Stackelberg equilibrium

Linear inverse demand: P = a - bQ, linear cost function C(q) = cq (0 < c < a) **Monopoly**: $\Pi(Q) = Q(a - bQ) - cQ$. Solve $\frac{\partial \Pi}{\partial Q} = 0$: $Q_M = \frac{1}{2} \left(\frac{a - c}{b}\right)$ $\Pi_M = \frac{(a - c)^2}{4b}$ **Duopoly**: $\begin{cases} \pi_1(q_1, q_2) = [a - b(q_1 + q_2)]q_1 - cq_1 \\ \pi_2(q_1, q_2) = [a - b(q_1 + q_2)]q_2 - cq_2 \end{cases}$ Solve $\frac{\partial \pi_1}{\partial q_1} = 0, \frac{\partial \pi_2}{\partial q_2} = 0$ Cournot-Nash equilibrium: $q_1^* = q_2^* = \frac{a - c}{3b}$ so $Q_d^* = \frac{2}{3} \left(\frac{a - c}{b}\right)$ and $\Pi_1^* = \Pi_2^* = \frac{(a - c)^2}{9b}$

Stackelberg. Perfect-information game where Firm 1 choose its output first and **commits to it** moves first and Firm 2 moves second.

Introduce a fixed cost.

$$P = 25 - Q \qquad \text{and} \qquad C(q) = \begin{cases} 0 & \text{if } q = 0 \\ q + 25 & \text{if } q > 0 \end{cases}$$

Multiple backward-induction solutions







Bargaining game between Ali and Baba.

They have \$100 to divide. Ali makes an offer to Baba. Offers can only be multiples of \$25. The minimum offer is \$25. Baba can accept or reject. If he rejects the money to be divided shrinks to \$50 and he makes an offer. If Ali rejects then they both get nothing. Thus only two rounds of offers.



Bargaining game between Ali and Baba.

They have \$100 to divide. Ali makes an offer to Baba. Offers can only be multiples of \$25. The minimum offer is \$25. Baba can accept or reject. If he rejects the money to be divided shrinks to \$50 and he makes an offer. If Ali rejects then they both get nothing. Thus only two rounds of offers.



Definition: A *strategy* for player i in a perfect-information game is a list of choices, one for each node that belongs to player i.







NOTE: Backward-induction solutions must be given in terms of *strategy profiles*, not in terms of actual choices.







Relationship between backward induction and Nash equilibrium



