Computing the mixed-strategy Nash equilibria

Theorem. At a Nash equilibrium in mixed strategies, a player must be indifferent between any two PURE strategies that she plays with positive probability.

		Player 2	
		C	*D*
Player 1	*A*	15, 3	36, 0
	B	34, 0	30, 2

Theorem. At a Nash equilibrium in mixed strategies, a player must be indifferent between any two strategies that she plays with positive probability.

Provides a necessary, *but not sufficient*, condition for a mixed-strategy profile to be a Nash equilibrium

		Player 2	
		D	E
Player 1	A	6, 0	0, 4
	B	0, 4	6, 0
	C	4, 0	4, 2

$$\left(\begin{pmatrix} A & B & C \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}, \begin{pmatrix} D & E \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}\right)$$

Theorem: A strategy can be played with positive probability at a Nash equilibrium only if it survives the IDSDS procedure.

		C	D	E
	A	1 , 4	4 , 2	0 , 8
1	B	4 , 0	2 , 1	2 , 0
	C	2 , 3	6 , 4	1 , 6

Player 2

		D		E		F	
Player 1	*A*	0	1	3	0	5	0
	B	1	2	1	4	2	3
	C	3	0	0	2	0	1

Player 2

		D		*E*		*F*	
	A	0	1	3	0	5	0
Player 1	*B*	1	2	1	4	2	3
	C	3	0	0	2	0	1

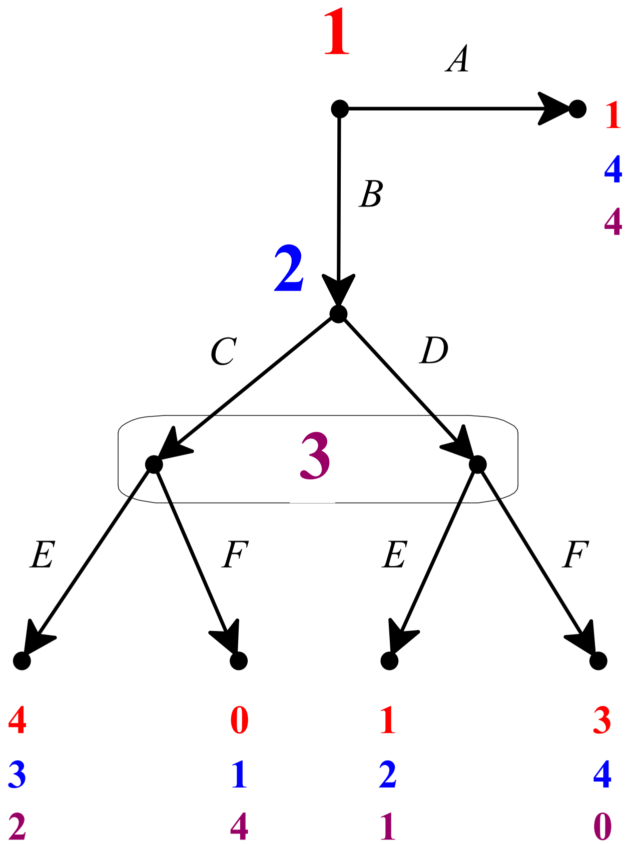
c,g

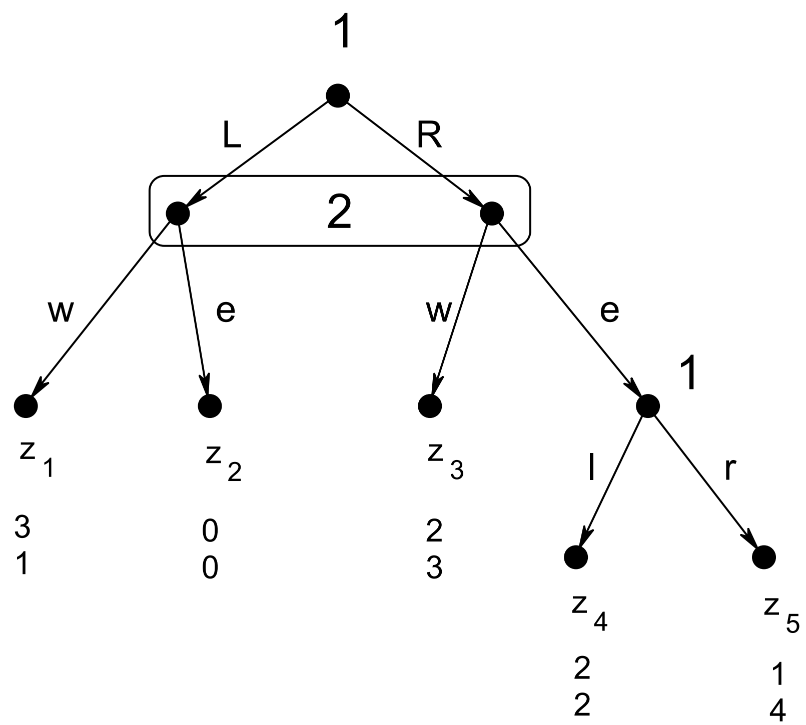
Cardinal IDSDS (Iterated Deletion of Strictly Dominated **pure** Strategies)

Player 2

Player 1

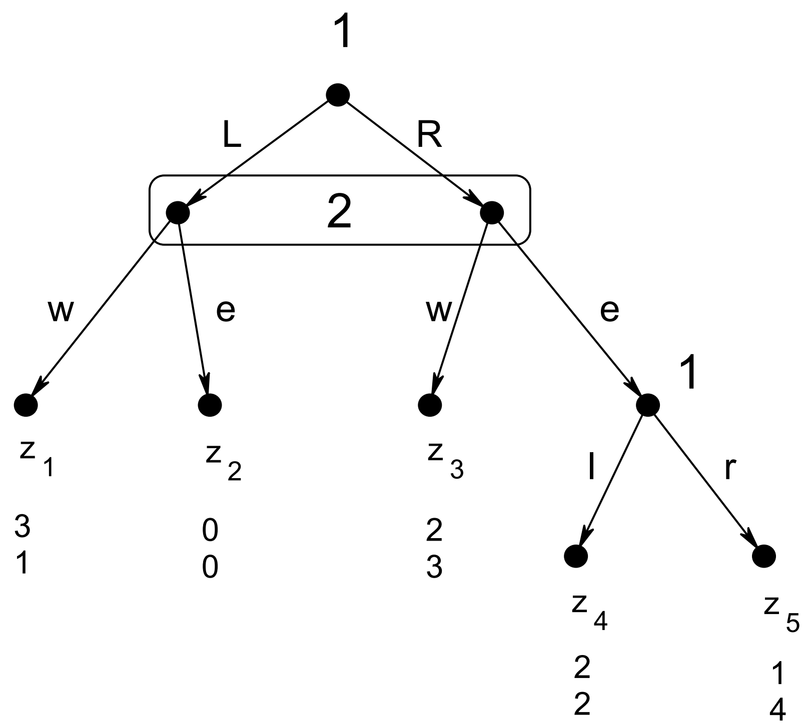
	D		*E*		*F*		*G*	
A	6	8	4	2	2	4	2	3
B	0	0	2	6	8	2	0	1
C	2	8	2	8	4	12	0	11


1
A
1
4
4
B
2
C
D
3
E
F
E
F
4
3
2
0
1
4
1
2
1
3
4
0



$$\left(\begin{pmatrix} Ll & Lr & Rl & Rr \\ \frac{1}{12} & \frac{4}{12} & \frac{2}{12} & \frac{5}{12} \end{pmatrix}, \begin{pmatrix} w & e \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \right)$$

outcome	z_1	z_2	z_3	z_4	z_5	z_6
probability						



$$\left(\begin{pmatrix} Ll & Lr & Rl & Rr \\ \frac{1}{12} & \frac{4}{12} & \frac{2}{12} & \frac{5}{12} \end{pmatrix}, \begin{pmatrix} w & e \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}\right)$$

outcome	z_1	z_2	z_3	z_4	z_5
probability	$\frac{5}{36}$	$\frac{10}{36}$	$\frac{7}{36}$	$\frac{4}{36}$	$\frac{10}{36}$

$$\left(\left(\begin{array}{cc|cc} L & R & l & r \\ \frac{5}{12} & \frac{7}{12} & \frac{2}{7} & \frac{5}{7} \end{array}\right), \begin{pmatrix} w & e \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}\right)$$

THEOREM (Kuhn, 1953). In an extensive game **with perfect recall**, mixed and behavior strategies are equivalent. [For a precise statement of "equivalence" see the textbook.]

What does Perfect Recall mean?

INSURANCE MARKETS

Consider an individual with

W	initial wealth
L	potential loss
p	probability of loss

With no insurance she faces the money lottery

An **insurance contract** is a pair (h,d)

h	premium
d	deductible
$L-d$	insured amount of the loss

With contract (h,d) the individual faces the lottery

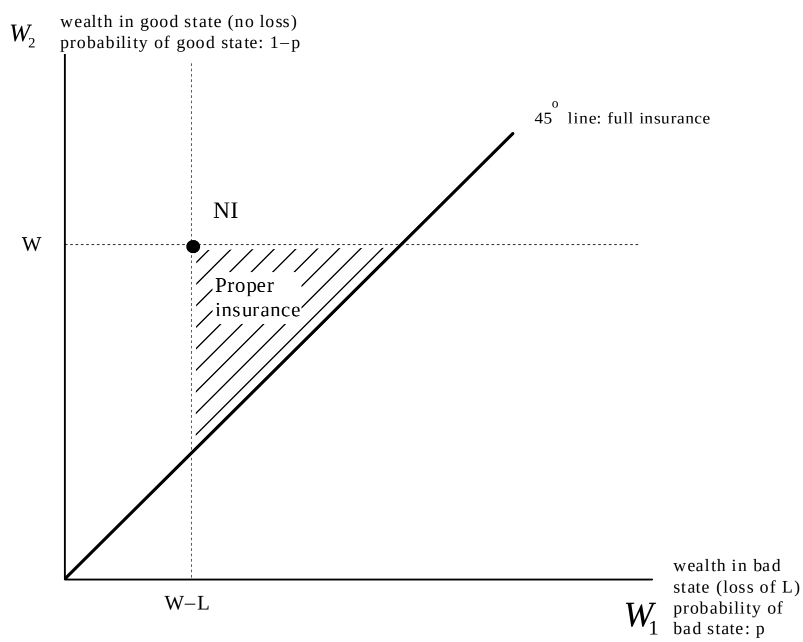
- If $d = 0$
- If $d > 0$

Would the individual purchase the full-insurance contract with $h = pL$?

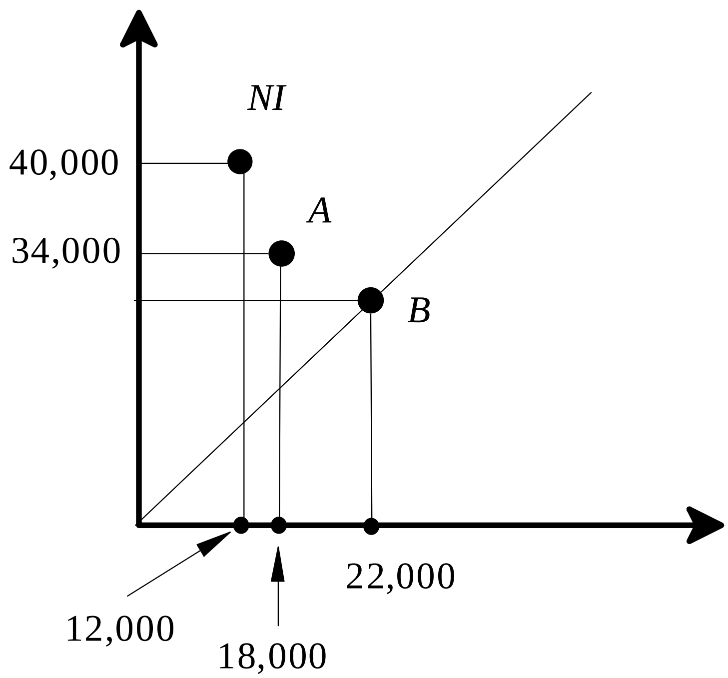
- If she is risk averse then

- If she is risk neutral then

- If she is risk loving then



A contract expressed as a pair (h,d) can be translated into a point in wealth space as follows:



Here we have: $W =$ $L =$

ISOPROFIT LINES

Assume that the **insurance company** is **risk neutral** so that it considers selling an insurance contract C = (h,d), corresponding to the lottery

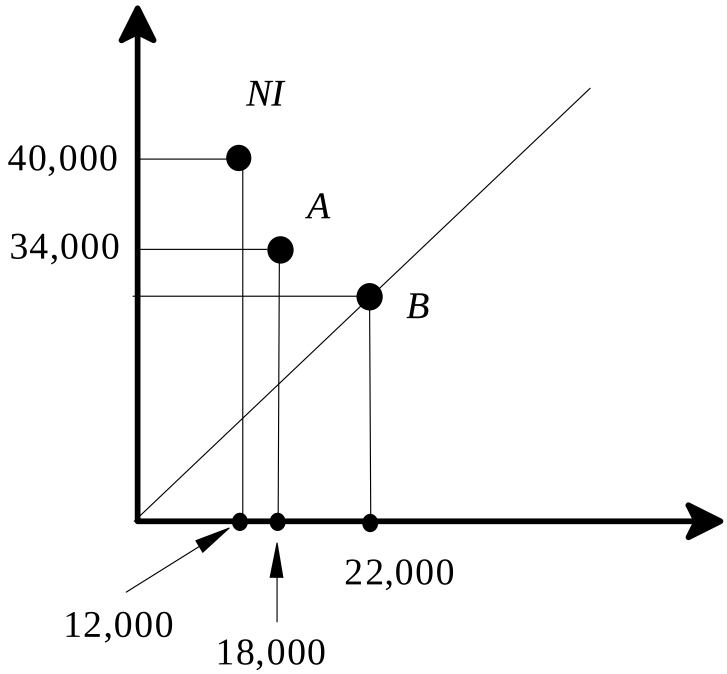
$C = \left(\qquad \right)$, as equivalent to getting its expected value

for sure: $\mathbb{E}[C] =$

We denote the expected profit from contract (h,d) by $\pi(h,d)$. Thus

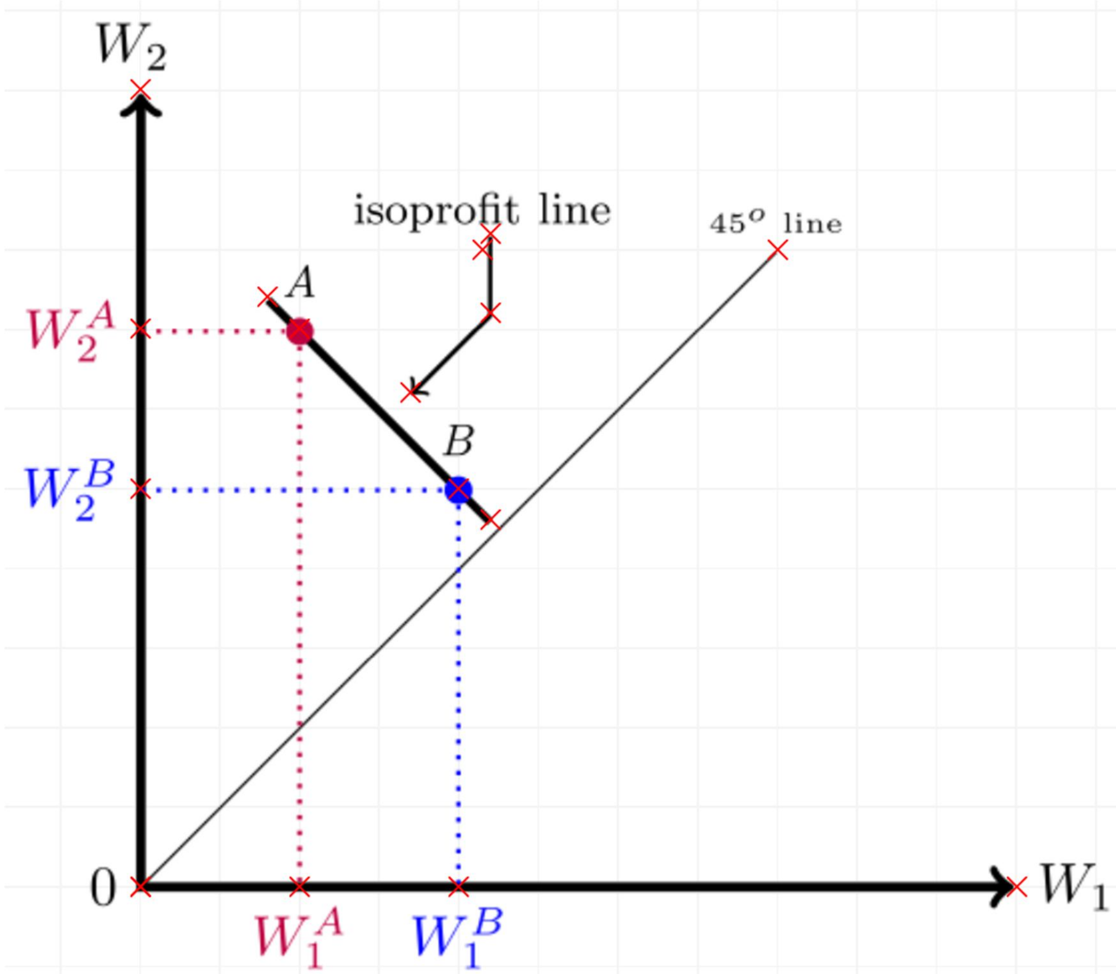
$$\pi(h,d) =$$

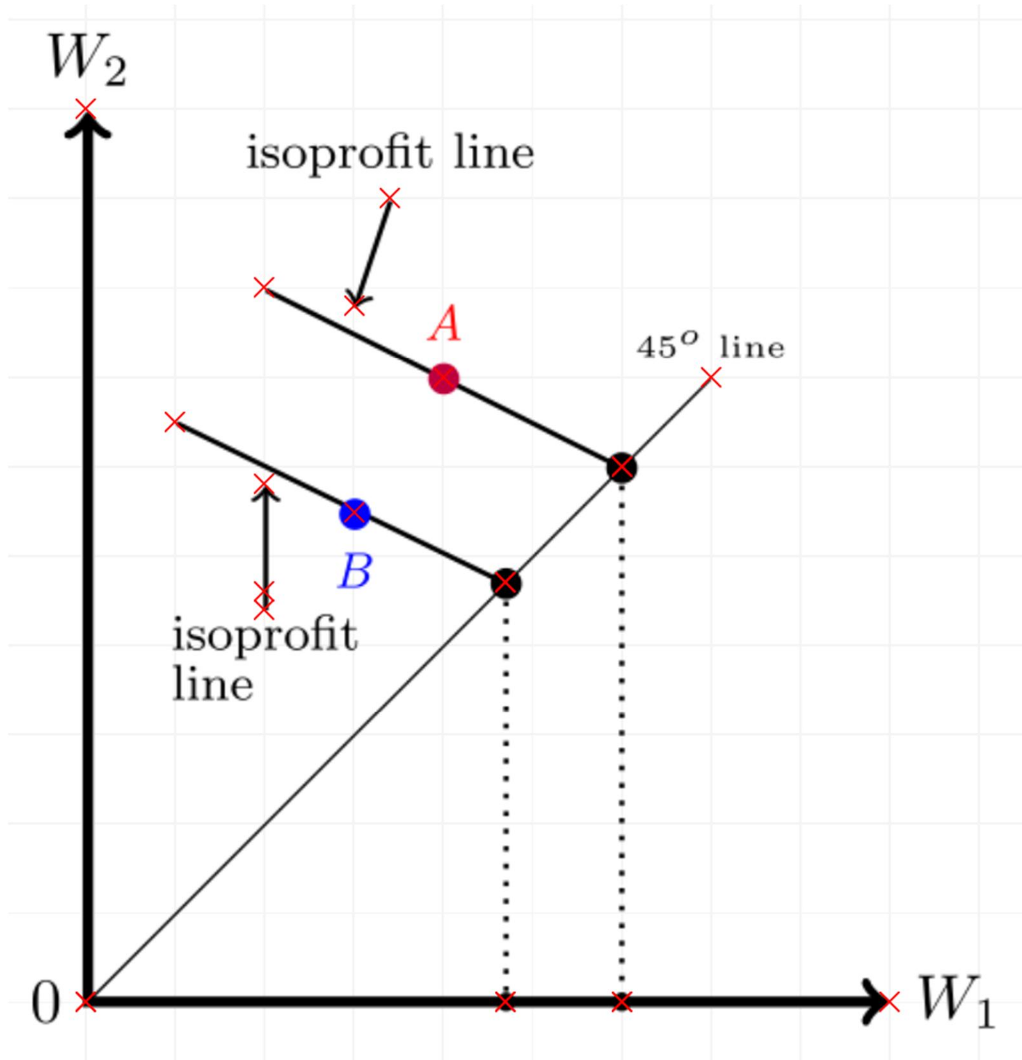
If the contract is expressed as a point (W_1, W_2) in wealth space then



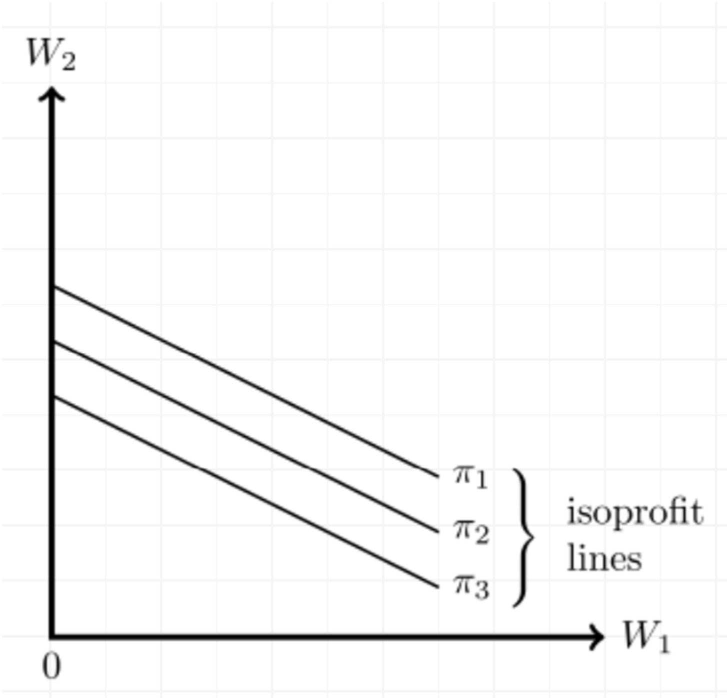
Suppose that $p = \frac{1}{10}$. What is $\pi(A)$? What is $\pi(B)$?

An **isoprofit line** is defined as a line joining contracts that give the same expected profit. Let $A = (W_1^A, W_2^A)$ and $B = (W_1^B, W_2^B)$ be such that $\pi(A) = \pi(B)$

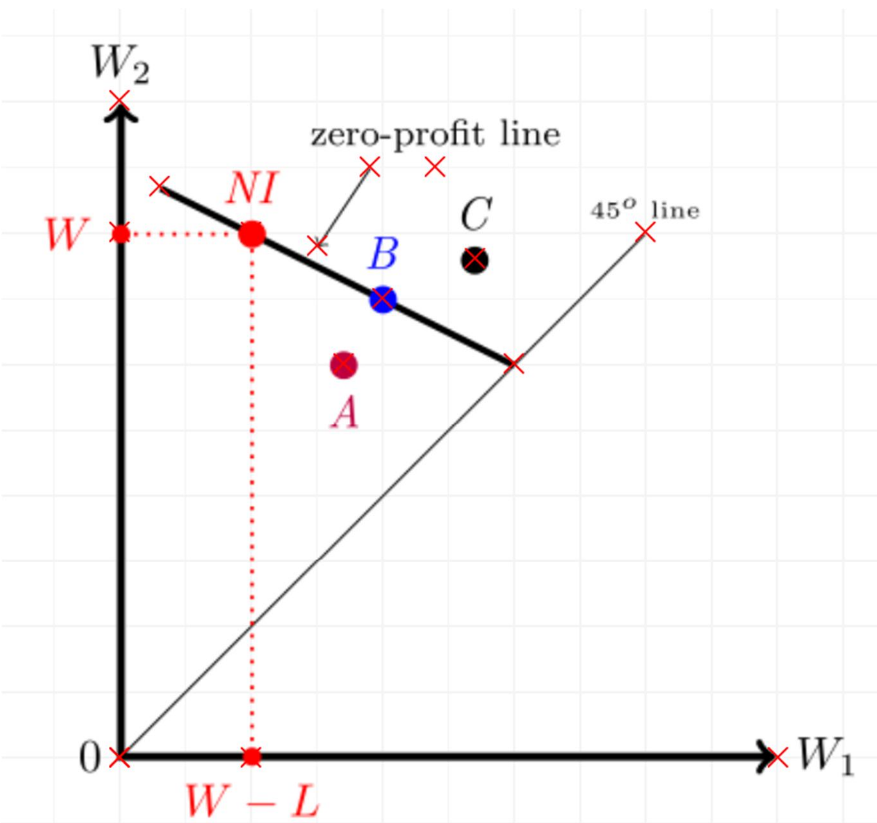




W_2
isoprofit line
A
45^o line
B
isoprofit
line
0
W_1

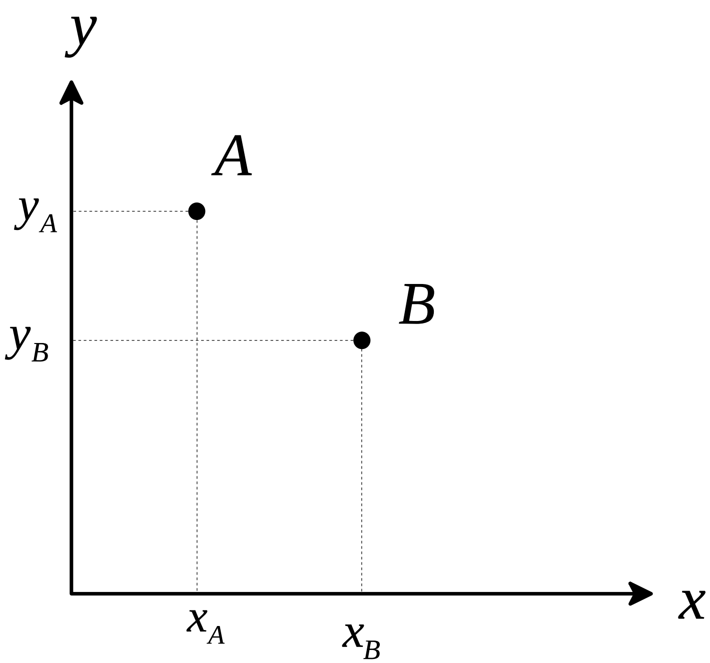


Since No Insurance can be thought of as the trivial contract h = 0 and d = L, which gives zero profits, the isoprofit line going through the NI point is the zero-profit line:



BINARY LOTTERIES

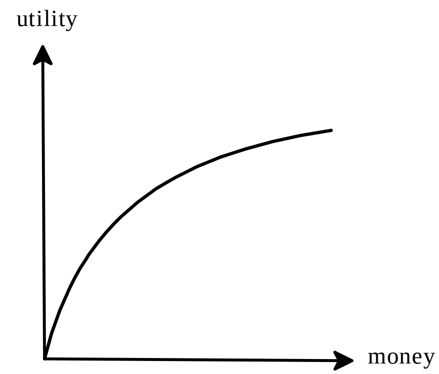
Lotteries of the form $\begin{pmatrix} \$x & \$y \\ p & 1-p \end{pmatrix}$ with p fixed and x and y allowed to vary.

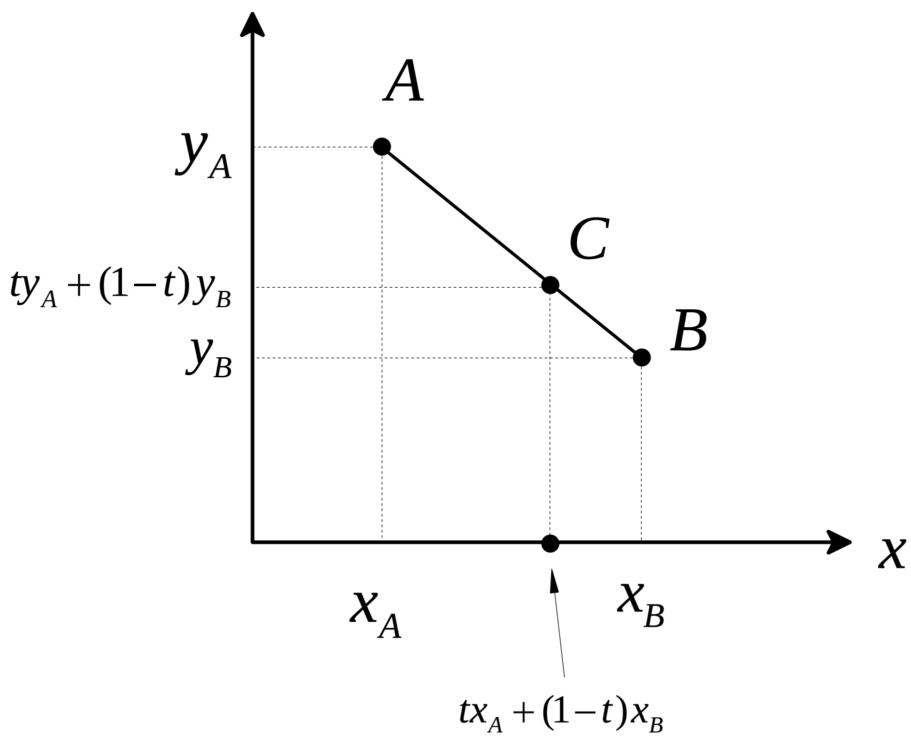


We want to draw indifference curves in this diagram.

Case 2: risk-averse agent

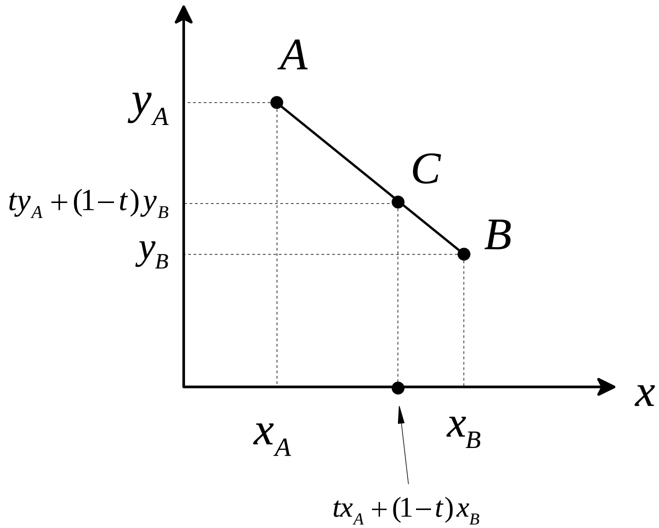
$U(m)$ is strictly concave:





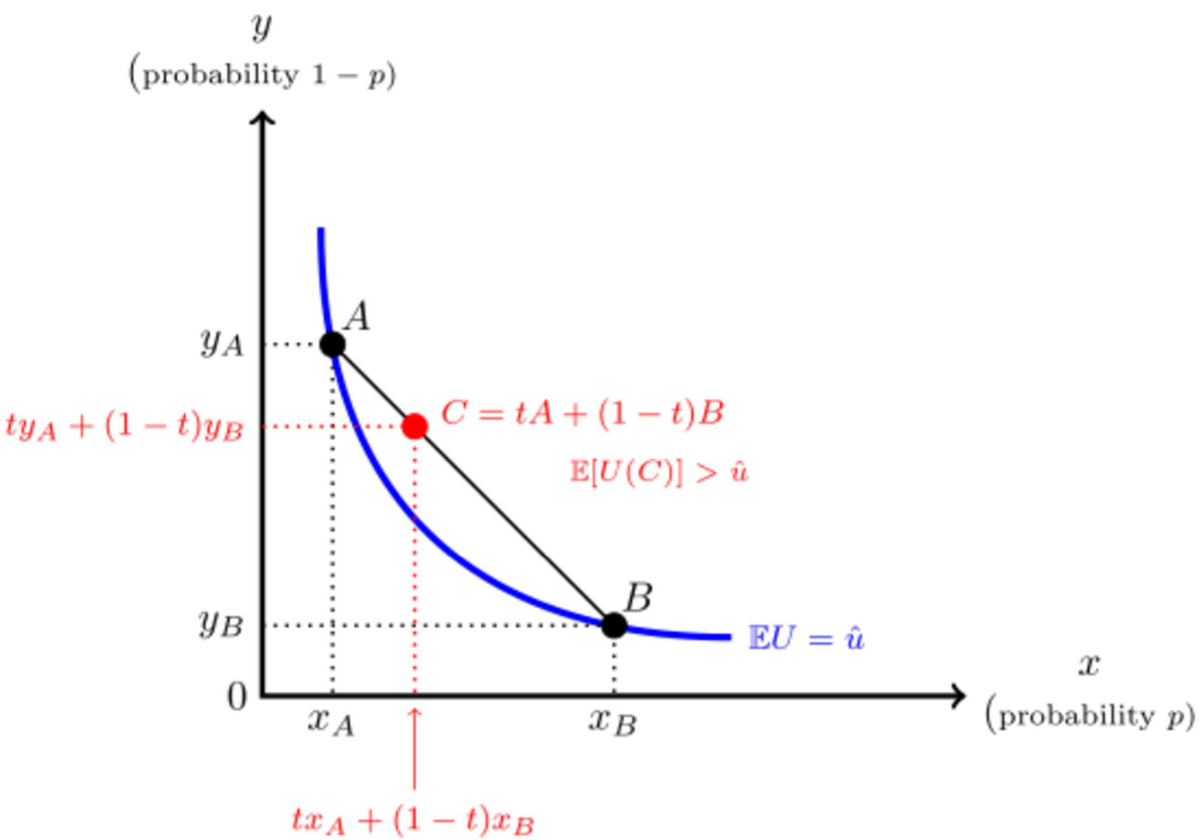
Suppose that $\mathbb{E}[U(A)] = \mathbb{E}[U(B)]$

Want to show that
$\mathbb{E}[U(C)] > \mathbb{E}[U(A)](= \mathbb{E}[U(B)])$



$\mathbb{E}[U(C)] =$

The indifference curve must lie below the straight-line segment joining A and B.


y
(probability $1-p$)
y_A
$ty_A + (1-t)y_B$
y_B
0
A
$C = tA + (1-t)B$
$\mathbb{E}[U(C)] > \hat{u}$
B
$\mathbb{E}U = \hat{u}$
x
(probability p)
x_A
x_B
$tx_A + (1-t)x_B$

Slope of indifference curve

Let A and B be two points that lie on the same indifference curve: $\mathbb{E}[U(A)] = \mathbb{E}[U(B)]$,

(*)

- Since x_B is close to x_A, $U(x_B) \simeq$
- Since y_B is close to y_A, $U(y_B) \simeq$

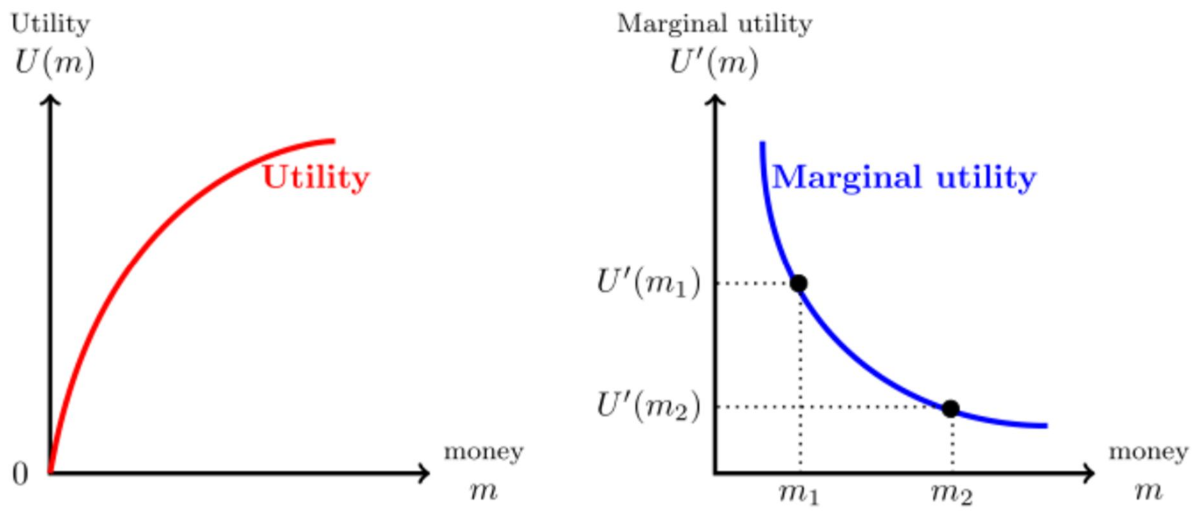
Thus the RHS of (*) can be written as

So (*) becomes

that is, which can be written as

Comparing the slope at a point with the ratio $\frac{p}{1-p}$

Look at the case of risk aversion but the other cases are similar.



- at a point **above** the 45^{o} line, where $x < y$,

- at a point **on** the 45^{o} line, where $x = y$,

- at a point **below** the 45^{o} line, where $x > y$,