The Cournot model of oligopoly (Antoine Augustin Cournot, 1838)

Example: DUOPOLY

$n = 2$

$$C_1(q_1) = cq_1 \ , \ \ C_2(q_2) = cq_2 \ , \ \ P(Q) = a - b\,Q \qquad a > 0,\ b > 0,\ a > c \qquad Q = q_1 + q_2$$

Properties of CNE

Theorem 1. Let $q^* = (q_1^*, \ldots, q_n^*)$ be a CNE with $q_i^* > 0 \quad \forall i = 1, \ldots, n$. Let $Q^* = \sum_{i=1}^{n} q_i^*$ and $P^* = P(Q^*)$. Then $P^* > MC_i^* \equiv \dfrac{dC_i}{dq_i}(q_i^*)$, $\quad \forall i = 1, \ldots, n$, that is price is greater than marginal cost for every firm.

Properties of CNE continued

Example. Two firms, each can produce either 1 or 2 units at zero cost. The demand function is:

P	Q
5	0
4	1
3	2
2	3
1.1	4

		Firm 2's output	
		1	2
Firm 1's output	1		
	2		

Theorem 2. Let $q* = (q*_1, \ldots, q*_n)$ be a CNE with $q_i^* > 0 \quad \forall i = 1, \ldots, n$. Then there exists a $\hat{q} \neq q*$ such that, $\hat{q}_i < q_i^*$ and $\pi_i(\hat{q}) > \pi_i(q^*)$, $\qquad \forall i = 1, \ldots, n$.

Linear demand and identical firms:

$P(Q) = a - bQ \qquad C_i(q_i) = c\, q_i \qquad (0 < c < a,\ b > 0)$

$$q_i^*(n) = \frac{a-c}{(n+1)b}$$ (output of each firm)

$$Q^*(n) = \frac{n(a-c)}{(n+1)b} = \frac{a-c}{\left(1+\frac{1}{n}\right)b}$$ (industry output)

$$P^*(n) = \frac{a+nc}{n+1} =$$ (price) $$\frac{dP^*}{dn} =$$ (since $a > c$), as $n \to \infty$, $P^* \to c$

$$\pi_i^*(n) = \frac{(a-c)^2}{(n+1)^2 b}$$ (profit per firm).

Existence of CNE

Existence theorem (sufficient conditions) for general games: if

S_i is convex and compact

π_i is continuous and concave in s_i

then a NE exists

$$\frac{\partial \pi_i}{\partial q_i} =$$

$$\frac{\partial^2 \pi_i}{\partial q_i^2} =$$

Joseph Bertrand (1883): what if we maintain the assumptions of Cournot's model but replace quantity competition with price competition? Assume that

- If all firms choose the same price, then consumers pick a firm at random so that each firm expects to get $\frac{1}{n}$ of the total demand (where *n* is the number of firms);
- If prices are different, then all consumers buy from the cheapest firm (if there is more than one cheapest firm, then consumers pick randomly among them).
- All firms have the same cost function given by $C_i(q_i) = c\,q_i$

Bertrand's theorem. Let $p^* = (p_1^*, \ldots, p_n^*)$ be a Bertrand-Nash equilibrium. Then,

(1) for all $i = 1, \ldots n$, $p_i \geq c$, and

(2) for at least two firms *j* and *k* ($j \neq k$), $p_j^* = p_k^* = c$.