Stackelberg equilibrium

Linear inverse demand: $P = a - bQ$, linear cost function $C(q) = cq$ $(0 < c < a)$

Monopoly: $\Pi(Q) = Q(a - bQ) - cQ$. Solve $\frac{\partial \Pi}{\partial Q} = 0$: $\boxed{Q_M = \frac{1}{2}\left(\frac{a-c}{b}\right)}$ $\boxed{\Pi_M = \frac{(a-c)^2}{4b}}$

Duopoly: $\begin{cases} \pi_1(q_1, q_2) = [a - b(q_1 + q_2)]q_1 - cq_1 \\ \pi_2(q_1, q_2) = [a - b(q_1 + q_2)]q_2 - cq_2 \end{cases}$ Solve $\frac{\partial \pi_1}{\partial q_1} = 0, \frac{\partial \pi_2}{\partial q_2} = 0$

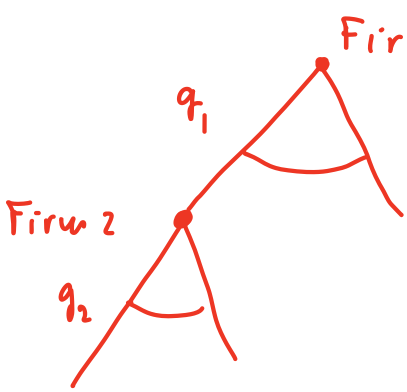
$\frac{2}{3}\left(\frac{25-1}{1}\right) =$

$= 16$

Cournot-Nash equilibrium: $q_1^* = q_2^* = \frac{a-c}{3b}$ so $\boxed{Q_d^* = \frac{2}{3}\left(\frac{a-c}{b}\right)}$ and $\boxed{\Pi_1^* = \Pi_2^* = \frac{(a-c)^2}{9b}}$

Stackelberg. Perfect-information game where Firm 1 choose its output first and **commits to it** moves first and Firm 2 moves second.

$a = 25 \quad b = 1 \quad c = 1$



$\pi_2(q_1, q_2) = [a - b(q_1 + q_2)]q_2 - cq_2$

$\frac{\partial \pi_2}{\partial q_2} = 0$ Solution $q_2 = 12 - \frac{q_1}{2}$

$\pi_1(q_1) = q_1\left[25 - \left(q_1 + 12 - \frac{q_1}{2}\right)\right] - q_1$ (**)

$\frac{\partial \pi_1}{\partial q_1} = 0$ $q_1 = \frac{a-c}{2b} = \frac{25-1}{2} = 12$

$q_2 = 12 - \frac{q_1}{2} = 12 - \frac{12}{2} = 6$

$Q = 18$

$a = 25 \quad b = 1$

$c = 1$

Introduce a fixed cost. $F = 25$

$\boxed{P = 25 - Q}$ and $\boxed{C(q) = \begin{cases} 0 & \text{if } q = 0 \\ q + 25 & \text{if } q > 0 \end{cases}}$

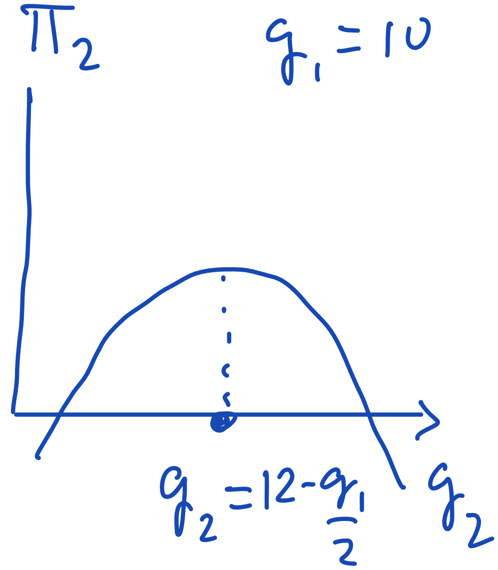
$$\pi_2 = q_2 [25 - (q_1 + q_2)] - q_2 - F$$

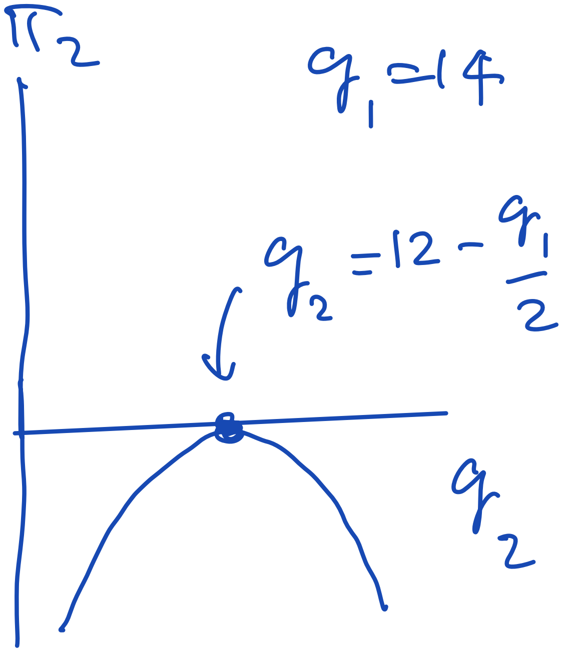
$$\frac{\partial \pi_2}{\partial q_2} = 0 \qquad q_2 = 12 - \frac{q_1}{2}$$

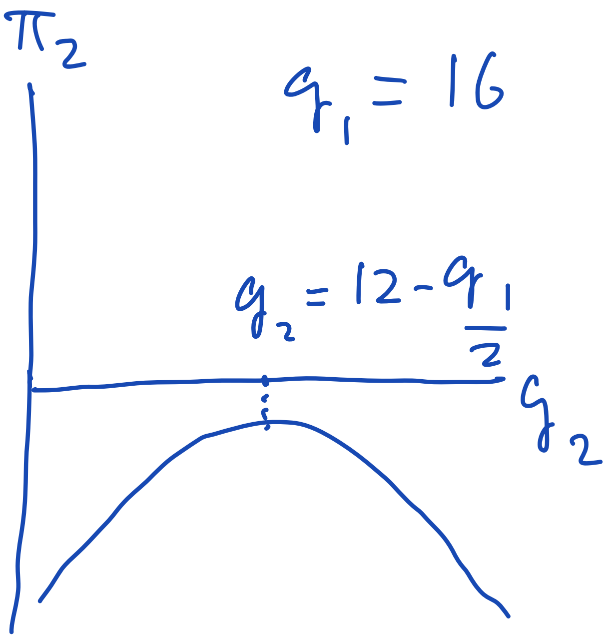
$$R_2(q_1) = \begin{cases} q_2 = 12 - \frac{q_1}{2} & \text{if it gives } \pi_2 \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\pi_2 = \left(12 - \frac{q_1}{2}\right)\left[25 - \left(q_1 + 12 - \frac{q_1}{2}\right)\right] - \left(12 - \frac{q_1}{2}\right) - 25 = 0$$

Solve $q_1 = 14$

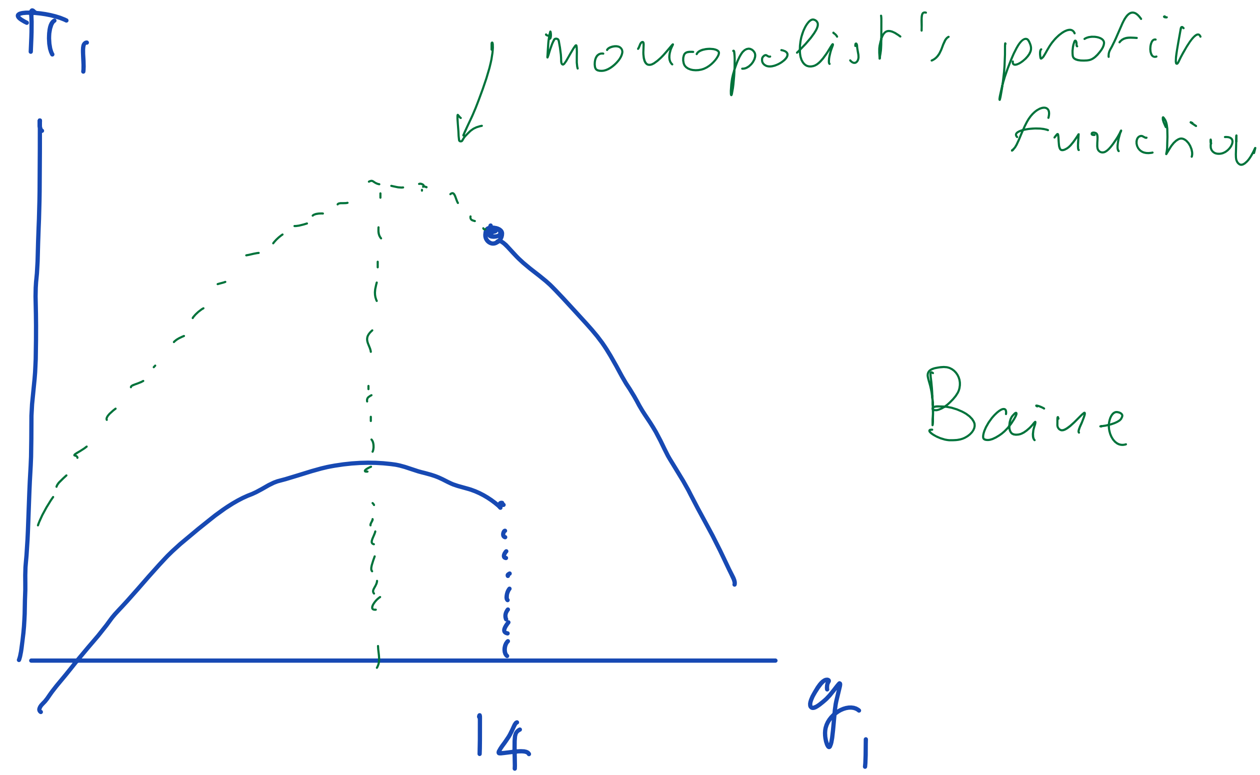






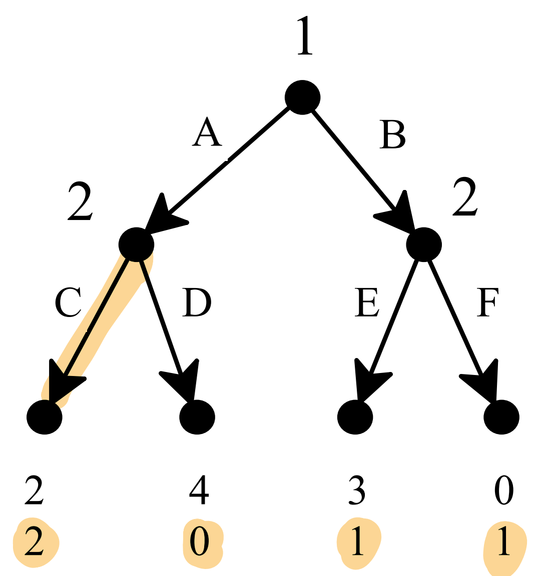
$$R_2(q_1) = \begin{cases} 0 & \text{if } q_1 \geq 14 \\ 12 - \frac{q_1}{2} & \text{if } q_1 < 14 \end{cases}$$

$$\pi_1(q_1) = \begin{cases} (**) & \text{if } q_1 < 14 \\ q_1[25 - q_1] - q_1 - F & \text{if } q_1 \geq 14 \end{cases}$$



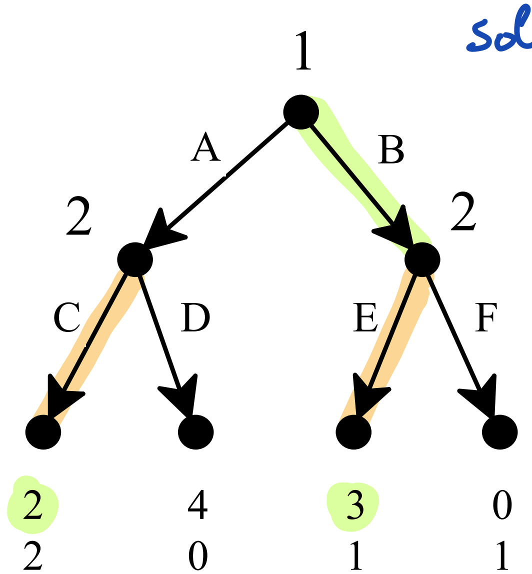
π1
monopolist's profit function
q1
14
Baine

Multiple backward-induction solutions



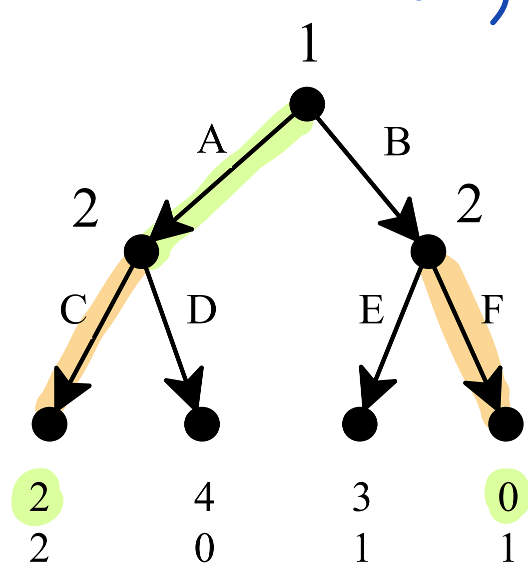
(B, CE)

"optimistic solution"



(A, CF)

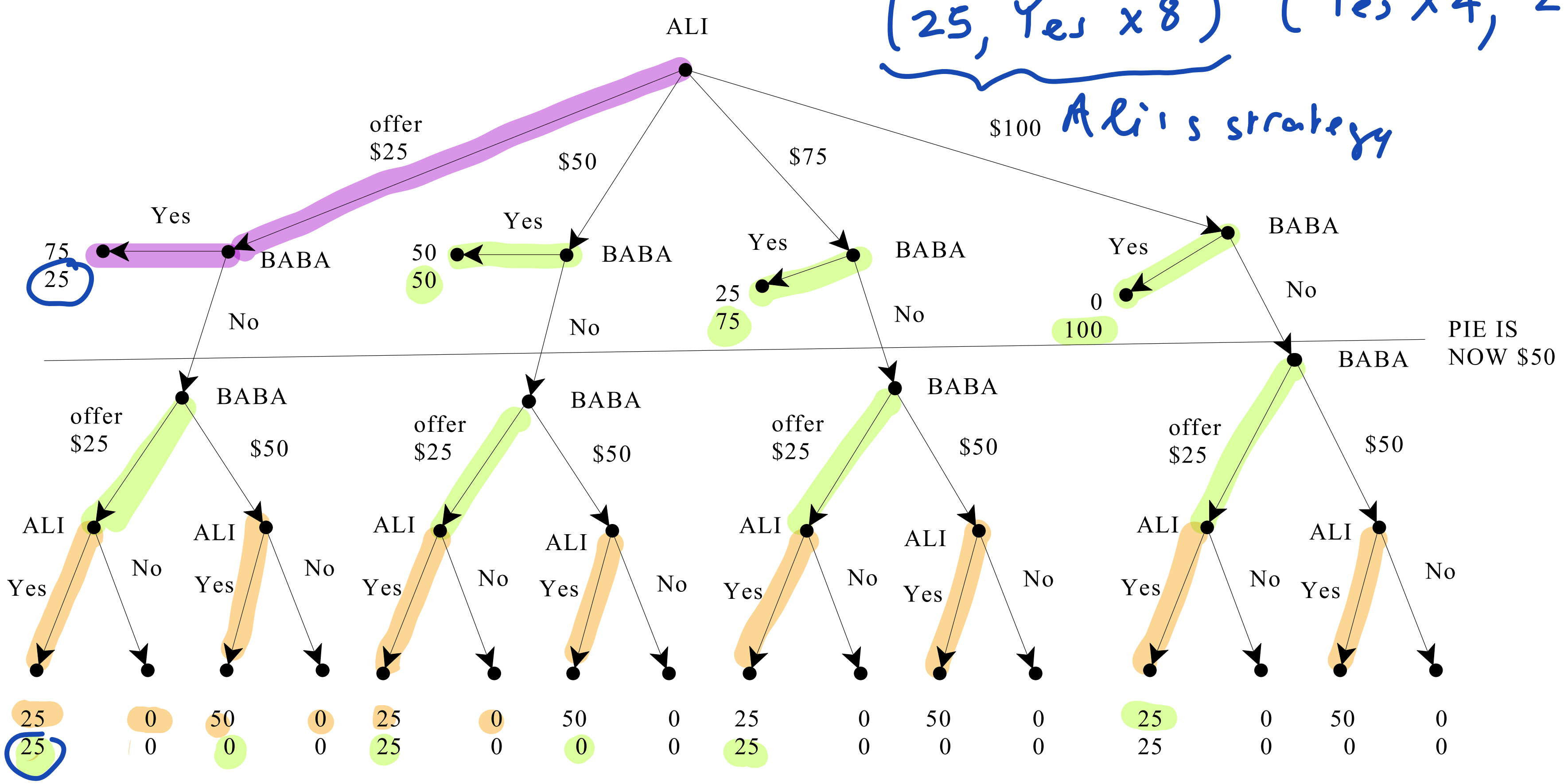
"pessimistic solution"



Bargaining game between Ali and Baba.

They have $100 to divide. Ali makes an offer to Baba. Offers can only be multiples of $25. The minimum offer is $25. Baba can accept or reject. If he rejects the money to be divided shrinks to $50 and he makes an offer. If Ali rejects then they both get nothing. Thus only two rounds of offers.

$\underbrace{(25, \text{Yes} \times 8)}_{\text{Ali's strategy}}$ (Yes x4, 25 x4)



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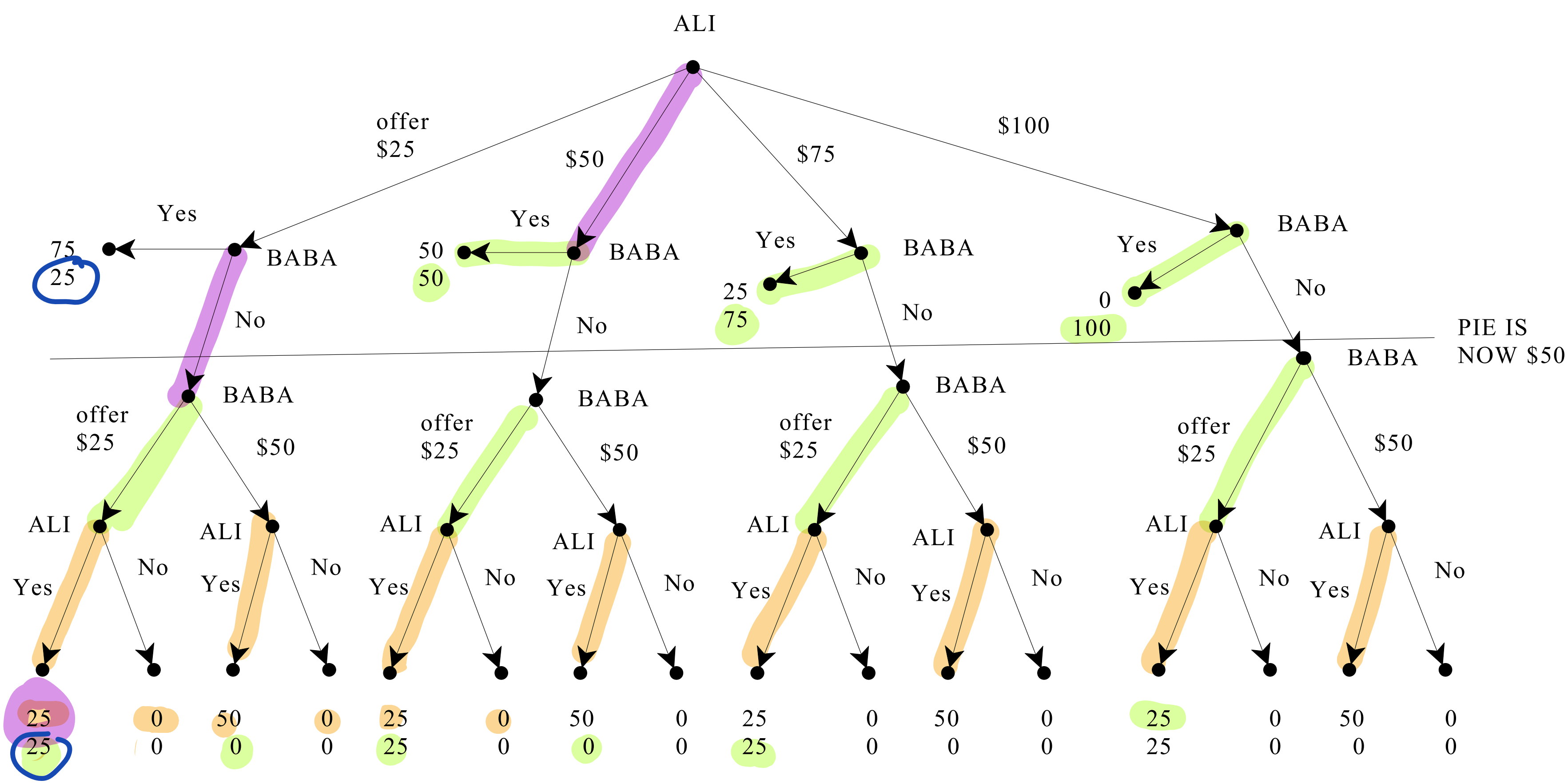
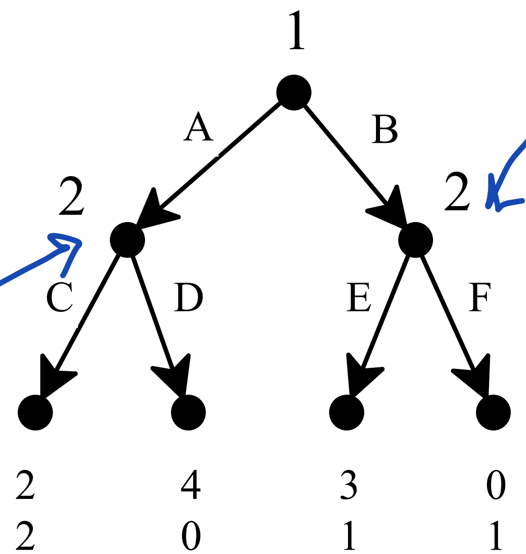


Definition: A *strategy* for player i in a perfect-information game is a list of choices, one for each node that belongs to player i .



Player 2 : (C or D ___ , ___ E or F)

$S_2 = \{(C,E), (C,F), (D,E), (D,F)\}$

$S_1 = \{A, B\}$

BI solutions:

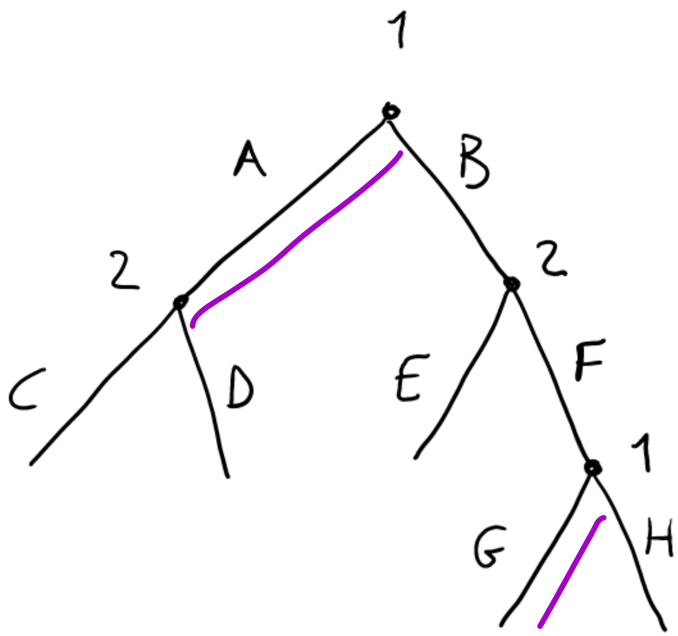
(A, CF)

(B, CE)

2

		CE	CF	DE	DF
1	A	2, 2	2, 2	4, 0	4, 0
	B	3, 1	0, 1	3, 1	0, 1

Both (A, CF) and (B, CE) are Nash equilibria

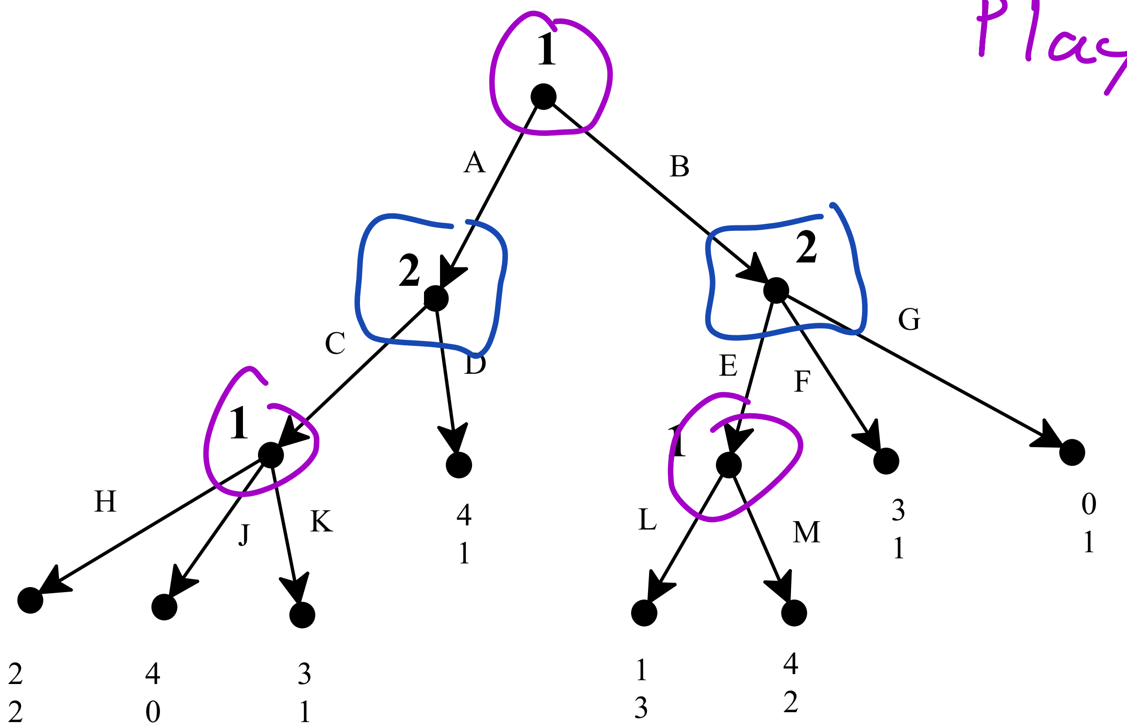


Strategy for Player 1:

$$\left(\underset{\substack{A\\ \text{or}\\ B}}{\underline{\quad\quad}} \;,\; \overset{G \text{ or } H}{\underline{\quad\quad}} \right)$$

$$S_1 = \{\underbrace{(A,G),\ (A,H)}_{A},\ (B,G),\ (B,H)\}$$

plan of action



Player 1

$\left(\frac{\quad}{2}, \frac{\quad}{3}, \frac{\quad}{2} \right)$

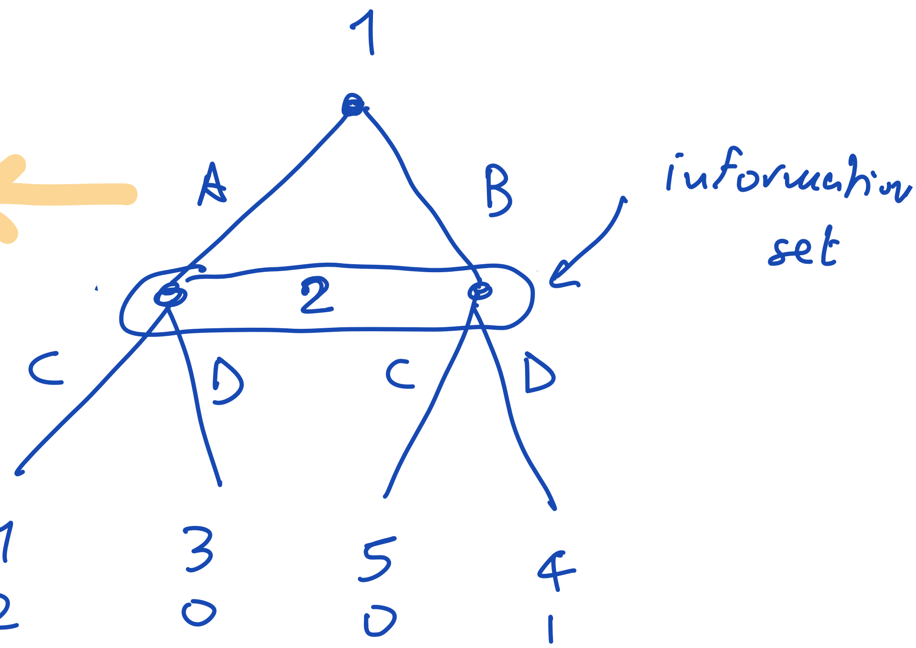
$2 \times 3 \times 2 = 12$

Player 2 : $\left(\frac{\quad}{2}, \frac{\quad}{3} \right)$

$2 \times 3 = 6$

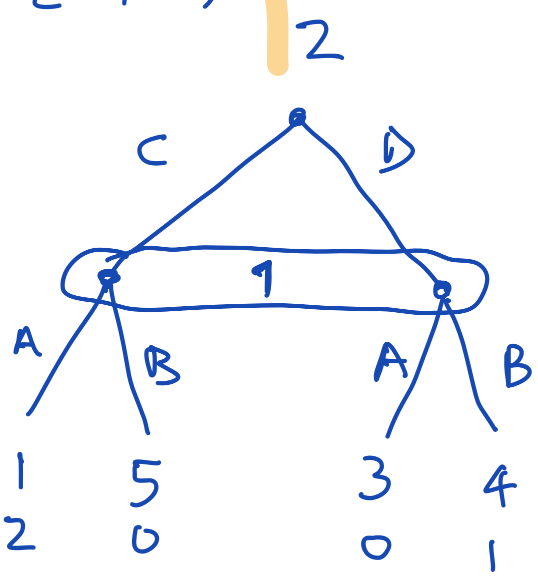
2

		C	D
1	A	1,2	3,0
	B	5,0	4,1

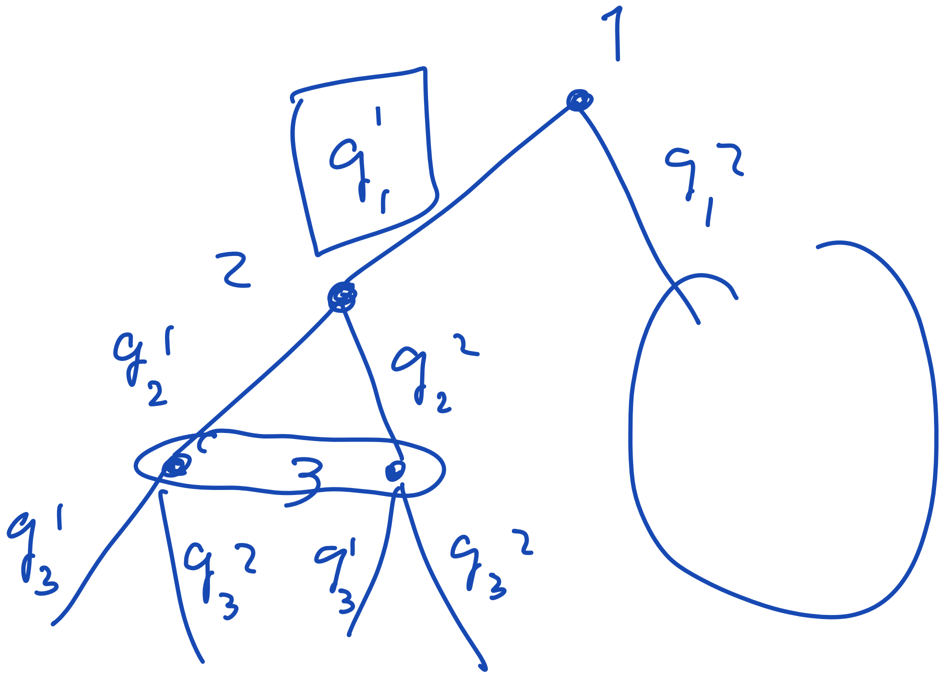


$S_1 = \{A, B\}$

$S_2 = \{C, D\}$



A strategy of player i is a list of choices, one for every ~~node~~ information set of player i



1
q_1^1
q_1^2
2
q_2^1
q_2^2
3
q_3^1
q_3^2
q_3^1
q_3^2