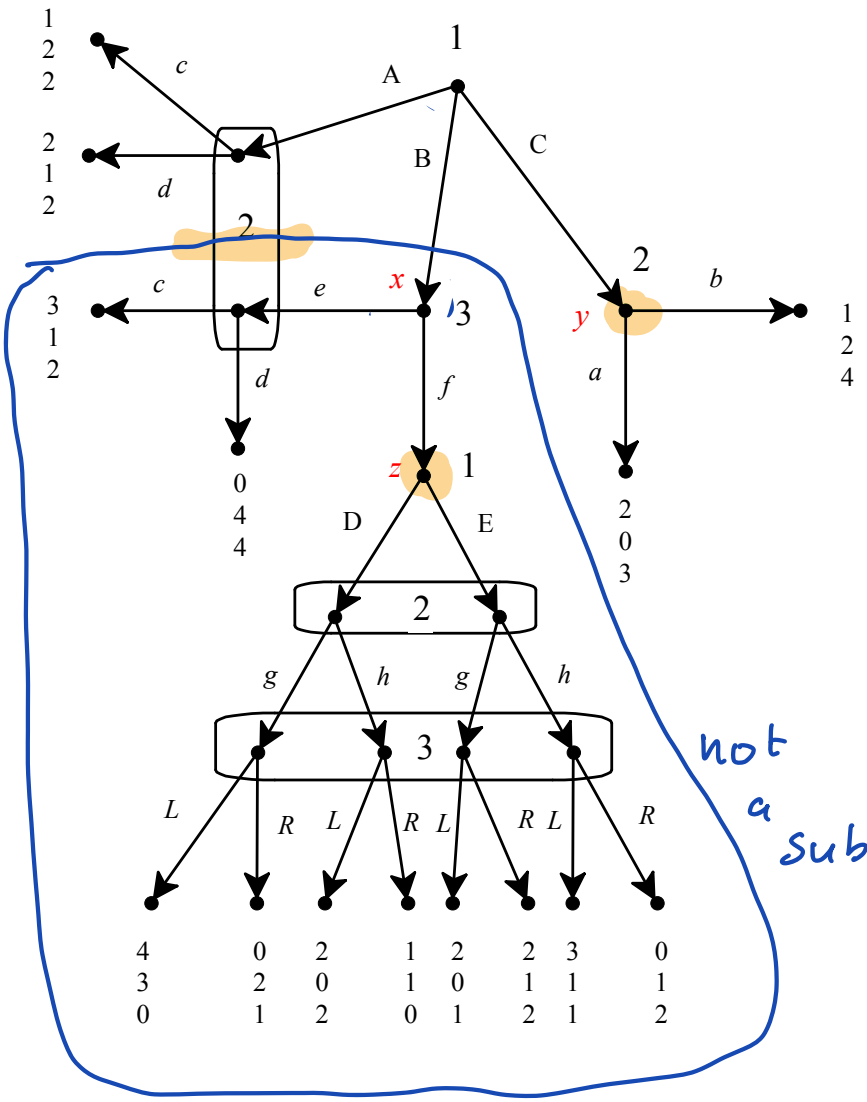
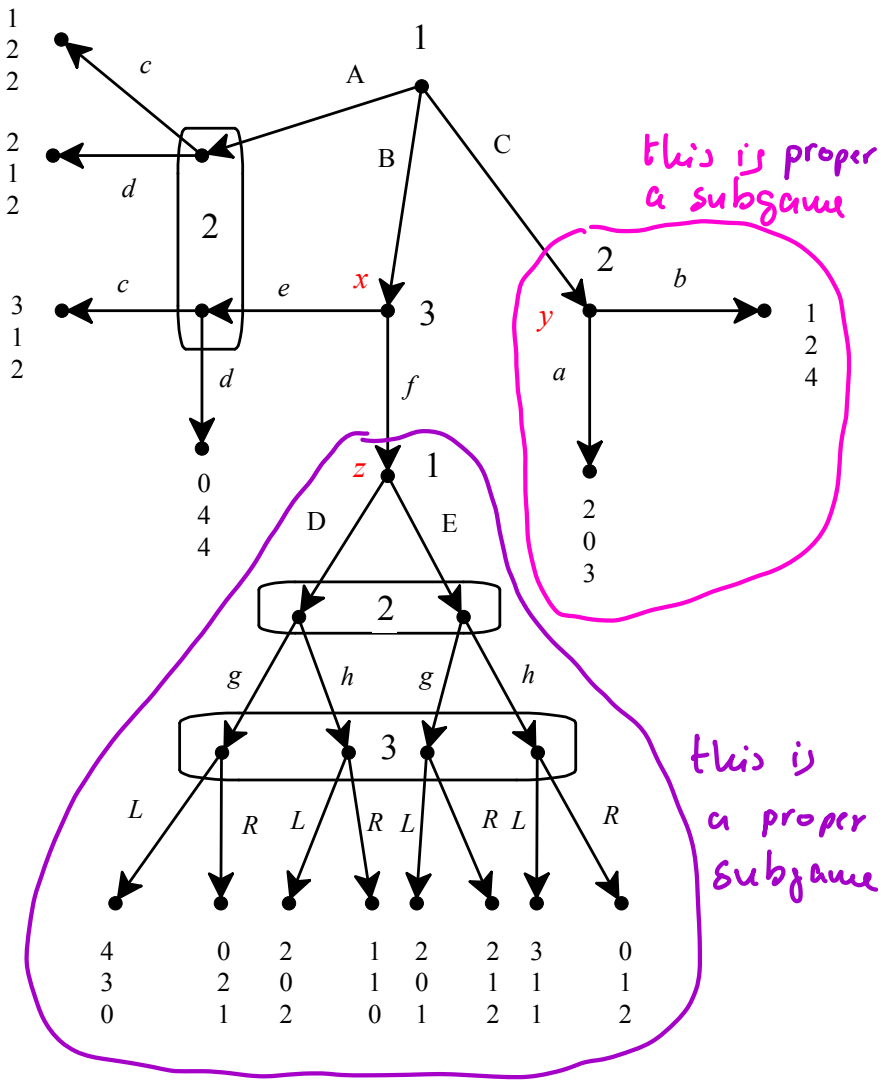
SUBGAME PERFECT EQUILIBRIUM (Selten)



Definition of **SUBGAME**.

- Start from a singleton information set
- Take the entire sub-tree from there
- If you don't cut any information sets then you have a subgame otherwise you don't



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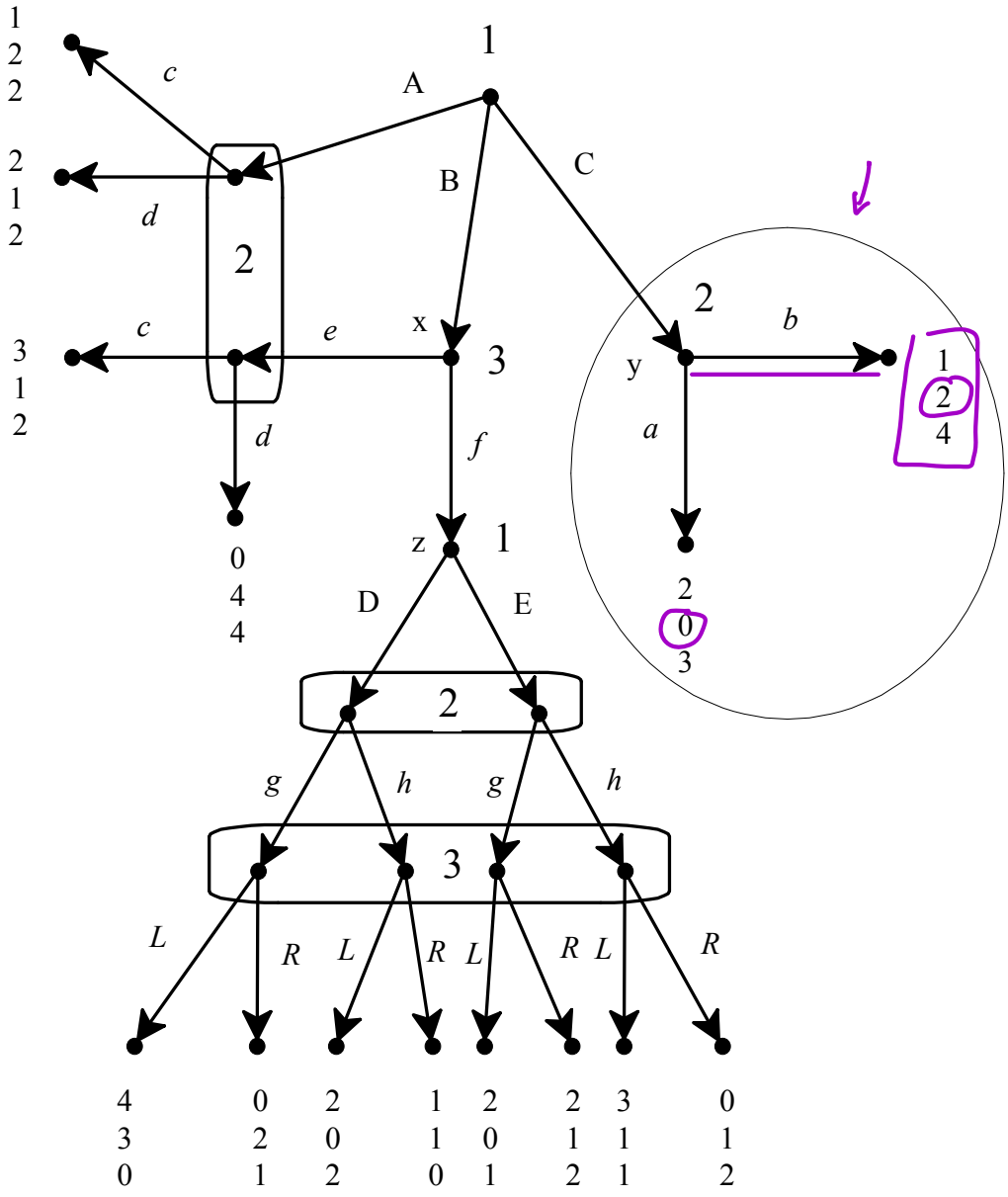
1
2
2
2
1
2
3
1
2
0
4
4
2
0
3
4
3
0
0
2
1
2
0
2
1
1
0
2
0
1
2
1
2
3
1
1
0
1
2
c
d
c
e
x
d
f
A
B
C
b
a
y
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D
E
g
h
g
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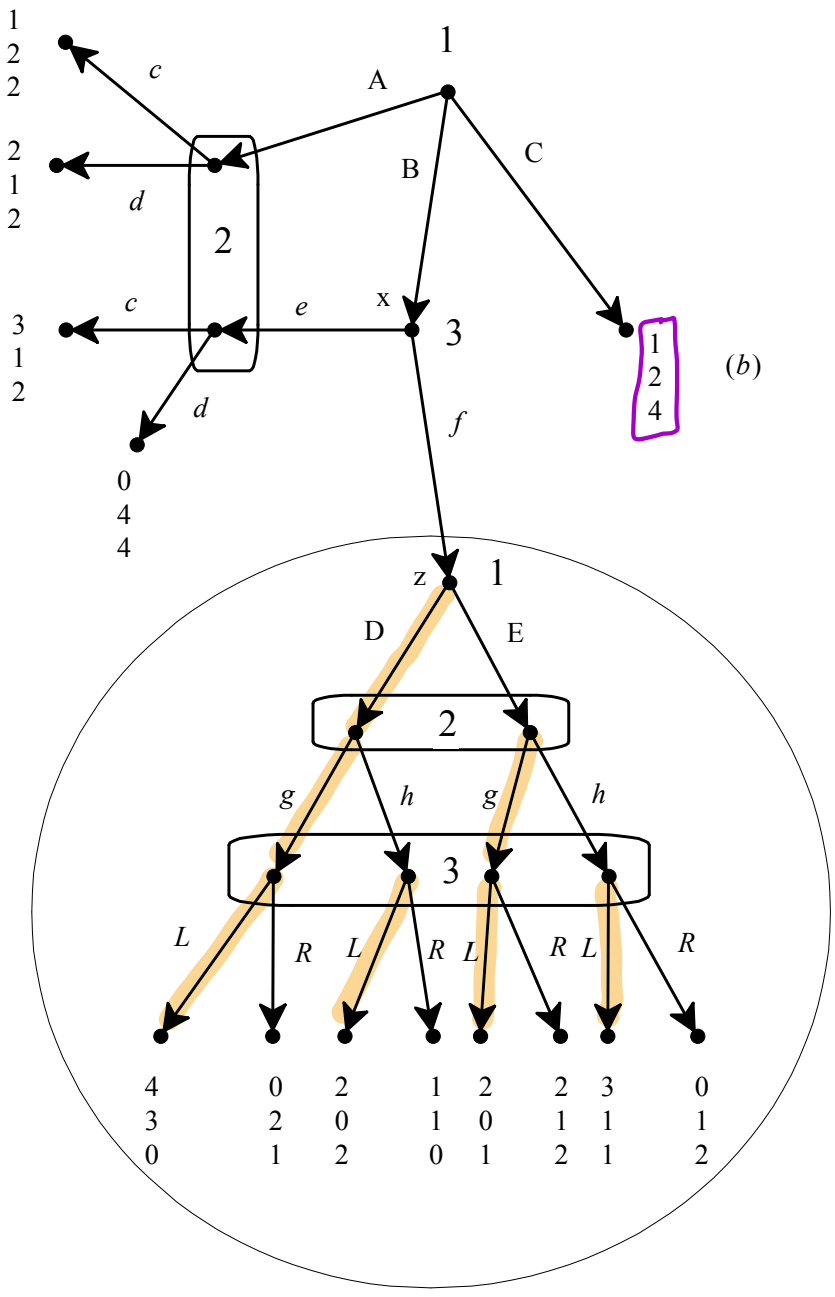
L
R
L
R
L
R
L
R



2

1		g			h		
D	4	3	0	2	0	2
E	2	0	1	3	1	1

Player 3: L

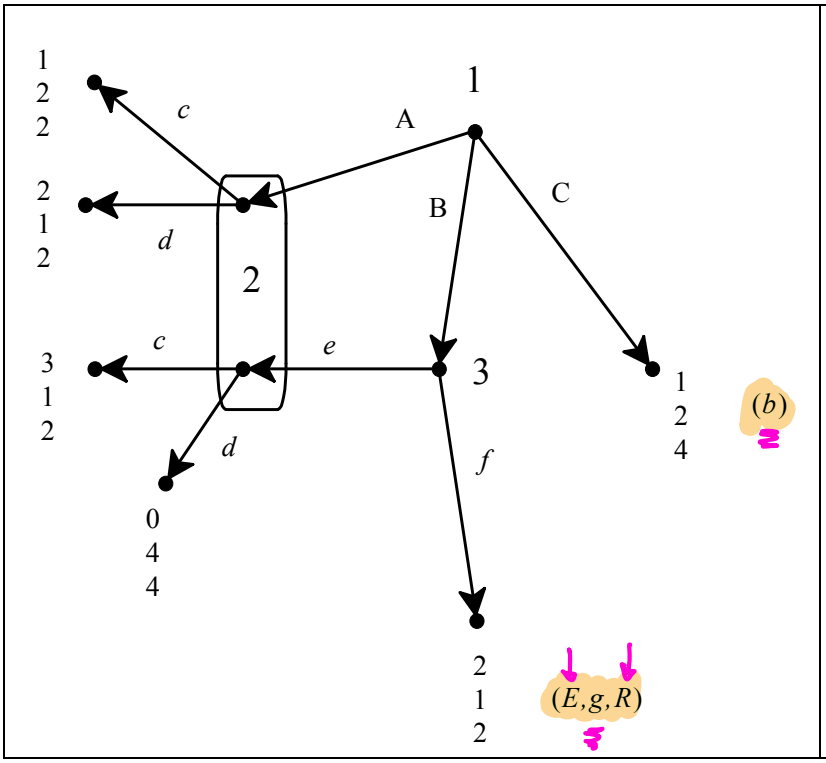
2

1		g			h		
D	0	2	1	1	1	0
E	2	1	2	0	1	2

Player 3: R

Unique N.E. : (E, g, R)

with payoffs : (2, 1, 2)



No more subgames

2

1		c		d	
A	1	2	2	2	1	2
B	3	1	2	0	4	4
C	1	2	4	1	2	4

Player 3: e

2

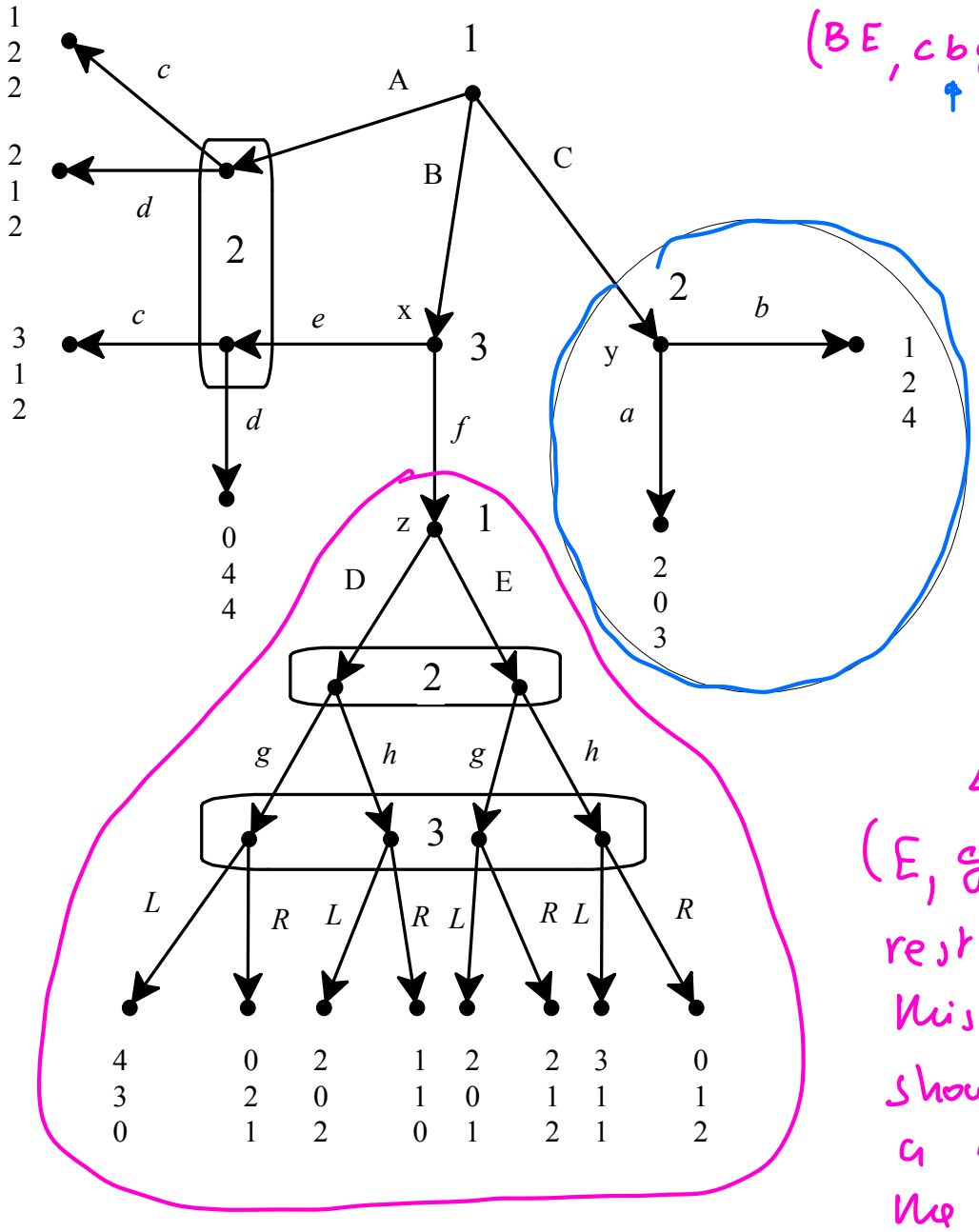
1		c		d	
A	1	2	2	2	1	2
B	2	1	2	2	1	2
C	1	2	4	1	2	4

Player 3: f

Unique N.E. : (B, c, f)

Subgame-Perfect Equilibrium of the original game: (BE, cbg, fR)

strategy profile of the original game called a SPE

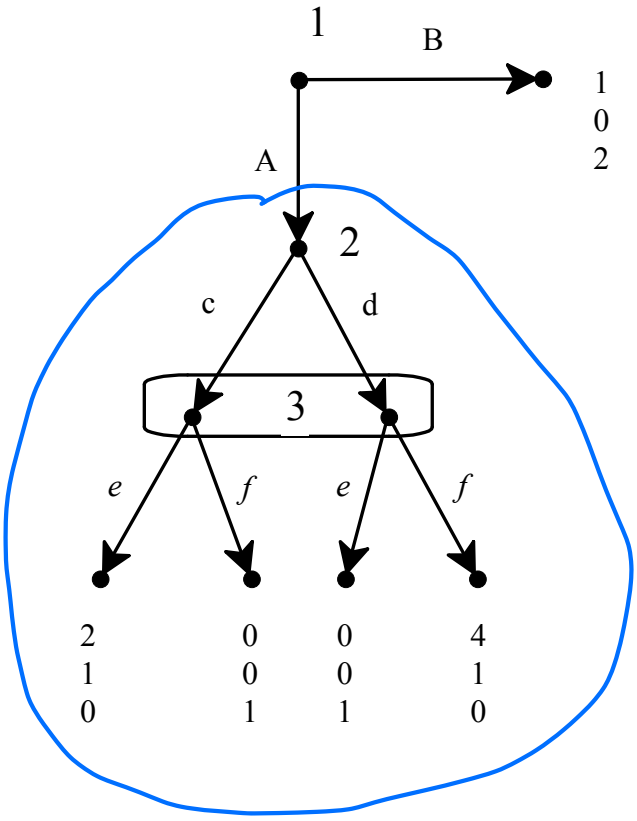


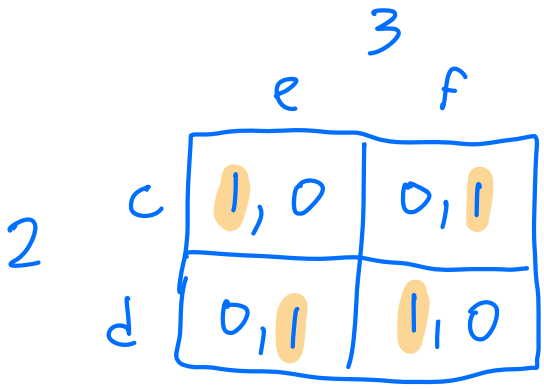
(BE, cbg, fR)

b is a NE of this subgame

(E, g, R) restriction to this subgame should be a NE of the subgame

There may be no subgame-perfect equilibria



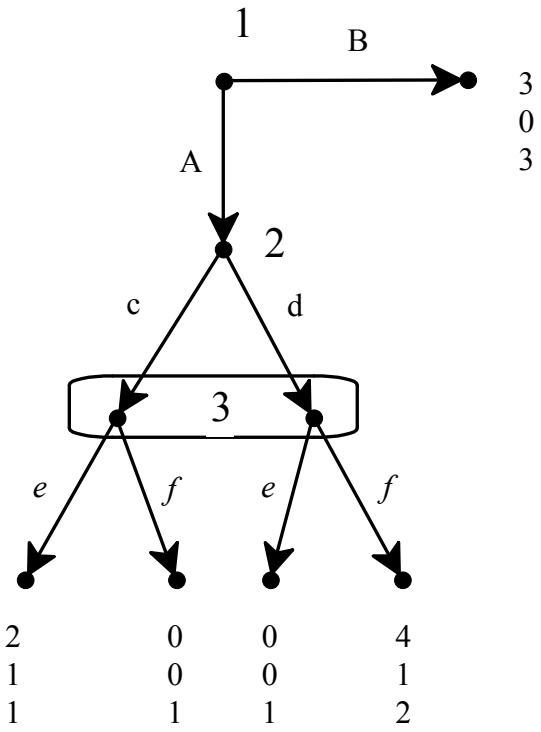


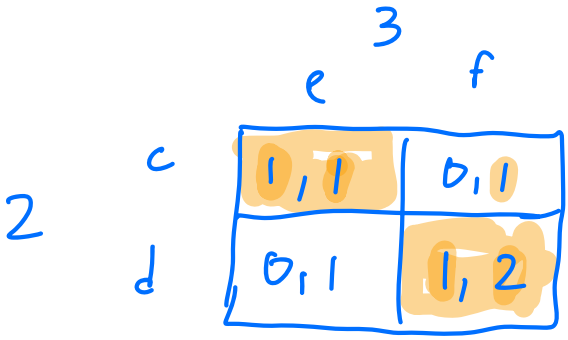
no N.E.

STOP.

The entire game does not have a SPE

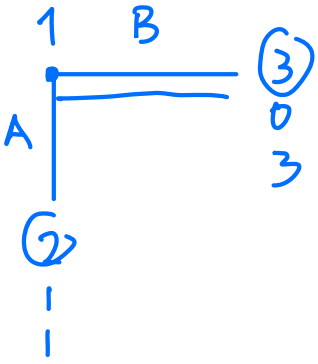
There may be several subgame-perfect equilibria





Two N.E. :

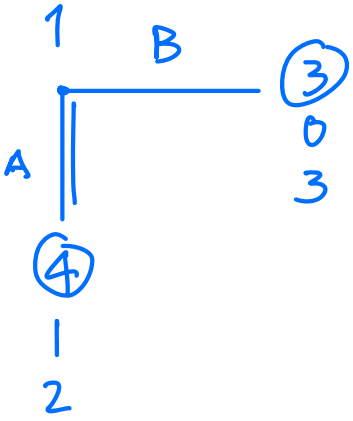
(c, e) and (d, f)



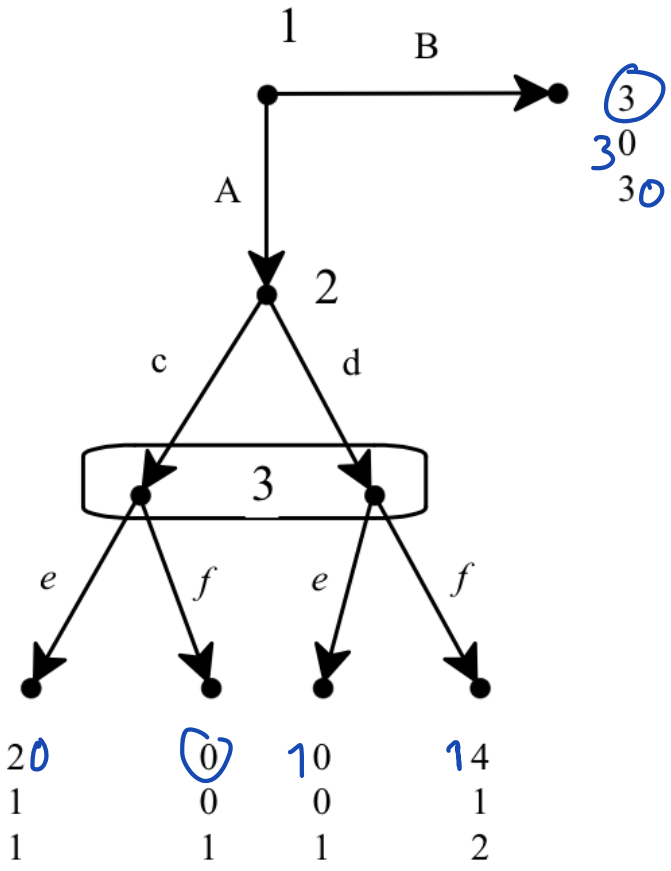
(c, e)

SPE : (B, c, e)

(B, c, f)

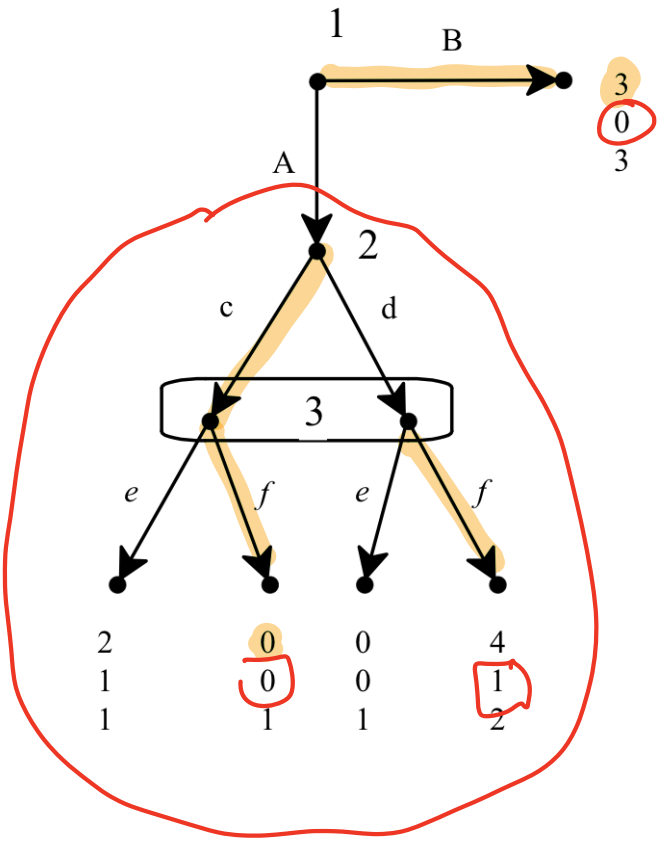


SPE : (A, d, f)



not a credible joint threat by (B, C, f) in 2 and 3

a NE



(B, c, f) is a N.E.

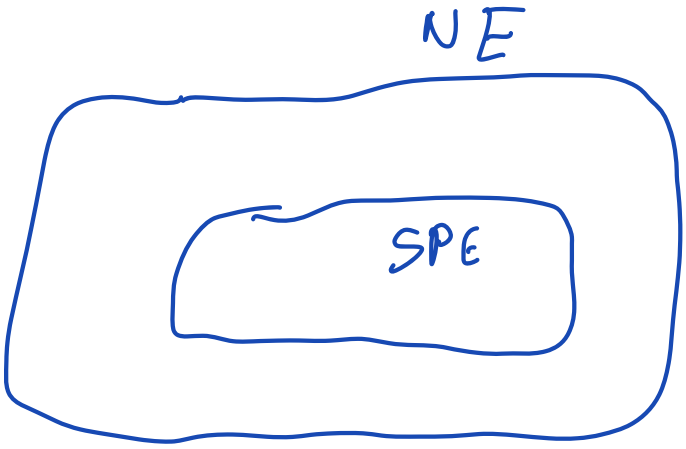
not SPE

because

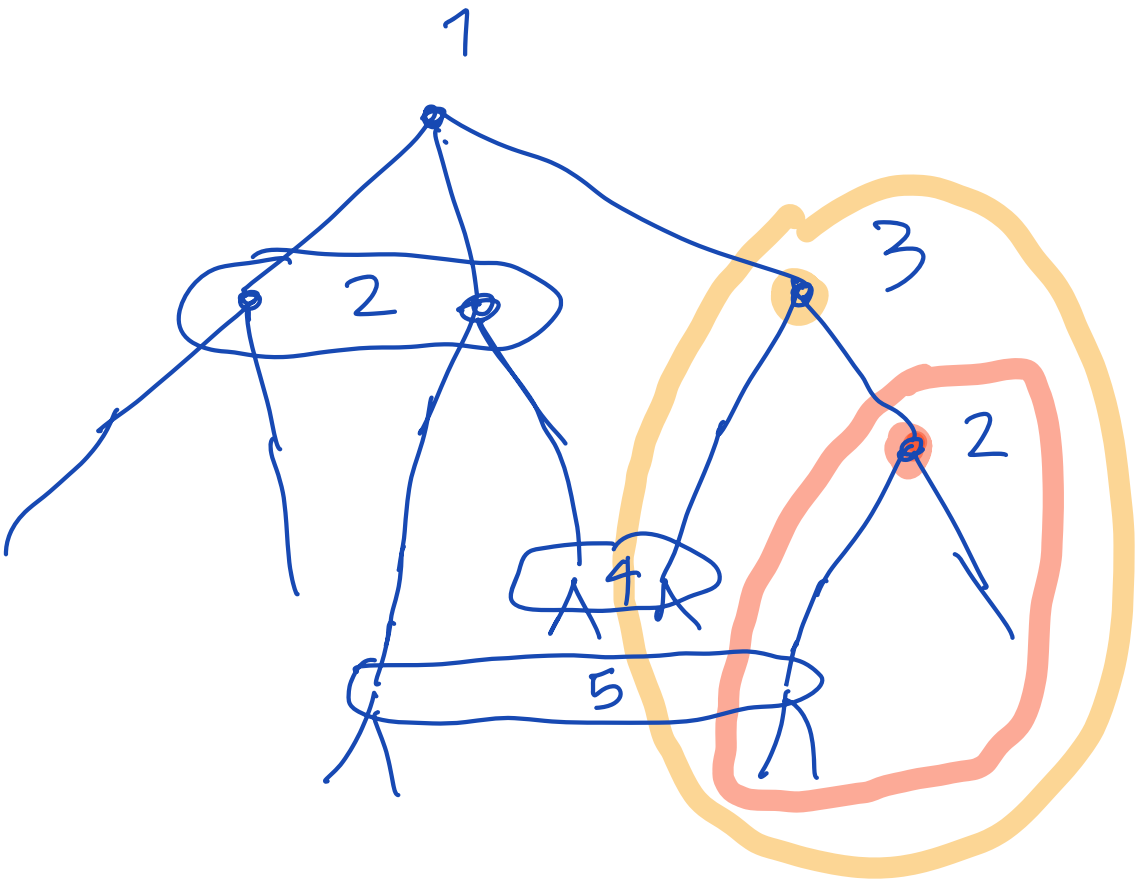
(c, F) is not a NE of the subgame

(c not a best reply to f)

Set of NE = set of SPE in the class of games that ...



have no proper subgames



A game with chance moves.

A coin is tossed twice. If the outcome is HH then Player 1 is informed that it was HH and if the outcome is any other outcome then Player 1 is only told that it was **not** HH. Then Player 1 chooses between *A* and *B*. Player 2 is not told what the outcome was, nor is she told what Player 1 chose and she has to choose between *C* and *D*. The outcomes are sums of money: the first is what Player 1 gets and the second what Player 2 gets:

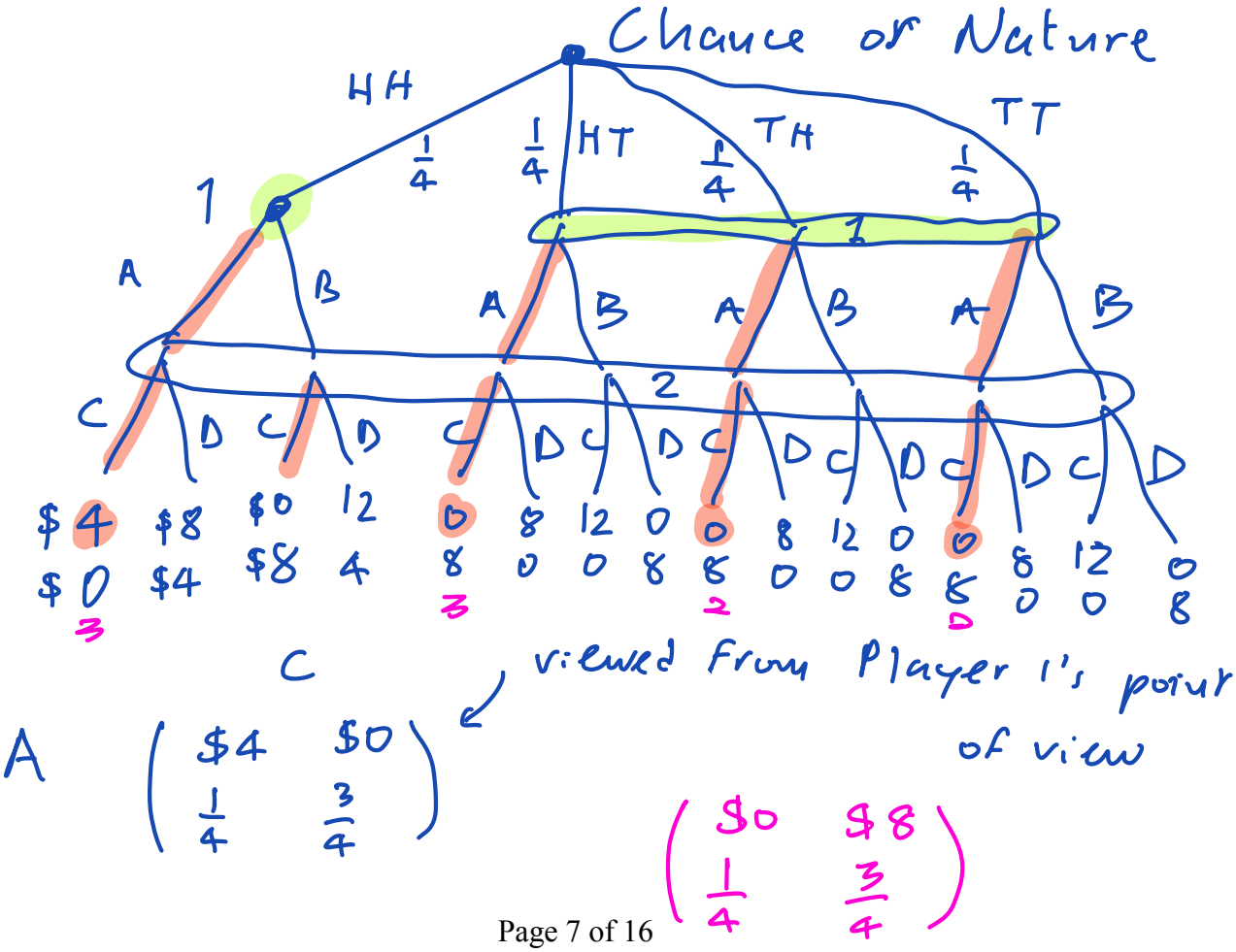
If the outcome is HH:

	C		*D*	
A	\$4	\$0	\$8	\$4
B	\$0	\$8	\$12	\$4

If the outcome is HT or TH or TT:

	C		*D*	
A	\$0	\$8	\$8	\$0
B	\$12	\$0	\$0	\$8

Players are selfish and greedy

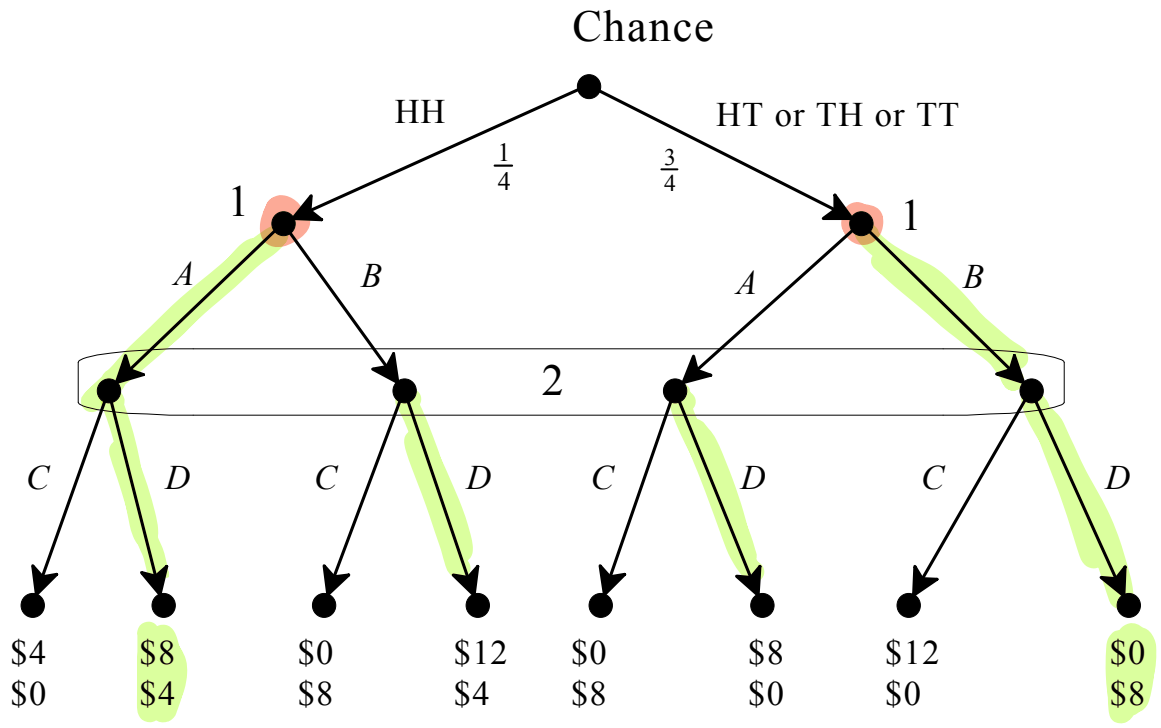


C

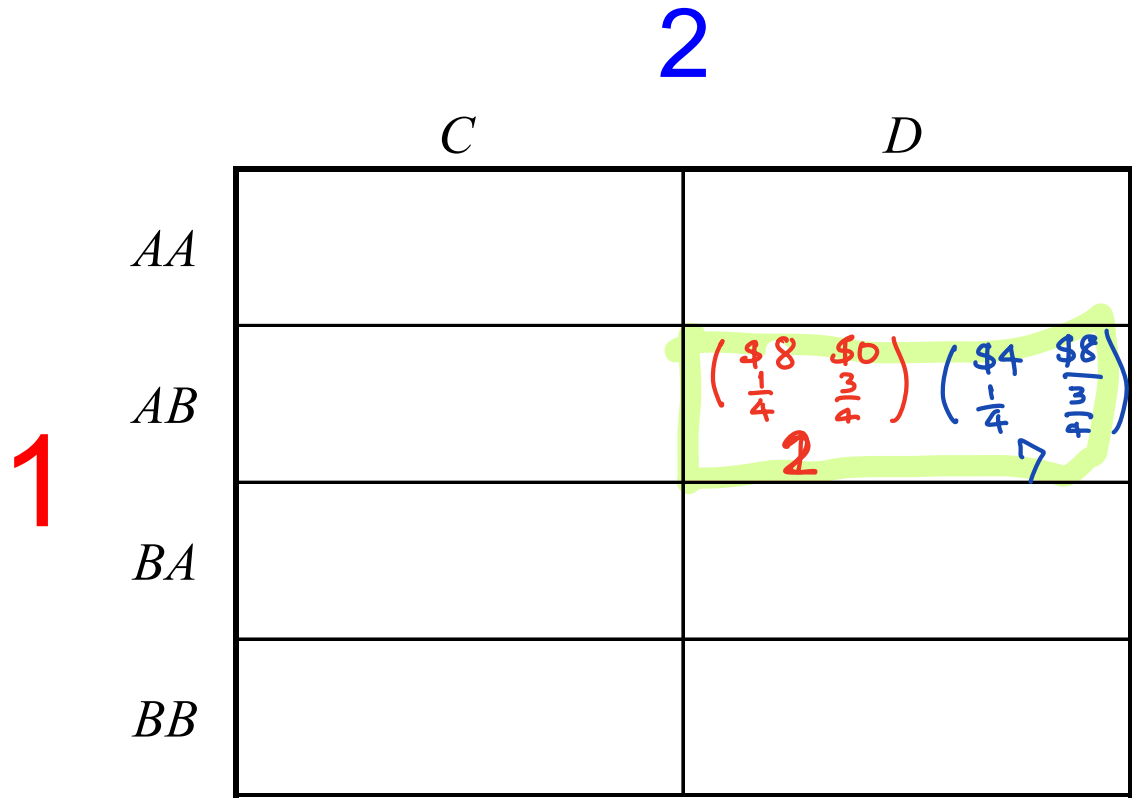
A (\$4 \$0)
(1/4 3/4)

viewed from Player 1's point of view

(\$0 \$8)
(1/4 3/4)



If each player is selfish and greedy then the associated strategic form is as follows:



If the players are risk neutral

$(\$8, \frac{1}{4}; \$0, \frac{3}{4}) \sim_1 \$2$ exp. value

$(\$4, \frac{1}{4}; \$8, \frac{3}{4}) \sim_2 \$7$ exp. value

2

1

	C	D
AA	($4 $0; 1/4 3/4) ($0 $8; 1/4 3/4)	$8 ($4 $0; 1/4 3/4)
AB	($4 $12; 1/4 3/4) $0	($8 $0; 1/4 3/4) ($4 $8; 1/4 3/4)
BA	$0 $8	($12 $8; 1/4 3/4) ($4 $0; 1/4 3/4)
BB	($0 $12; 1/4 3/4) ($8 $0; 1/4 3/4)	($12 $0; 1/4 3/4) ($4 $8; 1/4 3/4)

Now if we add the assumption that the players are risk neutral then the above strategic form can be simplified to the following:

Player 2

Player 1	C	D
AA		
AB		
BA		
BB		

Strategic entry deterrence