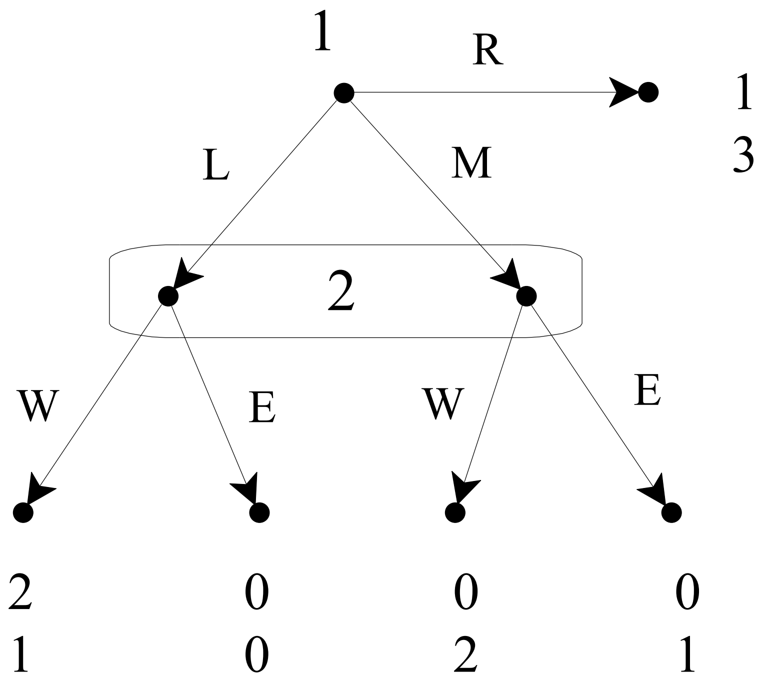
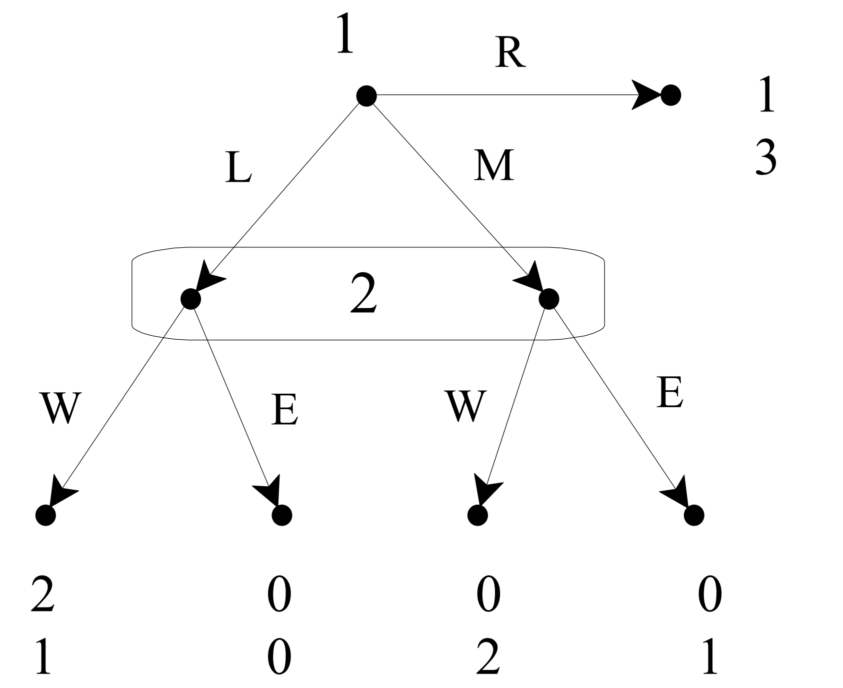
WEAK SEQUENTIAL EQUILIBRIUM



		Player 2	
		W	*E*
Player 1	*L*		
	M		
	R		

Definition. An ***assessment*** is a pair (σ,μ) where σ is a profile of behavioral strategies and μ is a "system of beliefs" defined as a collection of probability distributions, one for each information set over the set of nodes in that information set.

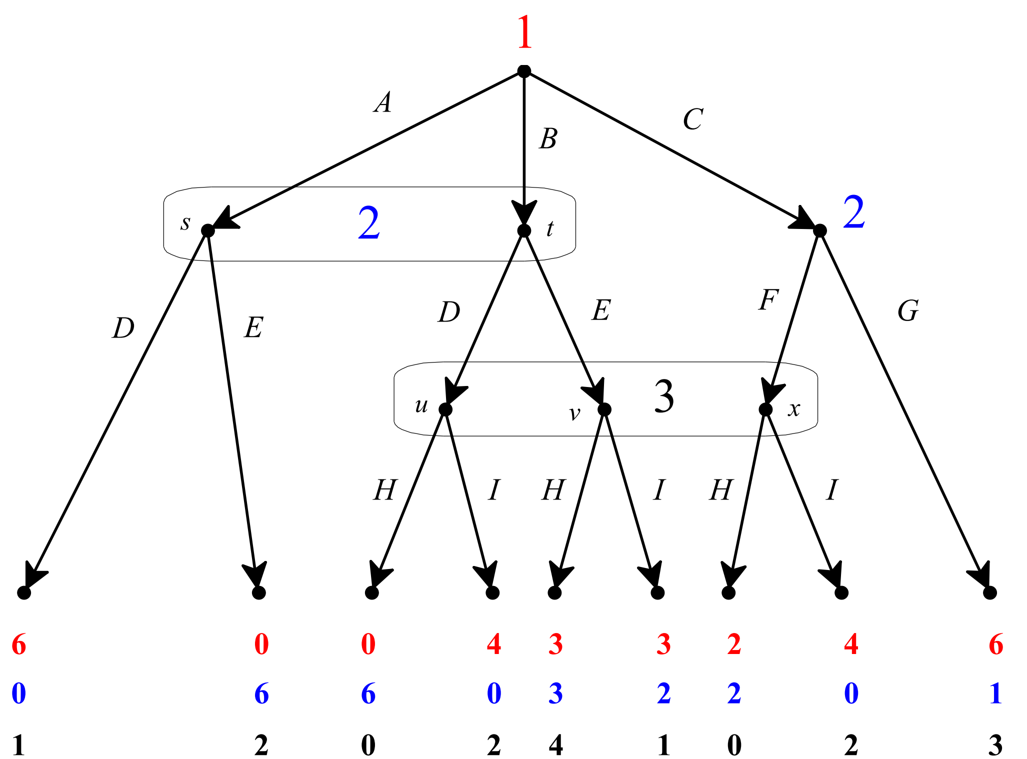


Nash and Subgame-Perfect equilibria: (R,E) and (L,W)

SEQUENTIAL RATIONALITY. At each information set, the action(s) taken by the player who has to move must be optimal given the player's beliefs at that information set and given the subsequent strategies (of his and of the other players).

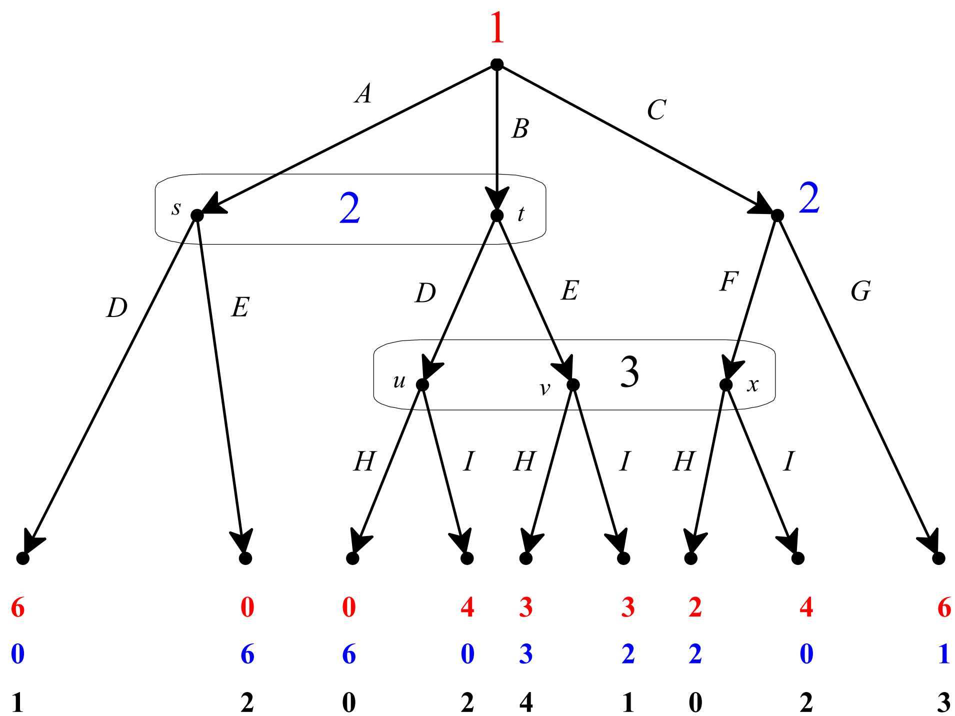
Definition. An ***assessment*** is a pair (σ, μ) where σ is a profile of behavioral strategies and μ is a "system of beliefs" defined as a collection of probability distributions, one for each information set over the set of nodes in that information set.

$$\sigma = \left(\begin{array}{ccc|cc:cc|cc} A & B & C & D & E & F & G & H & I \\ \frac{1}{6} & \frac{4}{6} & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & 1 & 0 & 1 & 0 \end{array} \right) \quad \mu = \left(\begin{array}{cc|ccc} s & t & u & v & x \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{4} & \frac{2}{4} & \frac{1}{4} \end{array} \right)$$



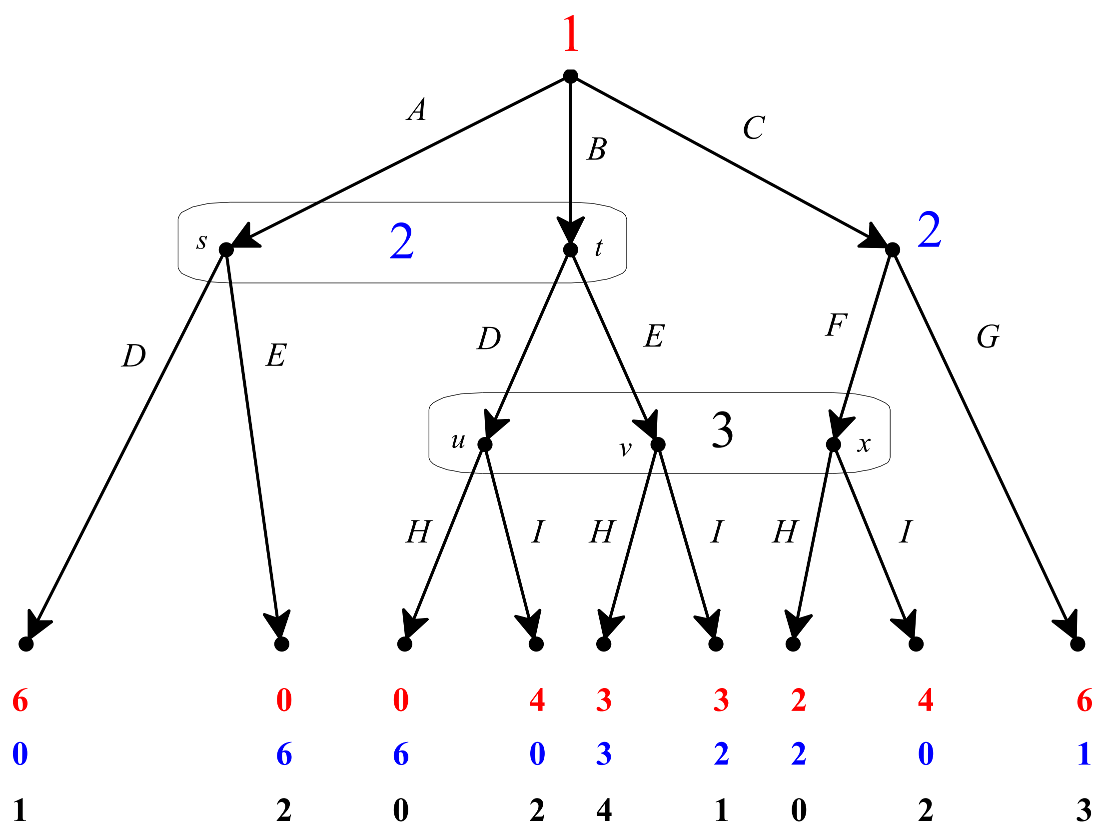
SEQUENTIAL RATIONALITY. At each information set, the action(s) taken by the player who has to move must be optimal given the player's beliefs at that information set and given the subsequent strategies (of his and of the other players).

Definition. An information set is **reachable** by the strategy profile σ if at least one node in it is reached with positive probability when the game is played according to σ.



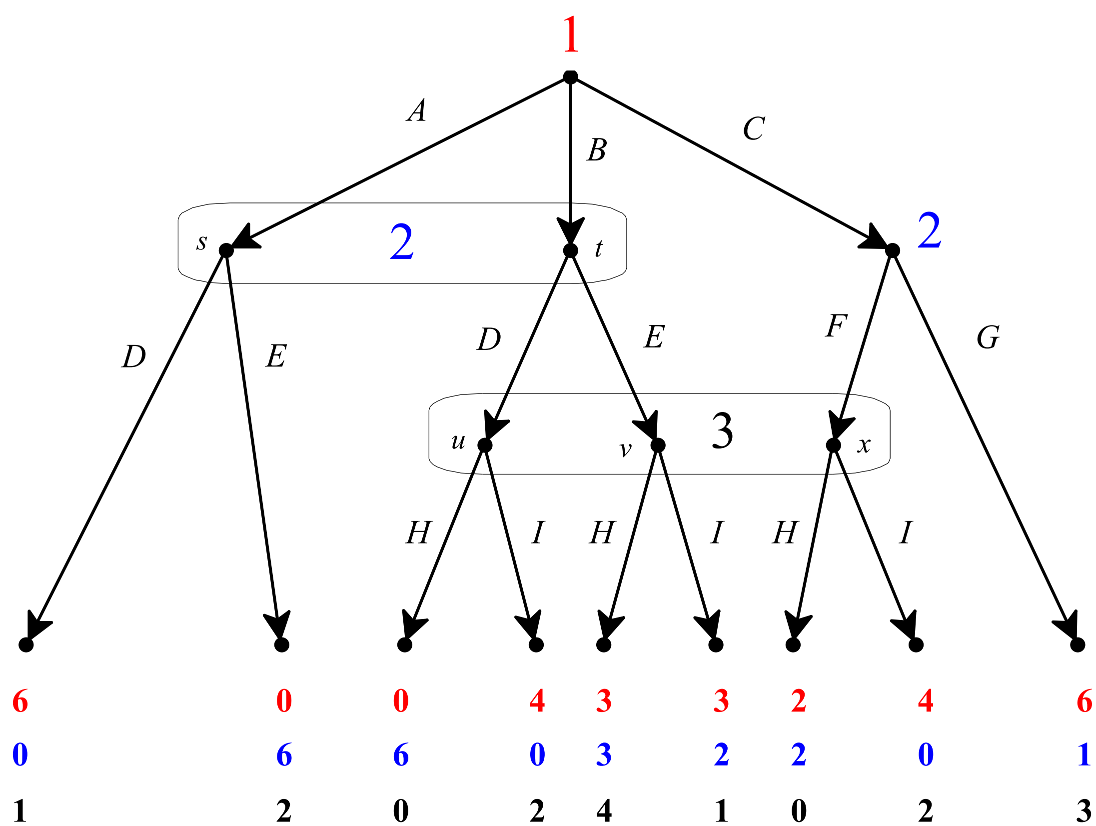
CONSISTENCY WITH BAYES' RULE: For information sets that are reachable, beliefs should be obtained using Bayes' rule.

$\sigma = \left(\begin{array}{ccc|cc|cc|cc} A & B & C & D & E & F & G & H & I \\ \frac{1}{6} & \frac{4}{6} & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & 1 & 0 & 1 & 0 \end{array} \right)$ $\mu = \left(\begin{array}{cc|ccc} s & t & u & v & x \\ \frac{1}{3} & \frac{2}{3} & \frac{1}{4} & \frac{2}{4} & \frac{1}{4} \end{array} \right)$ is **not** Bayes consistent because ...



Definition. An assessment (σ,μ) is a **Weak Sequential Equilibrium** if it satisfies the two requirements: sequential rationality and consistency with Bayes' rule (at reachable information sets).

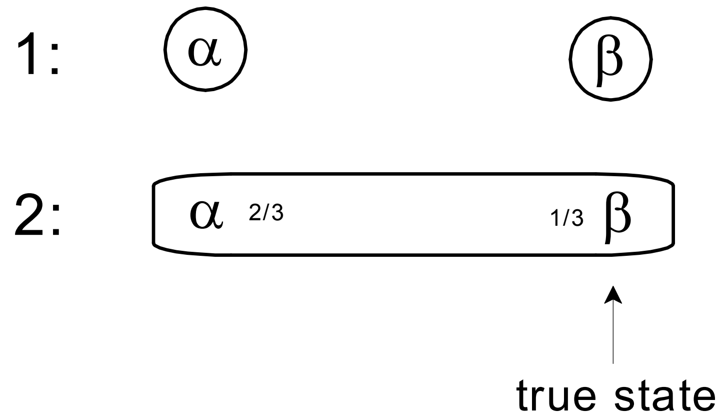
$\sigma = \left(\begin{array}{ccc|cc:cc|cc} A & B & C & D & E & F & G & H & I \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right)$ $\mu = \left(\begin{array}{cc|ccc} s & t & u & v & x \\ \frac{1}{3} & \frac{2}{3} & 0 & \frac{1}{4} & \frac{3}{4} \end{array}\right)$ is a weak sequential equilibrium.

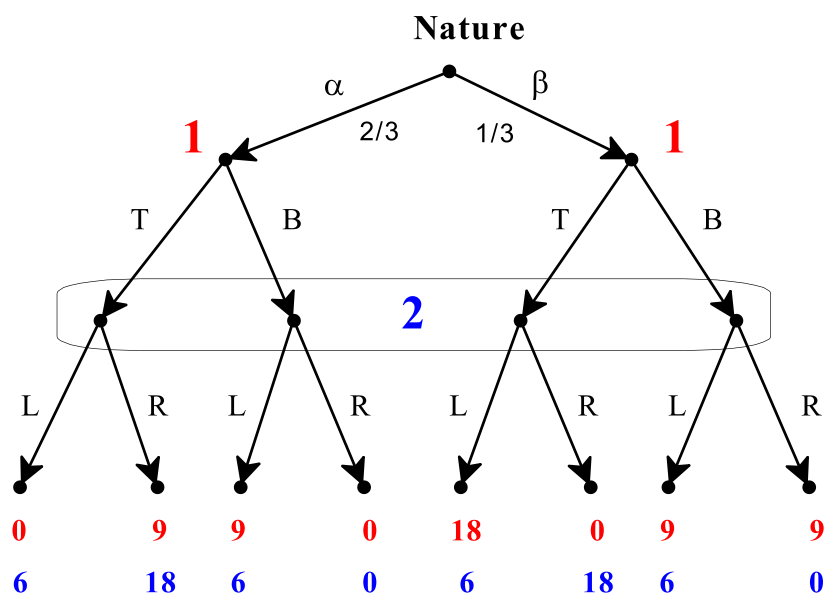


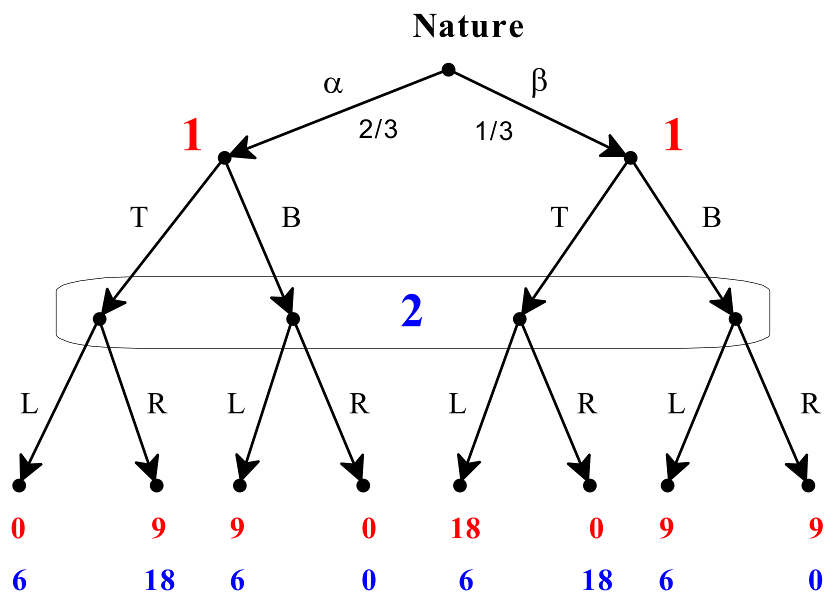
Theorem. If (σ,μ) is a weak sequential equilibrium then σ is a Nash equilibrium.

		2: L	2: R
1	T	0, 6	9,18
	B	9, 6	0, 0

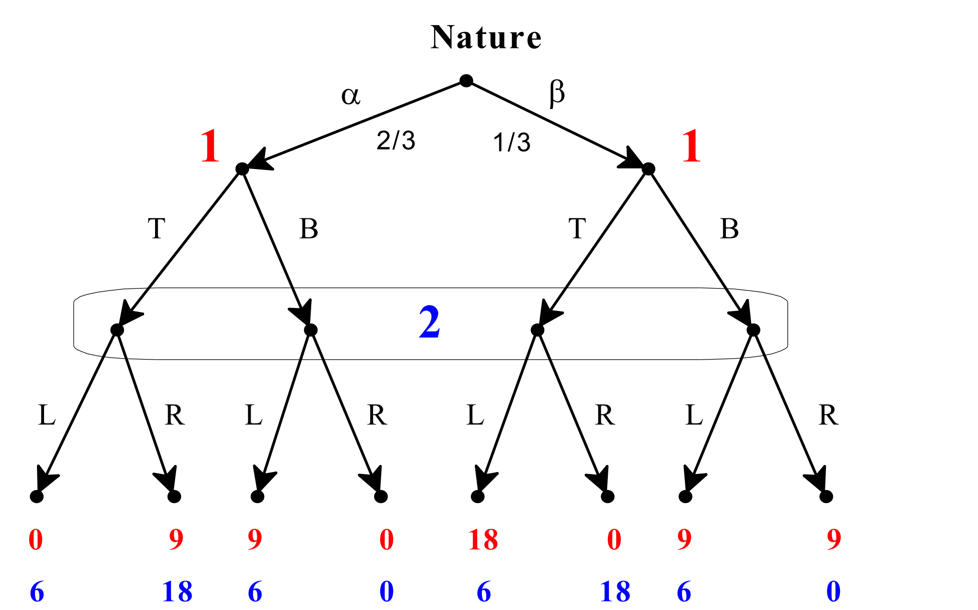
		2: L	2: R
1	T	18,6	0, 18
	B	9, 6	9, 0



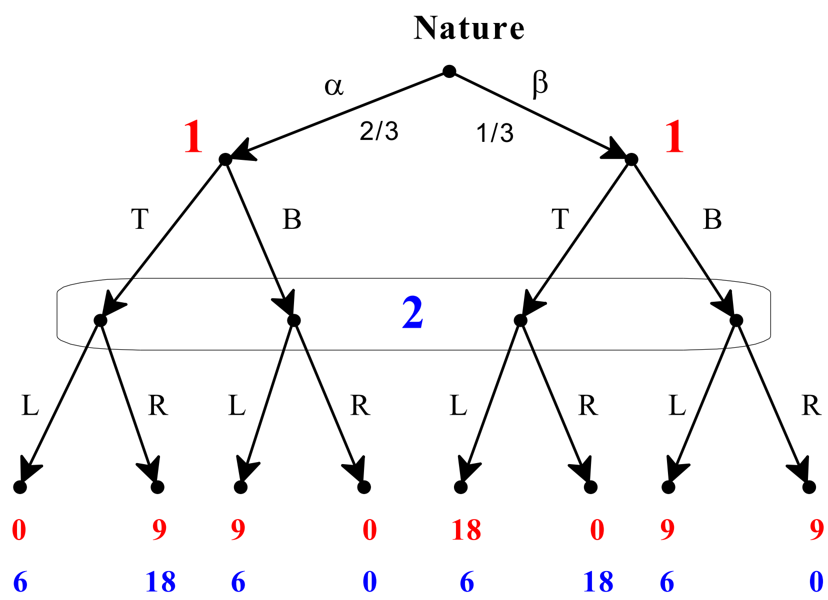




Suppose there is a pure-strategy WSE where Player 2's strategy is L:

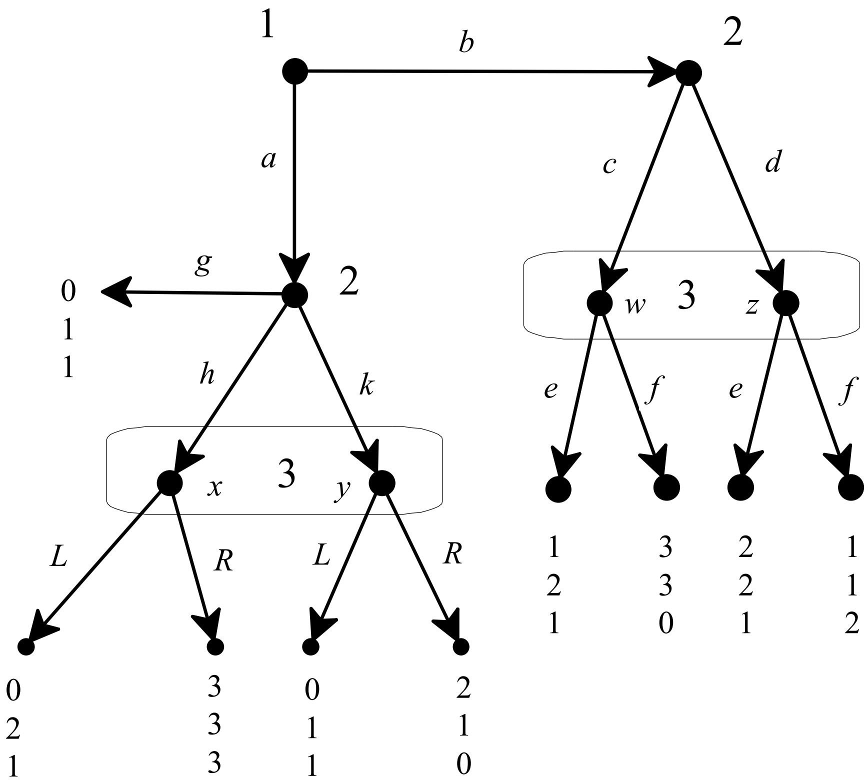


Suppose there is a pure-strategy WSE where Player 2's strategy is R:

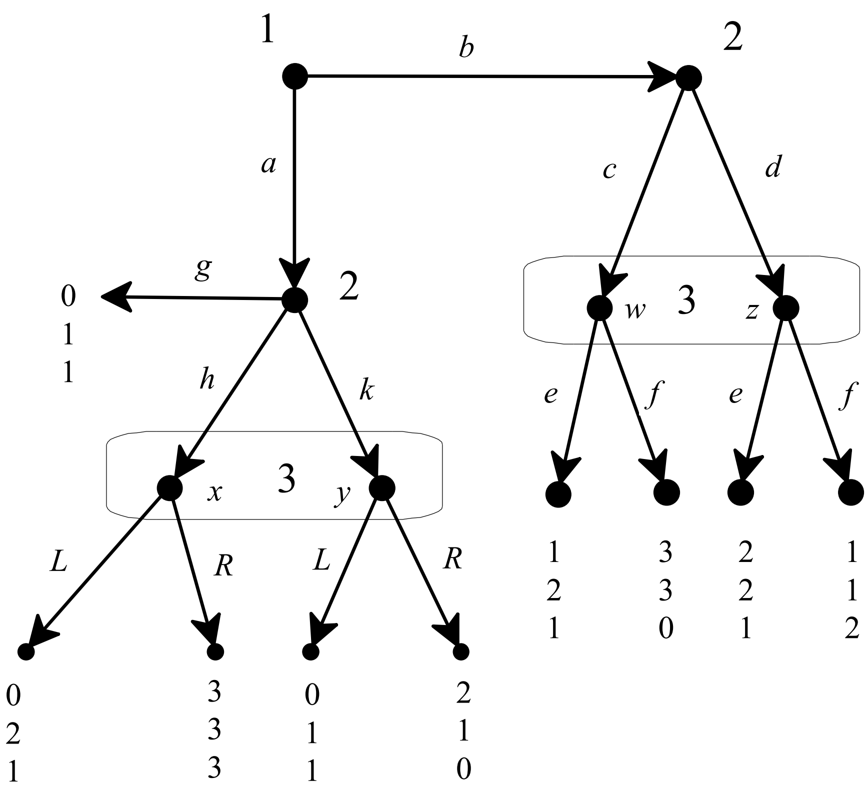


Hence both ***(TB,R)*** and ***(BT,L)*** are Nash equilibria.

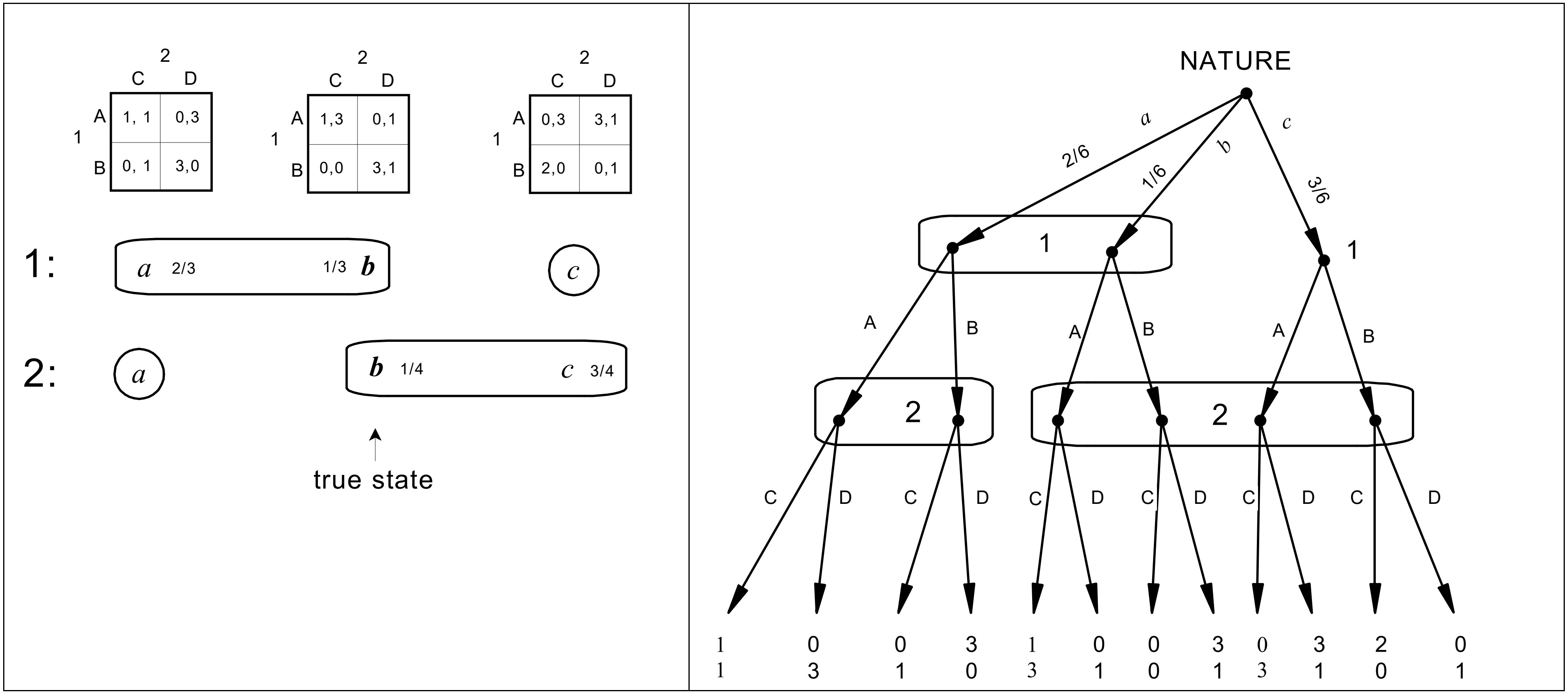
NOTE: in a game where, no matter what strategies the players choose, every information set is reached with positive probability, the set of Nash equilibria coincides with the set of weak sequential equilibria. Thus in the above game (TB,R) and (BT,L) are the only pure-strategy Nash equilibria

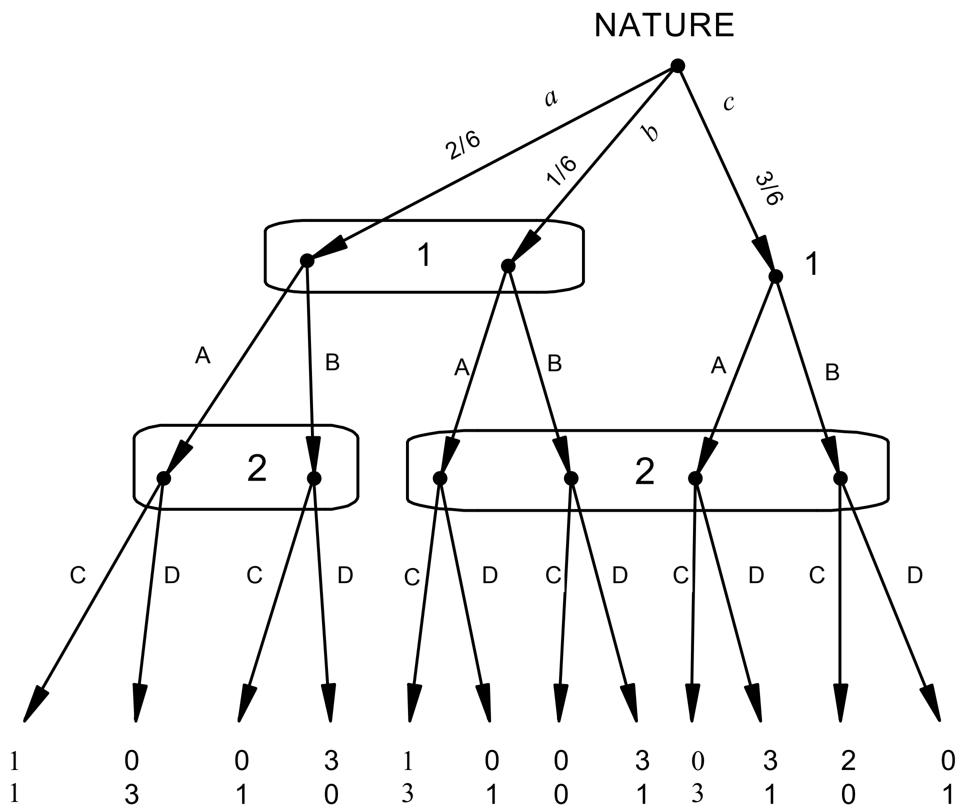


$$\sigma = (b, ch, eL) = \left(\begin{array}{cc|cc|ccc|cc|cc} a & b & c & d & g & h & k & e & f & L & R \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right) \quad \mu = \left(\begin{array}{cc|cc} x & y & w & z \\ 0 & 1 & 1 & 0 \end{array}\right)$$

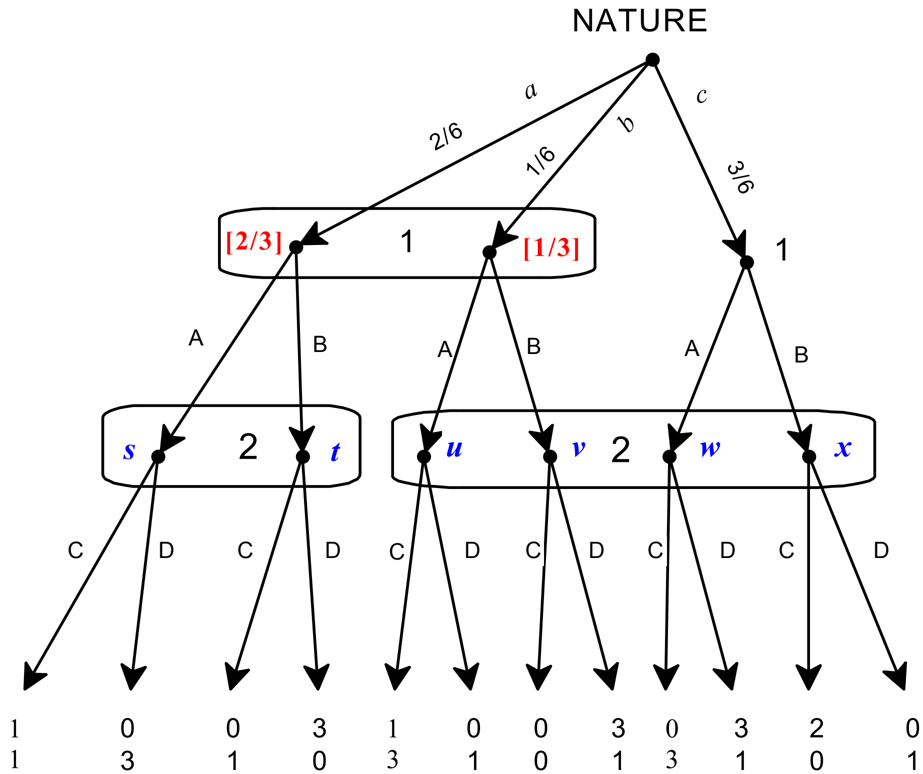


$$\sigma = (a, ch, eR) = \left(\begin{array}{cc|cc:ccc|cc:cc} a & b & c & d & g & h & k & e & f & L & R \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \quad \mu = \left(\begin{array}{cc|cc} x & y & w & z \\ 1 & 0 & 1 & 0 \end{array}\right)$$

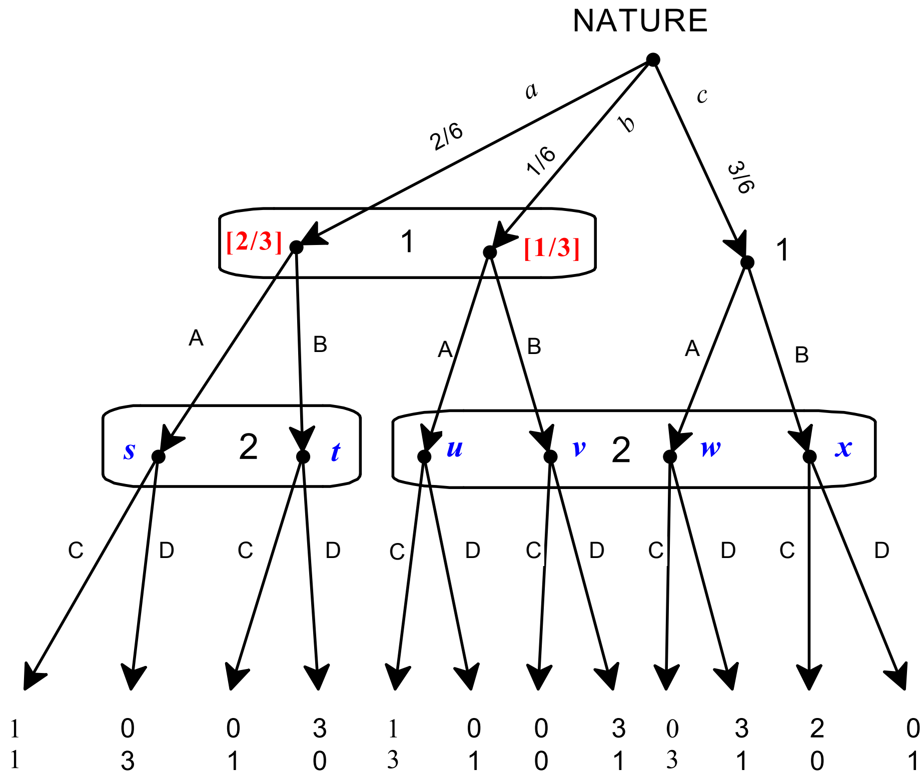

2
C D
A 1, 1 0,3
1
B 0, 1 3,0
2
C D
A 1,3 0,1
1
B 0,0 3,1
2
C D
A 0,3 3,1
1
B 2,0 0,1
1: a 2/3 1/3 b c
2: a b 1/4 c 3/4
true state
NATURE
a 2/6
b 1/6
c 3/6
1
1
A B A B A B
2
2
C D C D C D C D C D C D
1 0 0 3 1 0 0 3 0 3 2 0
1 3 1 0 3 1 0 1 3 1 0 1



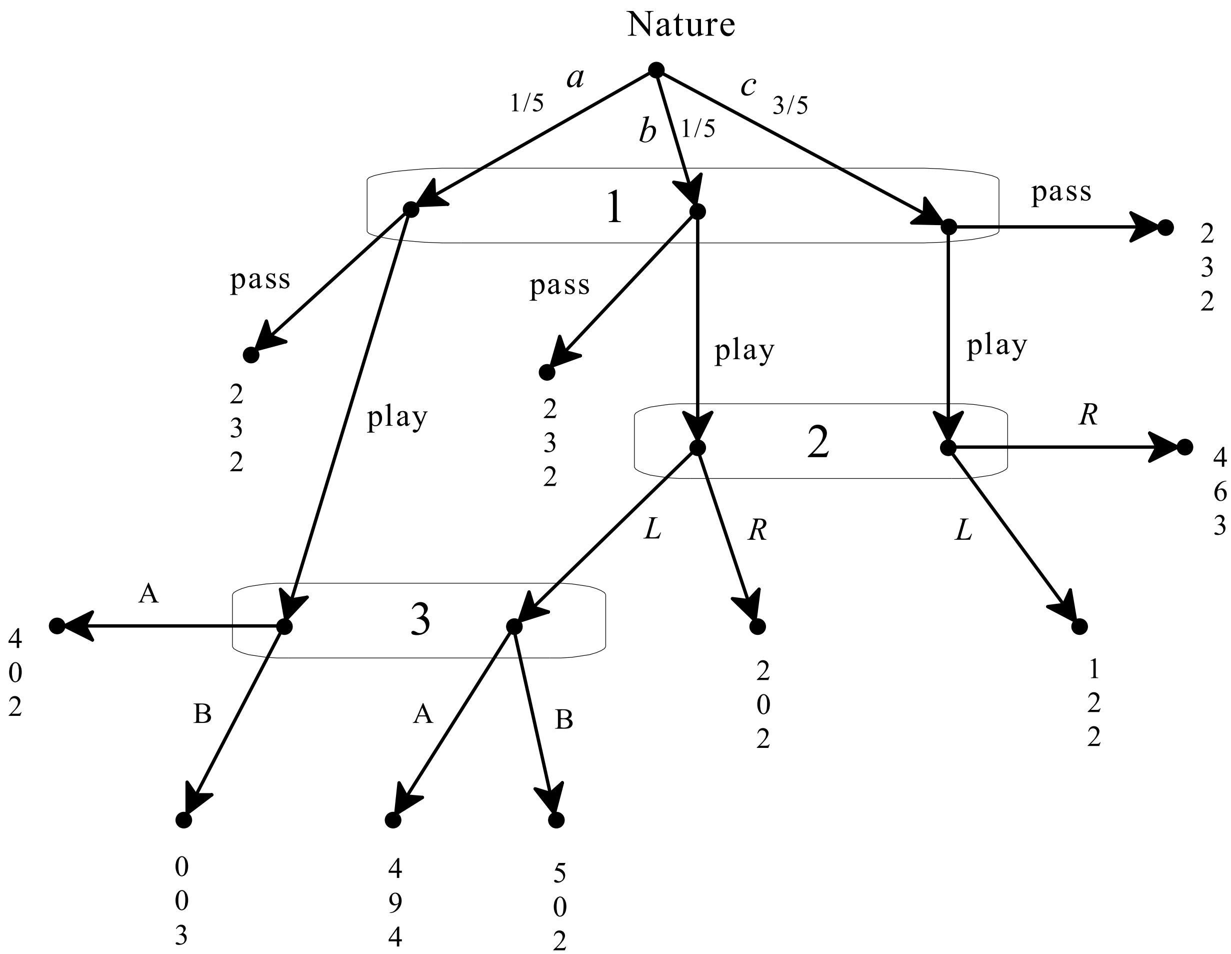
This game does not have pure-strategy Nash equilibria



Suppose that Player 1's pure strategy involves playing *A* at his left information set.



Thus Player 1's pure strategy must involve playing *B* at his left information set.


Nature
a
1/5
b
1/5
c
3/5
1
pass
2
3
2
pass
2
3
2
play
pass
2
3
2
play
play
2
R
4
6
3
L
R
L
3
A
4
0
2
B
0
0
3
A
4
9
4
B
5
0
2
2
0
2
1
2
2