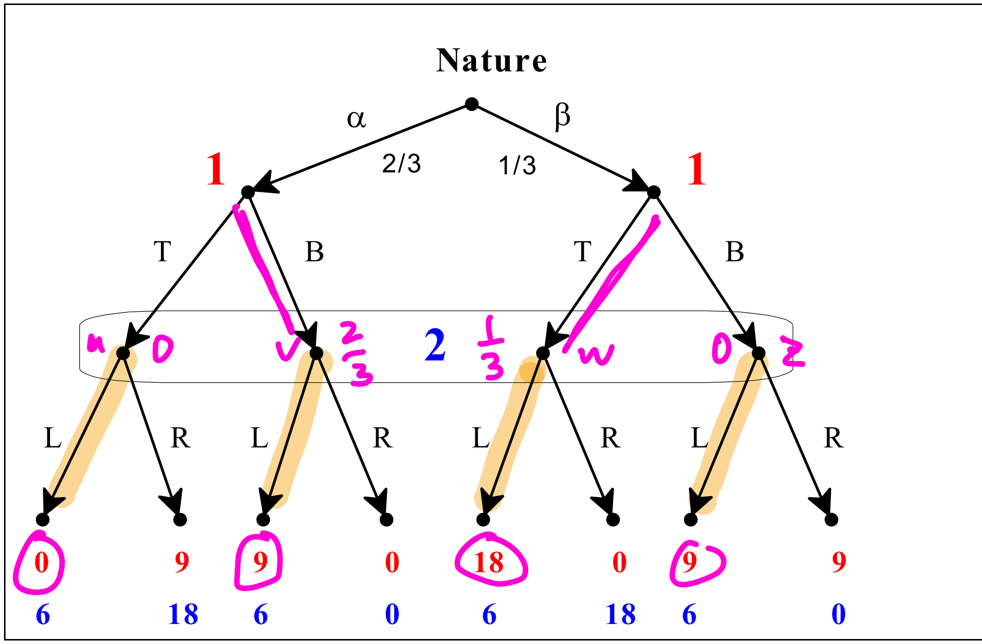
1 \ 2	L	R
T	0, 6	9, 18
B	9, 6	0, 0

G_2

1 \ 2	L	R
T	18, 6	0, 18
B	9, 6	9, 0







Suppose there is a pure-strategy WSE where Player 2's strategy is L:

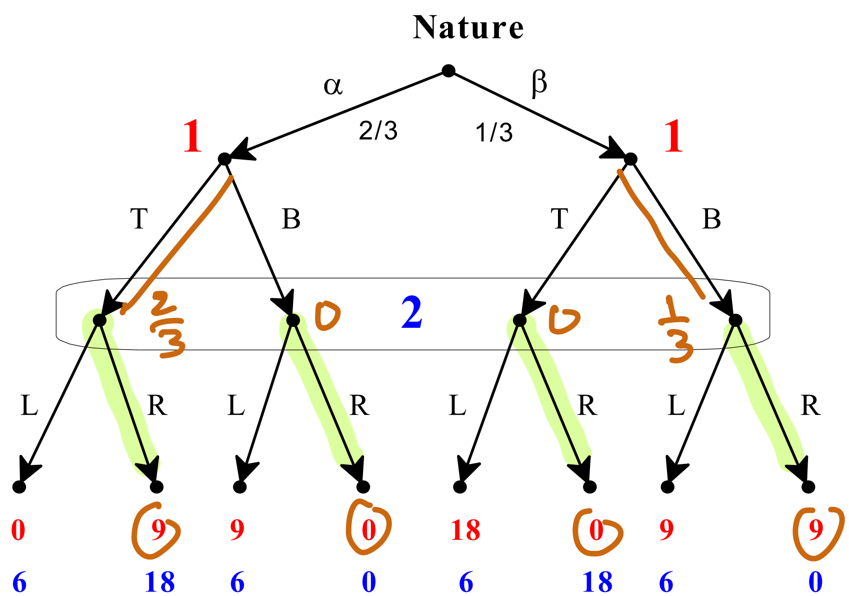
$$L \to \frac{2}{3}6 + \frac{1}{3}6 = 6$$

$$R \to \frac{2}{3}0 + \frac{1}{3}18 = 6$$

$$\underbrace{\sigma = (BT, L) \qquad \mu = \begin{pmatrix} u & v & w & z \\ 0 & \frac{2}{3} & \frac{1}{3} & 0 \end{pmatrix}}_{WSE}$$

Theorem: if (σ, μ) is a WSE then σ is a N.E.

So (BT, L) is a N.E.



Suppose there is a pure-strategy WSE where Player 2's strategy is R:

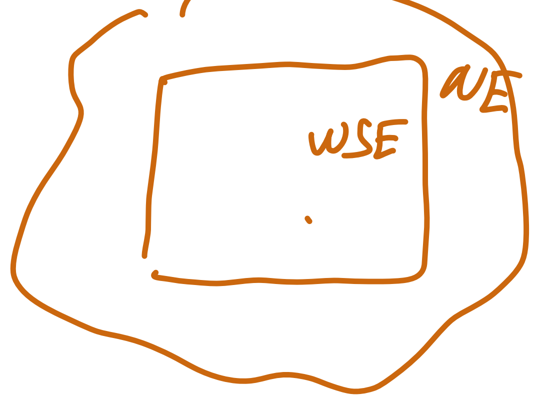
$R \rightarrow \frac{2}{3} 18 + \frac{1}{3} 0 = 12$

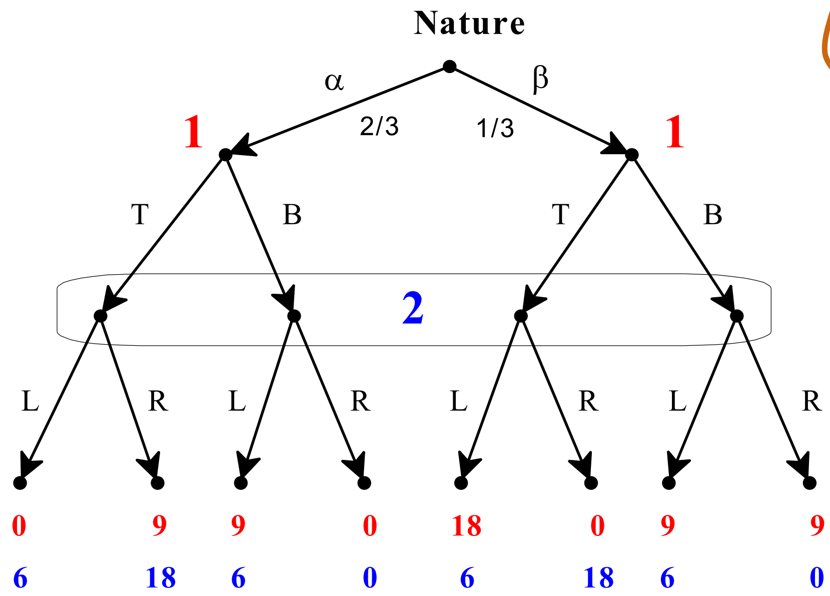
$L \rightarrow \frac{2}{3} 6 + \frac{1}{3} 6 = 6$

$\sigma = (TB, R)$ $\mu = \left(\frac{2}{3} \; 0 \; 0 \; \frac{1}{3}\right)$

WSE

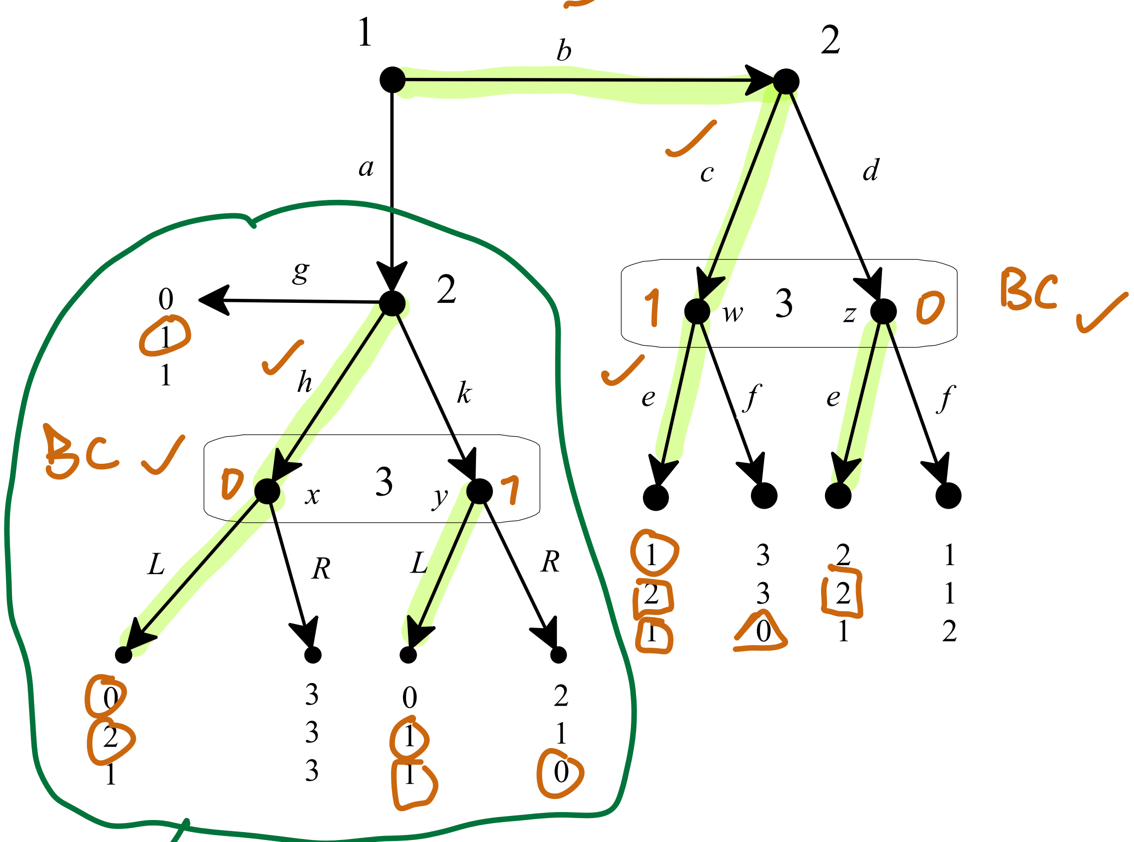
So (TB, R) is a NE

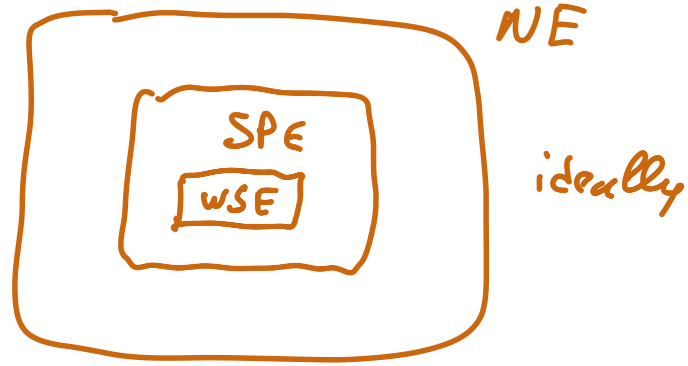


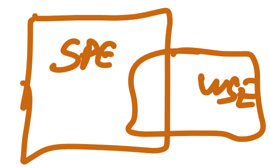


Hence both **(*TB*,*R*)** and **(*BT*,*L*)** are Nash equilibria.

NOTE: in a game where, no matter what strategies the players choose, every information set is reached with positive probability, the set of Nash equilibria coincides with the set of weak sequential equilibria. Thus in the above game (TB,R) and (BT,L) are the only pure-strategy Nash equilibria







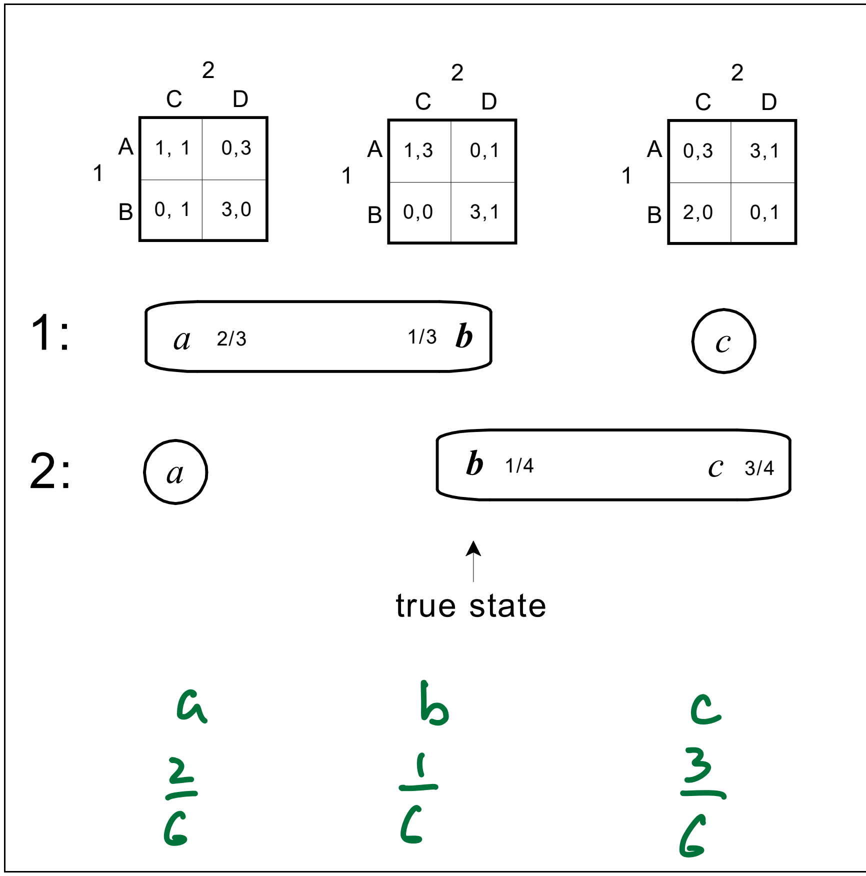
$$\sigma = (b, ch, eL) = \underbrace{\left(\begin{array}{cc|cc|ccc|cc|cc} a & b & c & d & g & h & k & e & f & L & R \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right)}_{\text{but not SPE}} \quad \mu = \left(\begin{array}{cc|cc} x & y & w & z \\ 0 & 1 & 1 & 0 \end{array}\right)$$

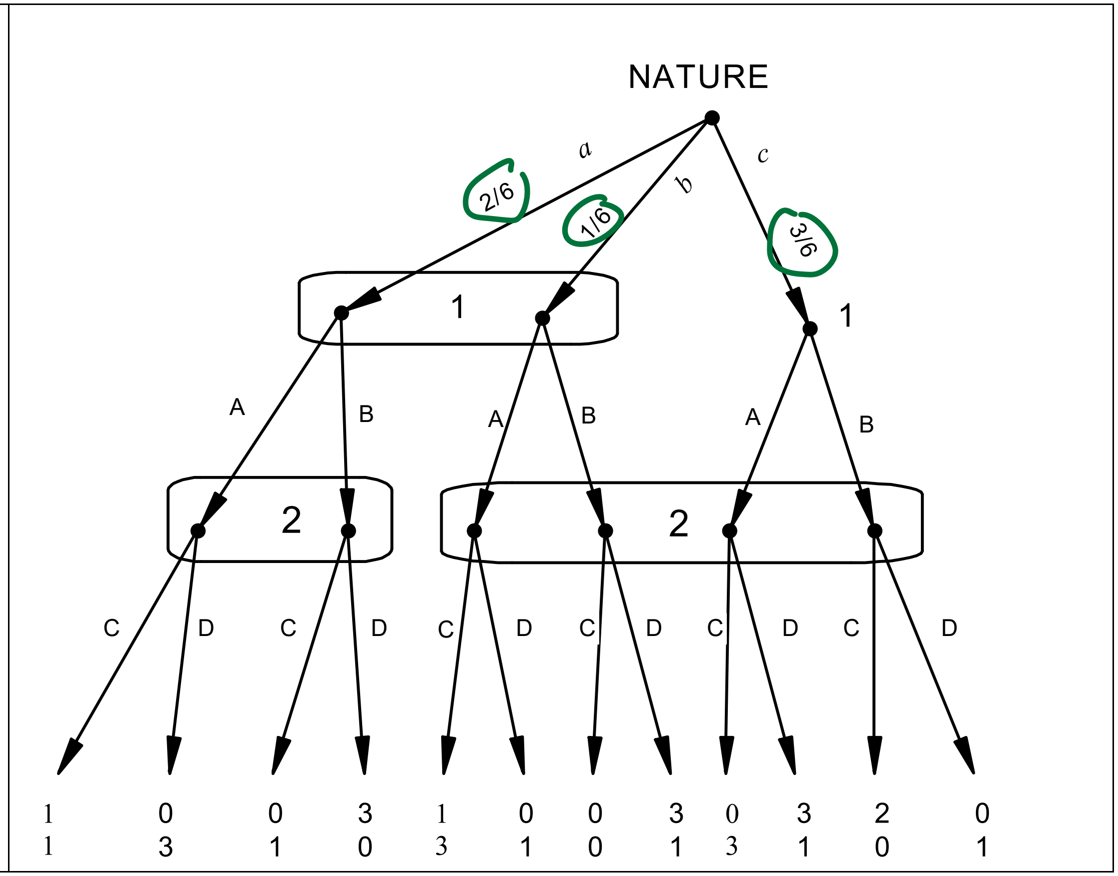
is a WSE

so σ is a NE

(h, L) not a NE of this subgame because L not a best reply to h

WSE ≈ Perfect Bayesian equilibrium





This game does not have any pure-strategy (Nash equilibria) WSE hence no pure strategy NE