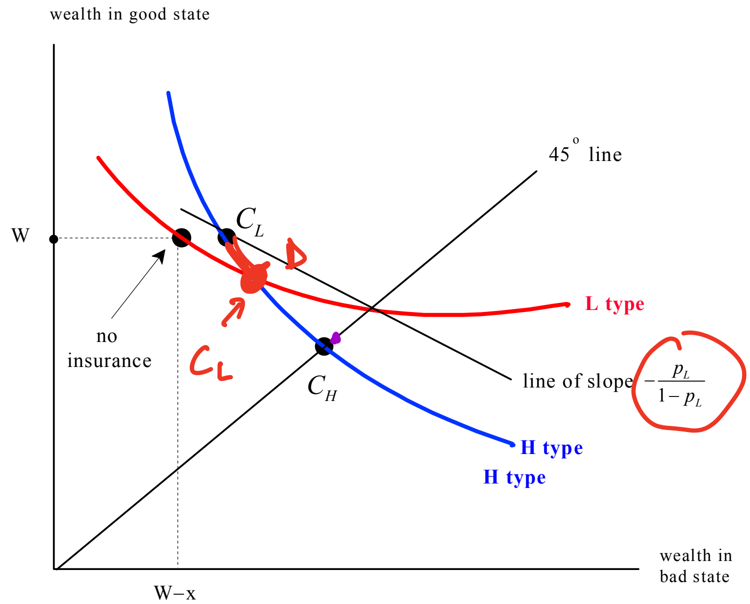
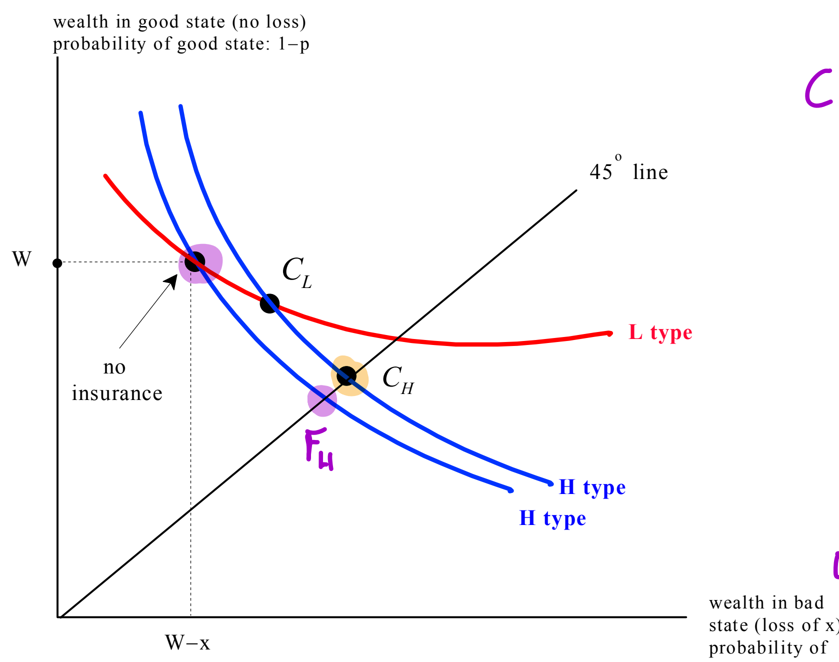
(IR_L) must be satisfied as an equality.



C_H must be a full-insurance contract

C_L given by intersection of H-indiff. curve through C_H

and L-indiff. curve through NI



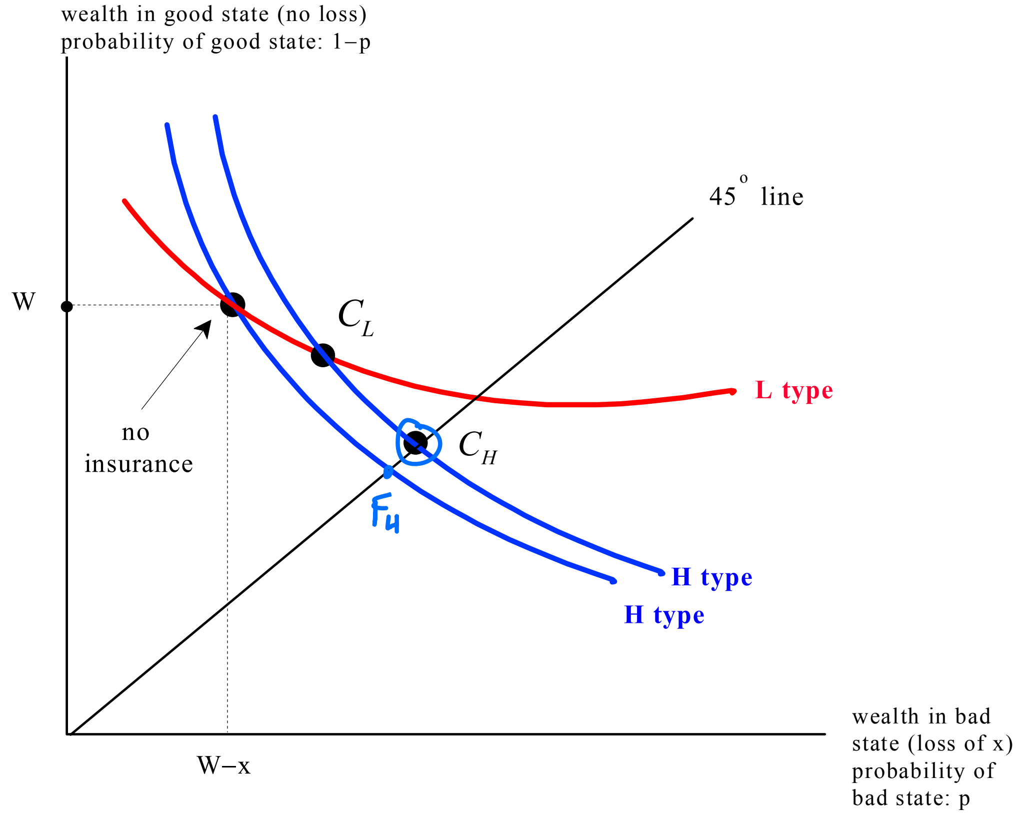
Special case:

Option 1

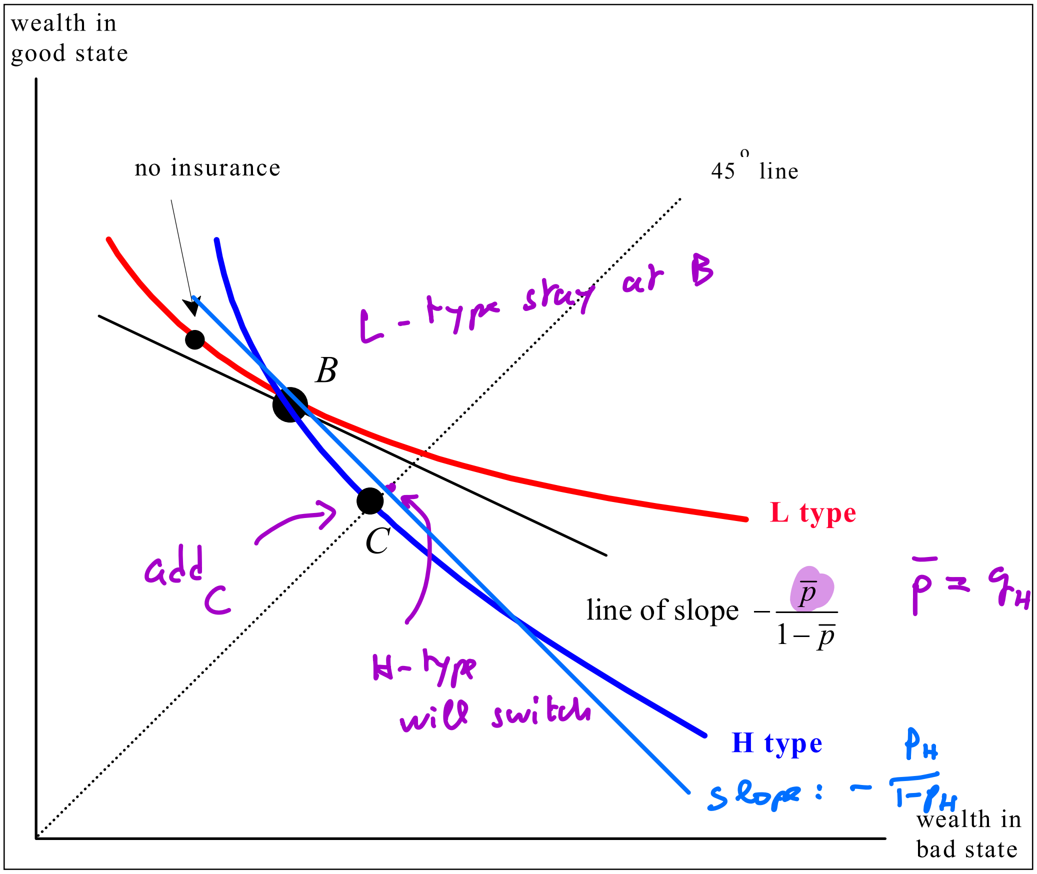
$$\begin{cases} C_H = (h_H^*, d_H = 0) \\ C_L = (h_L = 0, d_L = \ell) \end{cases}$$

(IC_L) **is not binding: it is always satisfied as a strict inequality**.

Option 1 is a special case of Option 3



Option 3 yields higher profits than Option 2: $\pi_2^* < \pi_3^*$



$\bar{p} = q_H p_H + (1-q_H) p_L$

In conclusion, the monopolist will always choose Option 3, although in some cases (namely when q_H is close to 1) **the outcome is the same as in Option 1.**

$$\max_{h_H} \quad \Pi_3 = q_H N[h_H - p_H \ell] + (1-q_H) N[h_L - p_L(\ell - d_L)]$$

$$\text{s.t.} \begin{cases} \underbrace{U(W-h_H)}_{\text{H utility from } C_H} = \underbrace{p_H U(W-h_L-d_L) + (1-p_H) U(W-h_L)}_{\text{H utility from } C_L} \\ \underbrace{p_L U(W-\ell) + (1-p_L) U(W)}_{\text{L utility from NI}} = p_L U(W-h_L-d_L) + (1-p_L) U(W-h_L) \end{cases}$$

Solution

$h_L(h_H)$

$d_L(h_H)$

$$\max_{h_H} \quad \Pi_3 = q_H N[h_H - p_H \ell] + (1-q_H) N[h_L(h_H) - p(\ell - d_L(h_H))]$$

EXAMPLE. $W = 1{,}600$, $\ell = 700$, $p_H = \frac{1}{5}$, $p_L = \frac{1}{10}$, $U(m) = \sqrt{m}$.

h_H^* is given by the solution to $\sqrt{1{,}600 - h} = \frac{1}{5}\sqrt{1600-700} + \frac{4}{5}\sqrt{1600}$

$$h_H^* = 156$$

Thus under $\boxed{\textbf{Option 1}}$ profits are: $q_H N\left[156 - \frac{1}{5}700\right]$

h_L^*

Now **Option 3**. Let $h_H \in [79, 156]$ be the premium for the full-insurance contract targeted to the H type To find c_L solve:

Fix h_H solve

$$\sqrt{1600 - h_H} = \frac{1}{5}\sqrt{1600 - h_L - d_L} + \frac{4}{5}\sqrt{1600 - h_L}$$

$$\frac{1}{10}\sqrt{1600-700} + \frac{9}{10}\sqrt{1600} = \frac{1}{10}\sqrt{1600 - h_L - d_L} + \frac{9}{10}\sqrt{1600 - h_L}$$

$$h_L(h_H) = h_H + 156\sqrt{1600 - h_H} - 6{,}084$$

$$d_L(h_H) = 80h_H + 5460\sqrt{1600 - h_H} - 219{,}260$$

We can solve the two equations in terms of h_H:

- $h_L(h_H) = h_H + 156\sqrt{1{,}600 - h_H} - 6{,}084$
- $d_L(h_H) = 80h_H + 5{,}460\sqrt{1{,}600 - h_H} - 219{,}260$

Then the monopolist will choose h_H to maximize

$$\max_{h_H} \quad \pi_3 = q_H N\left[h_H - \frac{1}{5}700\right] + (1-q_H)N\left[h_L(h_H) - \frac{1}{10}\left(700 - d_L(h_H)\right)\right]$$

This function is strictly concave and $\left.\frac{d\pi_3}{dh_H}\right|_{h_h=79} = q_H N > 0$ and

$\left.\frac{d\pi_3}{dh_H}\right|_{h_h=156} = \frac{47}{38}q_H - \frac{9}{38}$. This is negative if and only if $q_H < \frac{9}{47}$. Thus,

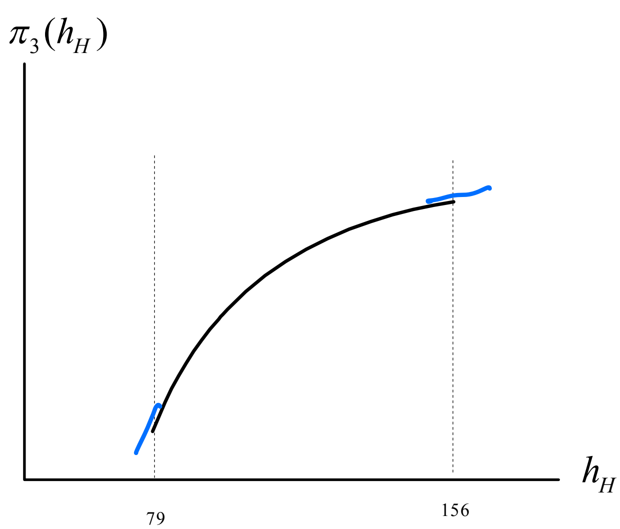


Figure 23a OPTION 1

$\boxed{q_H \geq \frac{9}{47}}$: the optimal solution is to offer only the full insurance contract at premium $h_H^* = 156$ (Option 1)

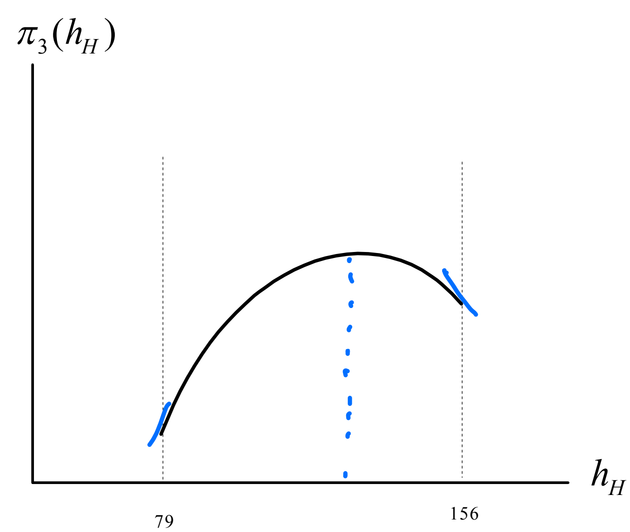


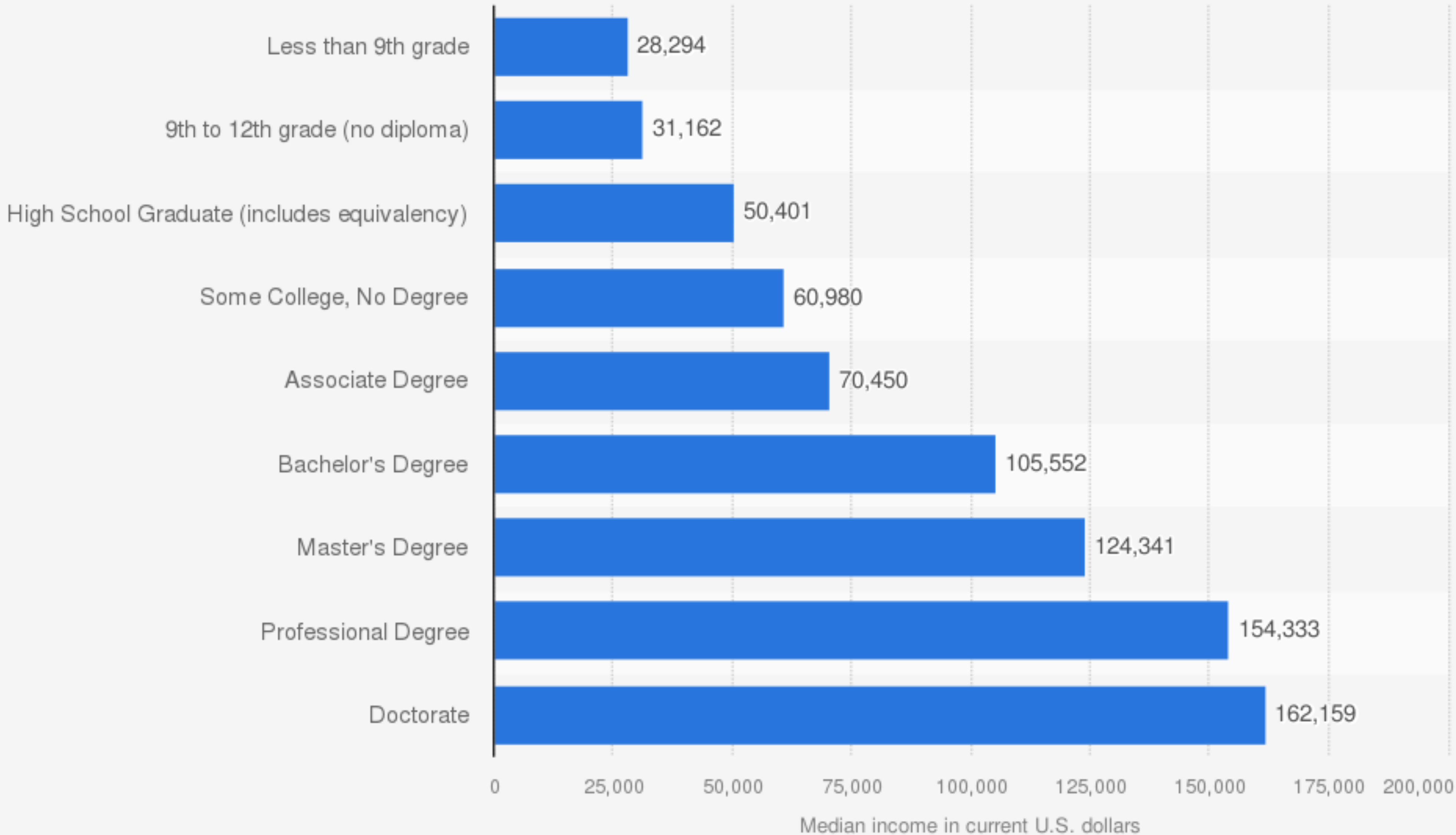
Figure 23b

$\boxed{q_H < \frac{9}{47}}$: the optimal solution is to offer two contracts (Option 3)

U.S. median household income 2021, by education level

Less than 9th grade	$28,294
9th to 12th grade (no diploma)	$31,162
High School Graduate	$50,401
Some College, No Degree	$60,980
Associate Degree	$70,450
Bachelor's Degree	$105,552
Master's Degree	$124,341
Professional Degree	$154,333
Doctorate	$162,159

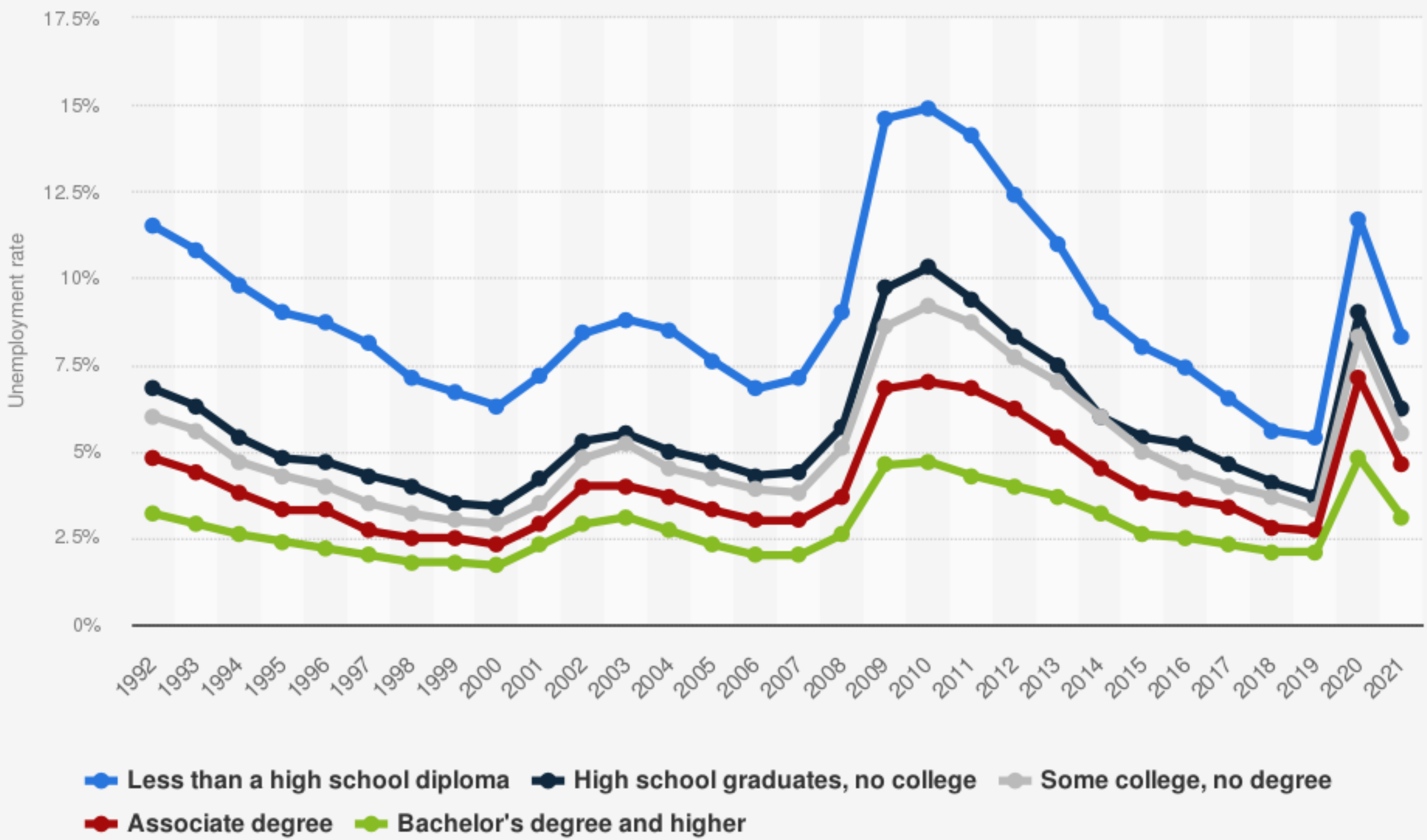
Median household income in the United States in 2021, by educational attainment of householder (in U.S. dollars)



Source
US Census Bureau

Additional Information:
United States; US Census Bureau; 2021

Unemployment rate in the United States from 1992-2021, by level of education



Source
Bureau of Labor Statistics

Additional Information:
United States; 1992 to 2021; 25 years and older

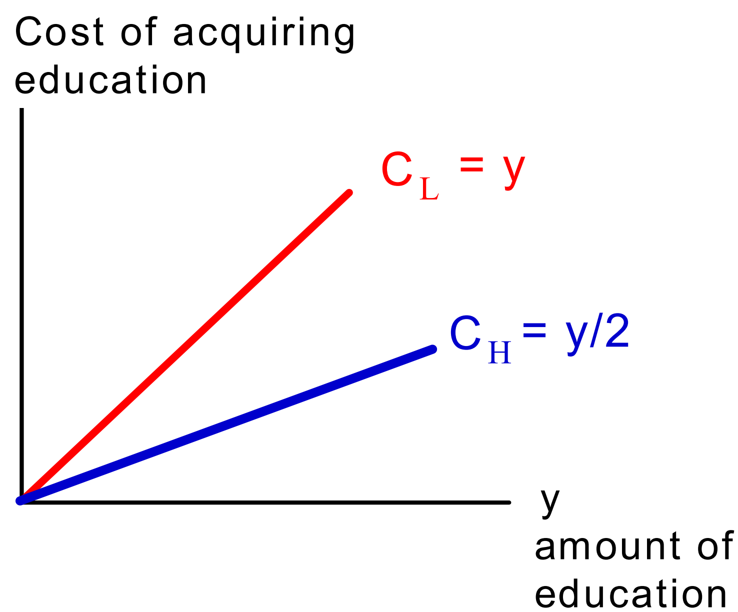
Suppose that there are two groups of individuals:

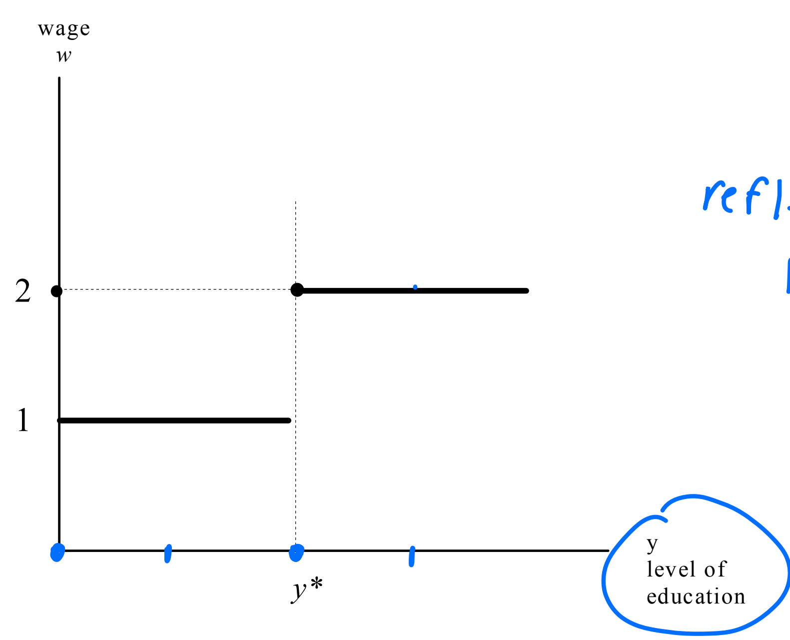
Constant, not a function of education

Group L	Group H
Marginal productivity = **1**	Marginal productivity = **2**
Proportion in population: q_L	Proportion in population: $1-q_L$

Constant

with $0 < q_L < 1$.





reflects wrong beliefs of the employer

For a GROUP L individual	**If choose $y = 0$**	get $w = 1$
		pay C = 0
		net wage = $1 - 0 = 1$
will choose $y=0$ if $1 > 2 - y^*$, $y^* > 1$	**If choose $y = y^*$**	get $w = 2$
		pay C = y^*
		net wage = $2 - y^*$

For a GROUP H individual	**If choose $y = 0$**	get $w = 1$
		pay C = 0
will choose y^*		net wage = 1
if $2 - \frac{y^*}{2} > 1$	**If choose $y = y^*$**	get $w = 2$
$1 > \frac{y^*}{2}$		pay C = $\frac{y^*}{2}$
		net wage = $2 - \frac{y^*}{2}$

$y^* < 2$

$1 < y^* < 2$ $\begin{cases} L \text{ choose } y=0 \\ H \text{ " } y=y^* \end{cases}$

Separating signaling equilibrium

Can a signaling equilibrium be Pareto inefficient?

everybody $y=0$

before gvnmt intervention

after gvnmt intervention

L-type : $y=0$ $w=1$

better off $U_L = 1$ $<$ $w = (1-q_H)1 + q_H 2$

H-type : $y = y^*$ $w=2$

$U_H = 2 - \frac{y^*}{2}$ $\gtrless$? $w = (1-q_H)1 + q_H 2$

possible that $1 - q_H + 2q_H$

$q_H + 1 > 2 - \frac{y^*}{2}$?

starting point

$\boxed{1 < y^* < 2}$

$2q_H + 2 > 4 - y^*$

$\boxed{y^* > 2 - 2q_H = 2(1-q_H)}$

1

e.g. q_H $\frac{1}{2}$

Signal

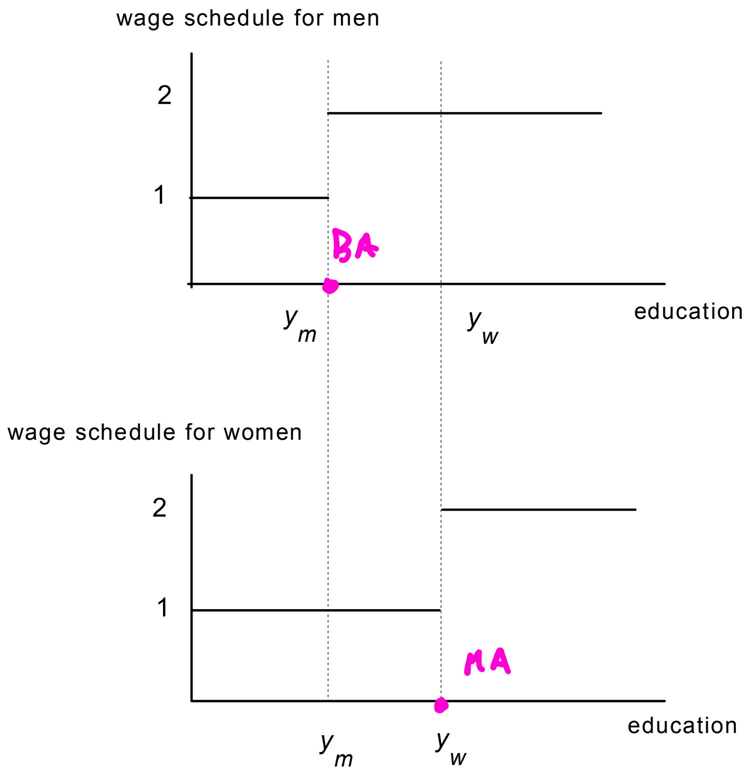
can be changed

index

cannot be changed

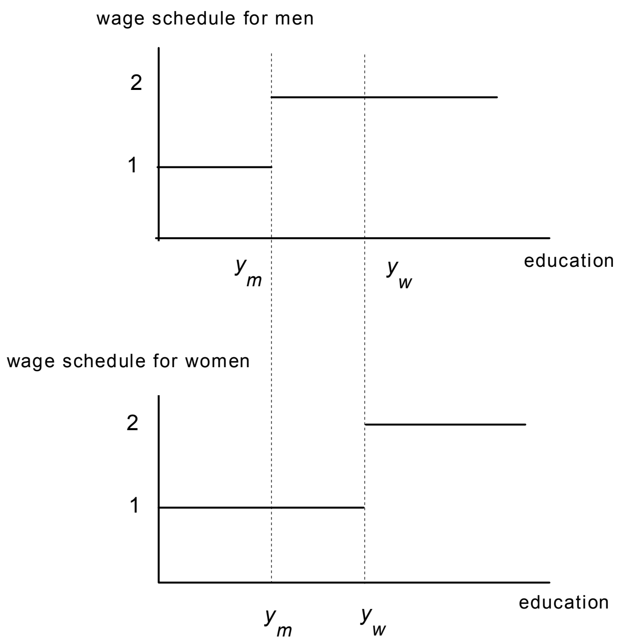
Index *vs* signal

	Women, L	Women, H	Men, L	Men, H
productivity	1	2	1	2
proportion	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
Cost of acquiring y units of education	y	$\frac{y}{2}$	y	$\frac{y}{2}$



	Women, L	Women, H	Men, L	Men, H
productivity	1	2	1	2
proportion	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
Cost of acquiring y units of education	y	$\frac{y}{2}$	y	$\frac{y}{2}$

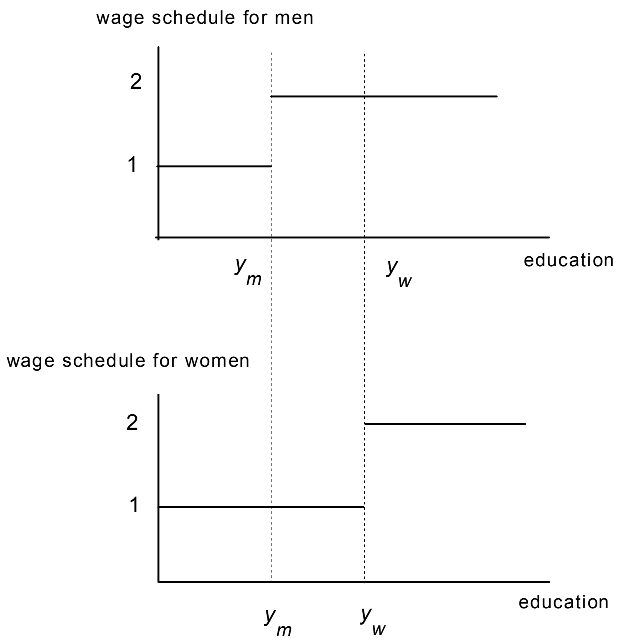
MEN's CALCULATIONS



$$1 < y_m < 2 \begin{cases} \text{L men choose } y = 0 \\ \text{H men choose } y = y_m \end{cases}$$

productivity	1	2	1	2
proportion	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
Cost of acquiring y units of education	y	$\frac{y}{2}$	y	$\frac{y}{2}$

WOMEN's CALCULATIONS



$1 < y_w < 2$

L women choose $y = 0$

H women choose $y = y_w$

$1 < y_m < y_w < 2$