

q 3 $1-q$
 E F

$P \rightarrow C$	$3, 2$	$1, 4$
$1-p \rightarrow D$	$2, 1$	$4, 0$

NE: $= \left(\underbrace{\begin{pmatrix} C & D \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}}_{\sigma_2}, \underbrace{\begin{pmatrix} E & F \\ \frac{3}{4} & \frac{1}{4} \end{pmatrix}}_{\sigma_3} \right)$

no pure-strategy NE

need : $\underline{3q + 1 - q = 2q + 4(1 - q)}$

Solution $q = \frac{3}{4}$

Solution $p = \frac{1}{3}$

need $2p + 1 - p = 4p$

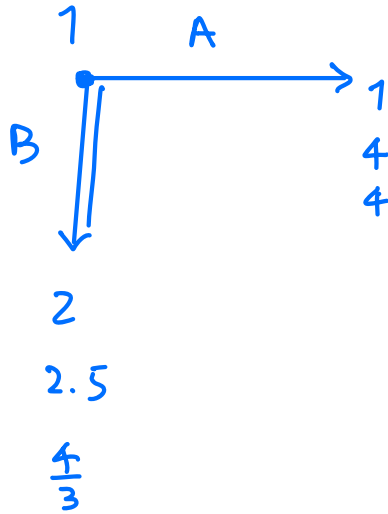
$\pi_2 = 3 \cdot \frac{3}{4} + 1 \cdot \frac{1}{4} = \frac{10}{4}$

$\pi_3 = 4 \cdot \frac{1}{3} = \frac{4}{3}$

By indifference theorem:
 $\pi_2(\sigma_2, \sigma_3) = \pi_2(C, \sigma_3)$

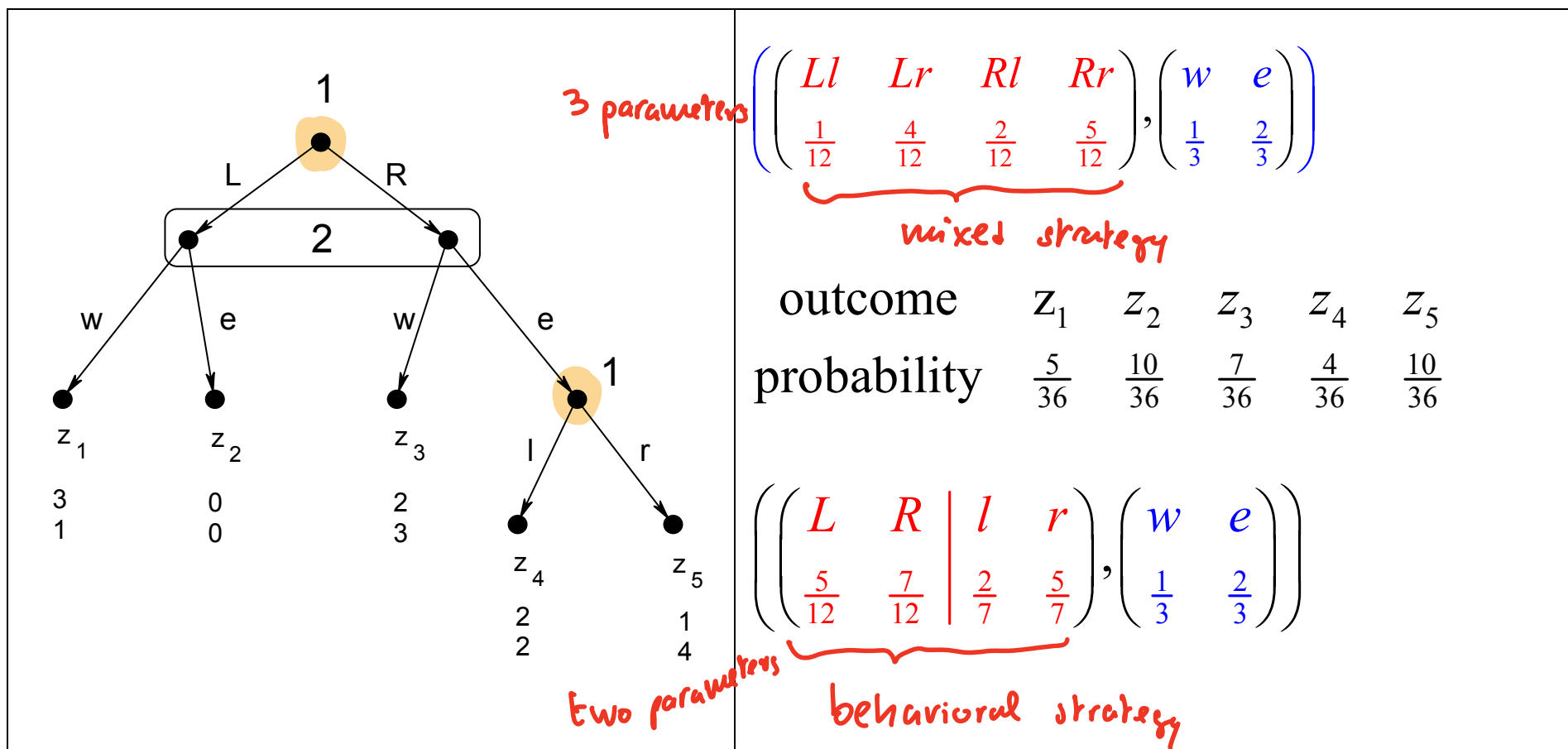
$\pi_3(\sigma_2, \sigma_3) = \pi_3(\sigma_2, F)$

$$\pi_1: \frac{1}{3} \cdot \frac{3}{4} \cdot 4 + \frac{1}{3} \cdot \frac{1}{4} \cdot 0 + \frac{2}{3} \cdot \frac{3}{4} \cdot 1 + \frac{2}{3} \cdot \frac{1}{4} \cdot 3 = 2$$



$$SPE: \left(\begin{pmatrix} A & B \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} C & D \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}, \begin{pmatrix} E & F \\ \frac{3}{4} & \frac{1}{4} \end{pmatrix} \right)$$

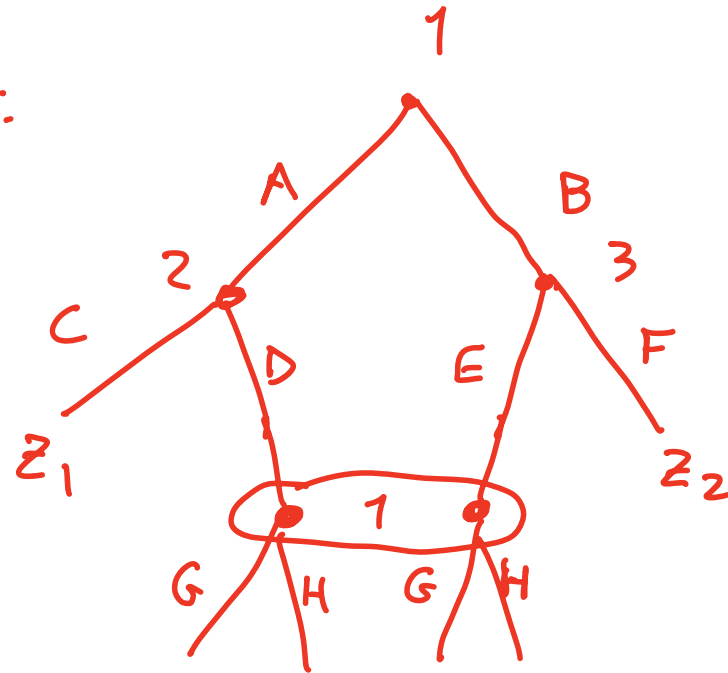
BEHAVIORAL STRATEGIES IN EXTENSIVE GAMES



THEOREM (Kuhn, 1953). In an extensive game **with perfect recall**, mixed and behavior strategies are equivalent. [For a precise statement of "equivalence" see the textbook.]

What does Perfect Recall mean?

Lack of perfect recall :



ASYMMETRIC INFORMATION

INSURANCE MARKETS

Consider an individual with

risk averse

W_0 initial wealth

L potential loss

p probability of loss

$$0 \leq L \leq W_0$$

With no insurance she faces the money lottery

$$NI = \begin{pmatrix} W_0 - L & W_0 \\ p & (1-p) \end{pmatrix}$$

An **insurance contract** is a pair (h, d)

h premium

d deductible

$L - d$ insured amount of the loss

$$\begin{pmatrix} W_0 - d - h & W_0 - h \\ p & (1-p) \end{pmatrix}$$

With contract (h, d) the individual faces the lottery

- If $d = 0$ full-insurance contract

$$d < L$$

- If $d > 0$ partial- " "

$$\text{loss : } \begin{pmatrix} L & 0 \\ p & 1-p \end{pmatrix}$$

$$\text{exp. value} = pL$$

$$h \geq pL$$

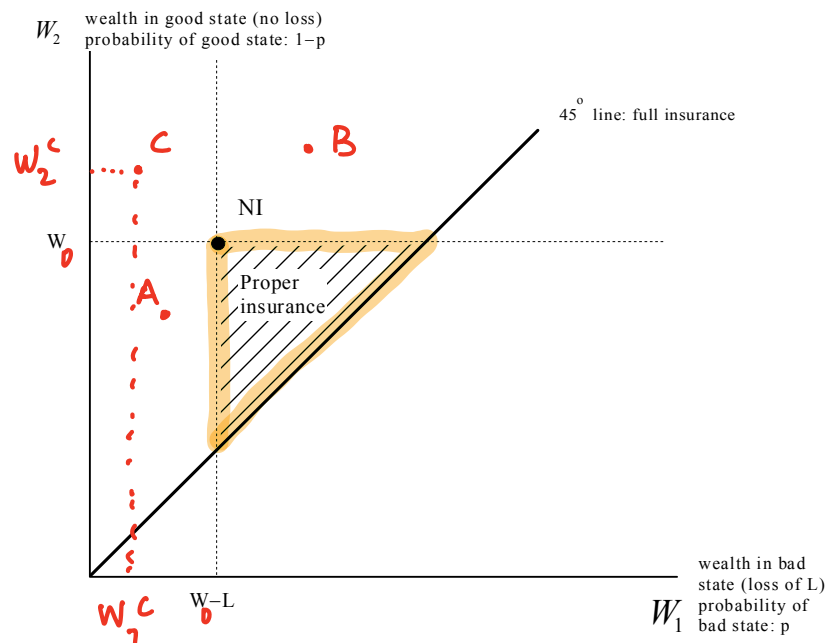
Would the individual purchase the full-insurance contract with $\underline{h = pL}$?

$$NI : \begin{pmatrix} W_0 - L & W_0 \\ p & 1-p \end{pmatrix}$$

$$E[NI] = pW_0 - pL + (1-p)W_0 = W_0 - pL$$

With contract $h = pL$ customer is guaranteed
 $d = 0$ $W_0 - pL$

- If she is risk averse then $U(W_0 - pL) > E[U(NI)]$
- If she is risk neutral then $U(W_0 - pL) = E[U(NI)]$
- If she is risk loving then $U(W_0 - pL) < E[U(NI)]$



A contract expressed as a pair (h, d) can be translated into a point in wealth space as follows:

Contract A:

$$h_A = 40,000 - 34,000 = 6,000 \quad W_2 \text{ (good state)}$$

$$d_A = 34,000 - 18,000 = 16,000$$

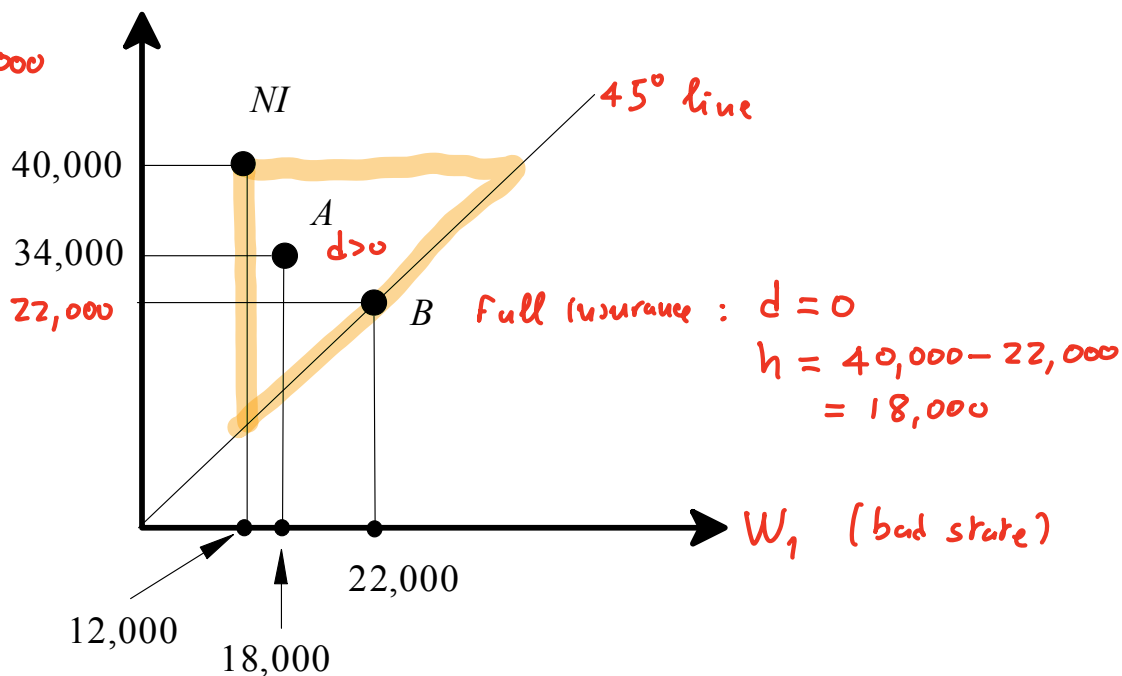
$$W_0 = 40,000$$

$$L = 40,000 - 12,000 = 28,000$$

(W_1, W_2)

$$h = W_0 - W_2$$

$$d = W_2 - W_1$$



Here we have: $W =$ $L =$

ISOPROFIT LINES

Assume that the **insurance company** is **risk neutral** so that it considers selling an insurance contract $C = (h, d)$, corresponding to the lottery

$$C = \begin{pmatrix} h - (L - d) & h \\ p & (1 - p) \end{pmatrix}, \text{ as equivalent to getting its expected value}$$

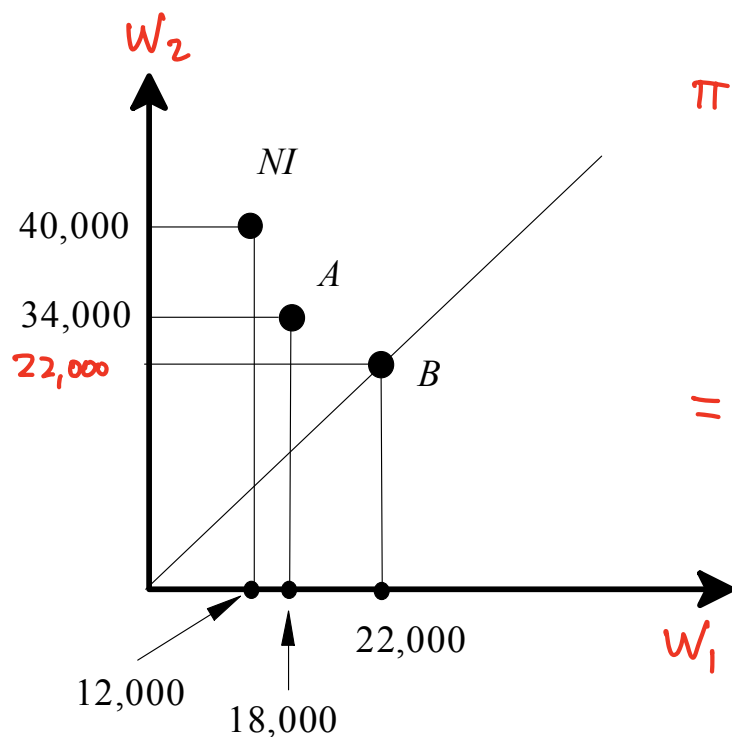
for sure: $\mathbb{E}[C] = p [h - (L - d)] + (1 - p) h = h - \underbrace{p(L - d)}_{\text{expected payment}}$

We denote the expected profit from contract (h, d) by $\pi(h, d)$. Thus

$$\pi(h, d) = h - \underbrace{p(L - d)}_{\text{expected payment}} = h - pL + pd$$

If the contract is expressed as a point (W_1, W_2) in wealth space then

$$\pi(W_1, W_2) = \underbrace{(W_0 - W_2)}_h - pL + p \underbrace{(W_2 - W_1)}_d$$



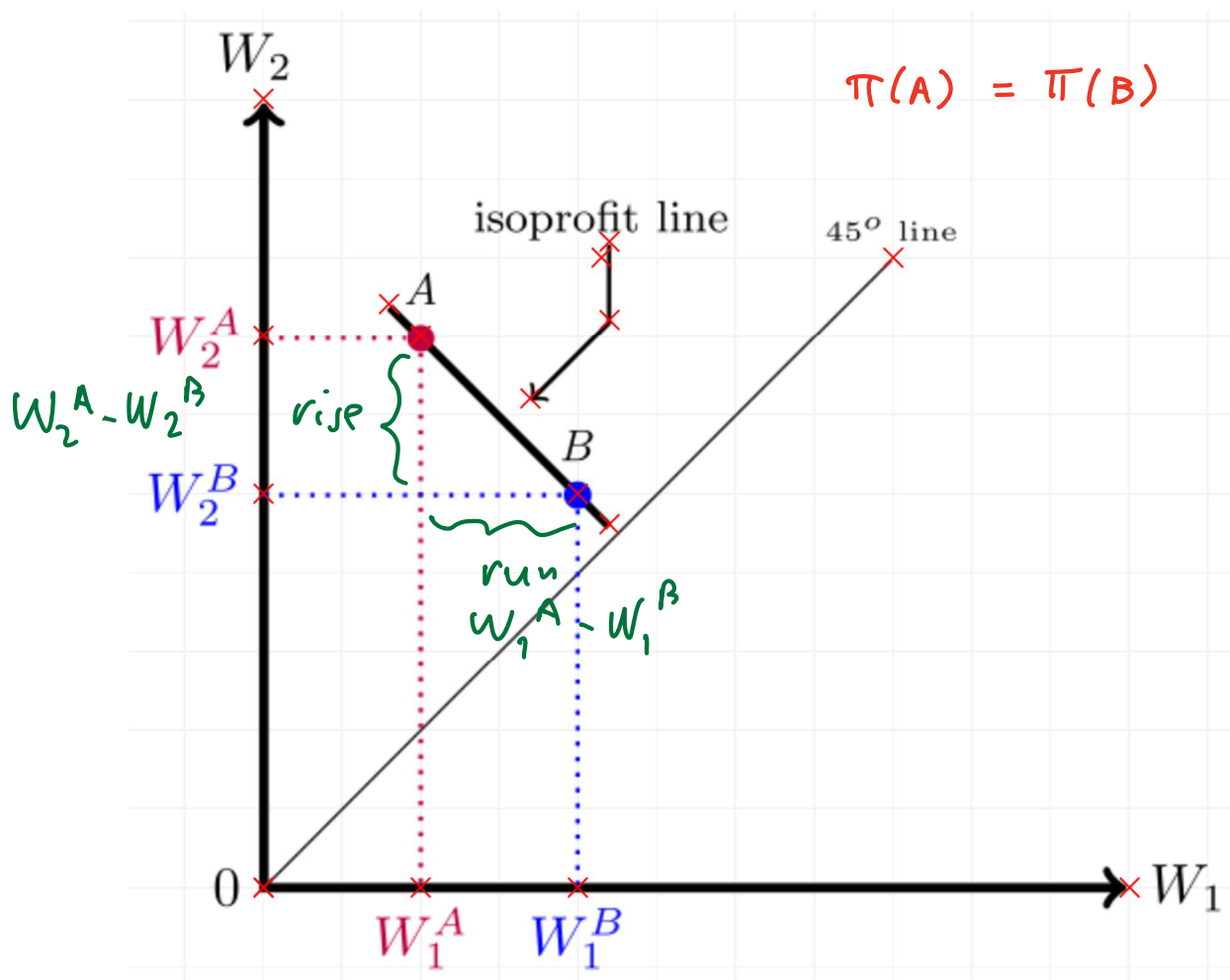
$$\pi(A) = \underbrace{W_0 - W_2}_h - \frac{1}{10} 28000 + \frac{1}{10} (W_2 - W_1)$$

$$= 40000 - 34000 - 2800 + \frac{1}{10} (34000 - 18000)$$

$$W_1 \quad \pi(B) = \underbrace{(40000 - 22,000)}_h - \frac{1}{10} 28000$$

Suppose that $p = \frac{1}{10}$. What is $\pi(A)$? What is $\pi(B)$?

An **isoprofit line** is defined as a line joining contracts that give the same expected profit. Let $A = (W_1^A, W_2^A)$ and $B = (W_1^B, W_2^B)$ be such that $\pi(A) = \pi(B)$



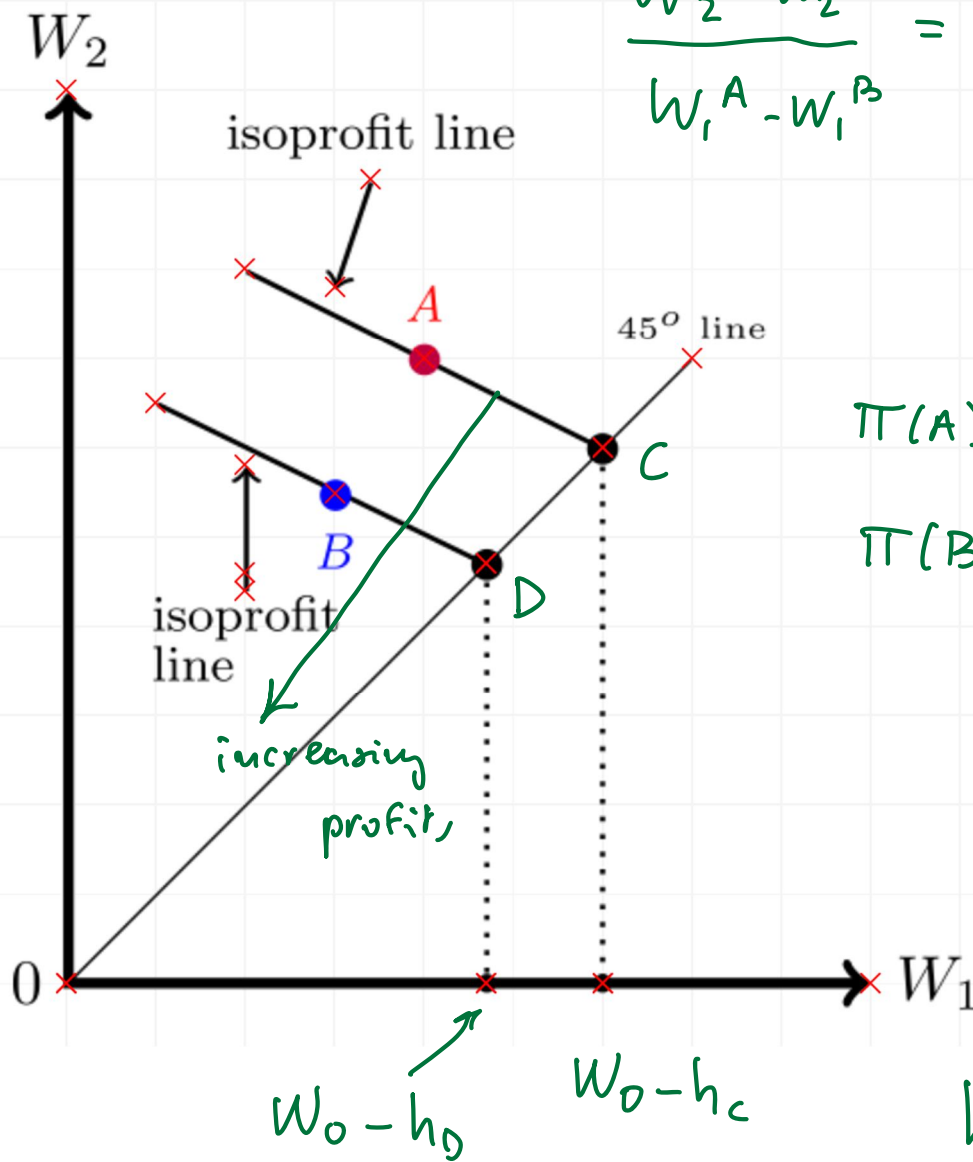
$$\pi(A) = \cancel{W_0} - W_2^A - \cancel{pL} + p(W_2^A - W_1^A)$$

$$\pi(B) = \cancel{W_0} - W_2^B - \cancel{pL} + p(W_2^B - W_1^B)$$

Set them equal

$$-W_2^A + p(W_2^A - W_1^A) = -W_2^B + p(W_2^B - W_1^B)$$

$$\frac{W_2^A - W_2^B}{W_1^A - W_1^B} = - \frac{p}{1-p}$$



$$\pi(A) = \pi(C)$$

$$\pi(B) = \pi(D)$$

$$h_D > h_C$$

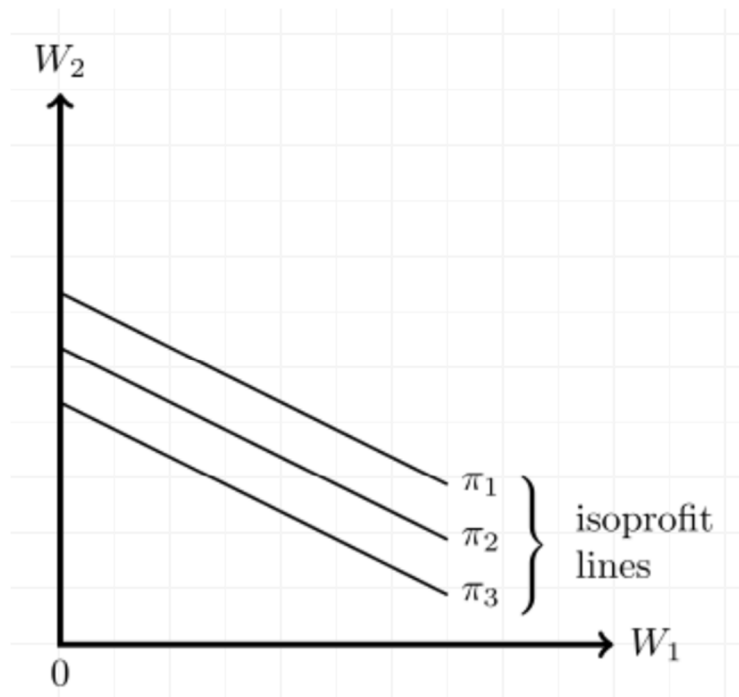
$$\pi(C) = h_C - pL$$

$$\pi(D) = h_D - pL$$

$$\pi(D) > \pi(C)$$

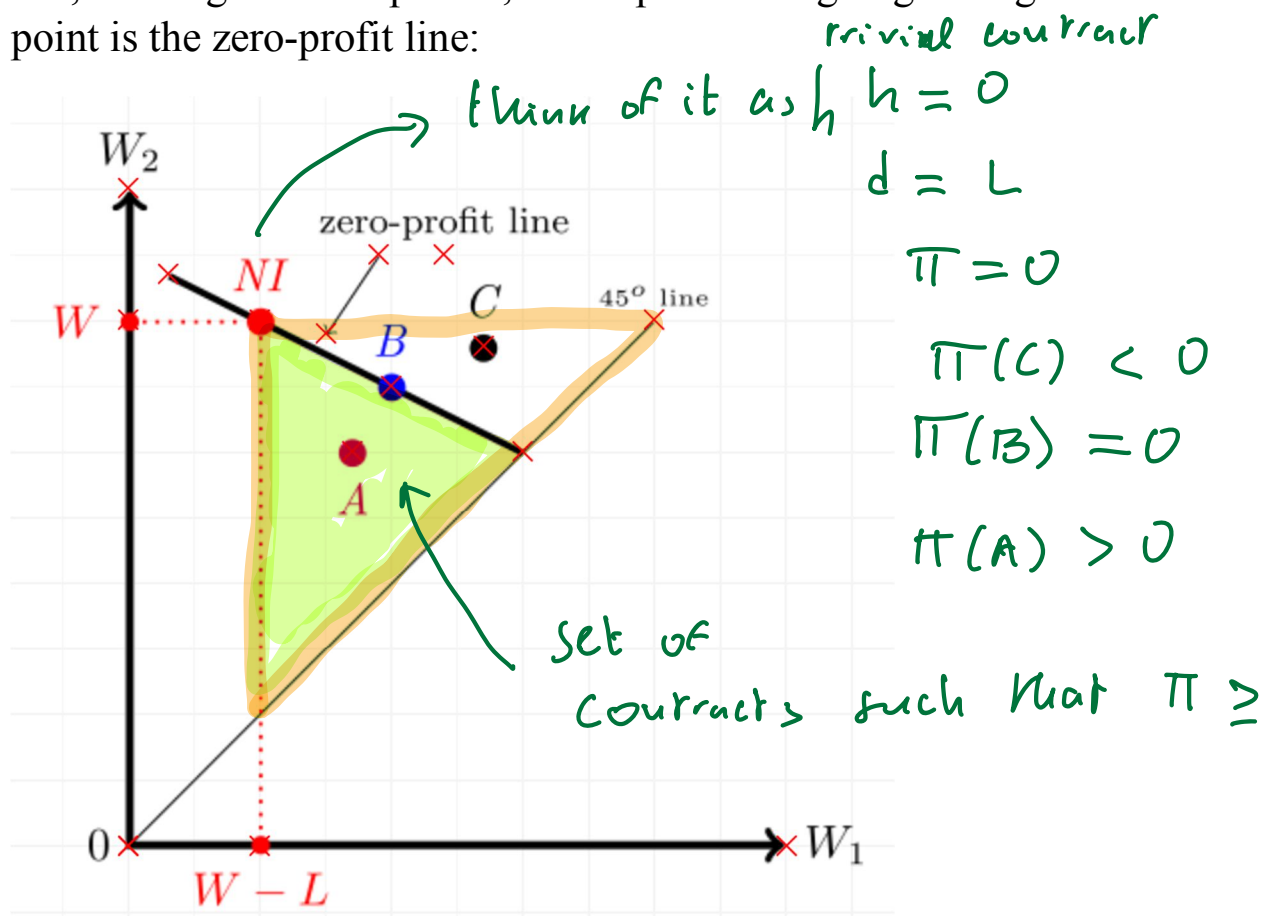
$$\parallel \parallel$$

$$\pi(B) > \pi(A)$$



$$\pi_3 > \pi_2 > \pi_1$$

Since No Insurance can be thought of as the trivial contract $h = 0$ and $d = L$, which gives zero profits, the isoprofit line going through the NI point is the zero-profit line:



Contract (W_1, W_2)

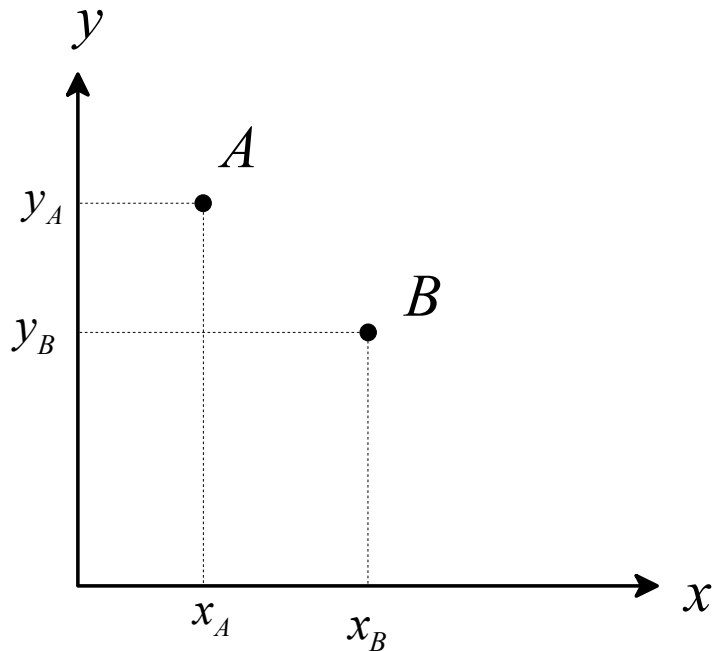
From consumer's point of view $\left(\begin{matrix} W_0 - (W_0 - W_2) - p \\ (W_2 - W_1) \end{matrix} \right)$

$$w_0 - (w_0 - w_2)$$

$$1-p$$

BINARY LOTTERIES

Lotteries of the form $\begin{pmatrix} \$x & \$y \\ p & 1-p \end{pmatrix}$ with p fixed and x and y allowed to vary.



$$U(\$u)$$

$$E[U(A)] = E[U(B)]$$

draw indifference curve through A

$$E[U(L)]$$

Risk averse if

We want to draw indifference curves in this diagram

$$U(t m_1 + (1-t) m_2) > t U(m_1) + (1-t) U(m_2)$$

$$E[L] \quad L = \begin{pmatrix} m_1 & m_2 \\ t & 1-t \end{pmatrix}$$

