2. Suppose the insurance industry is perfectly competitive

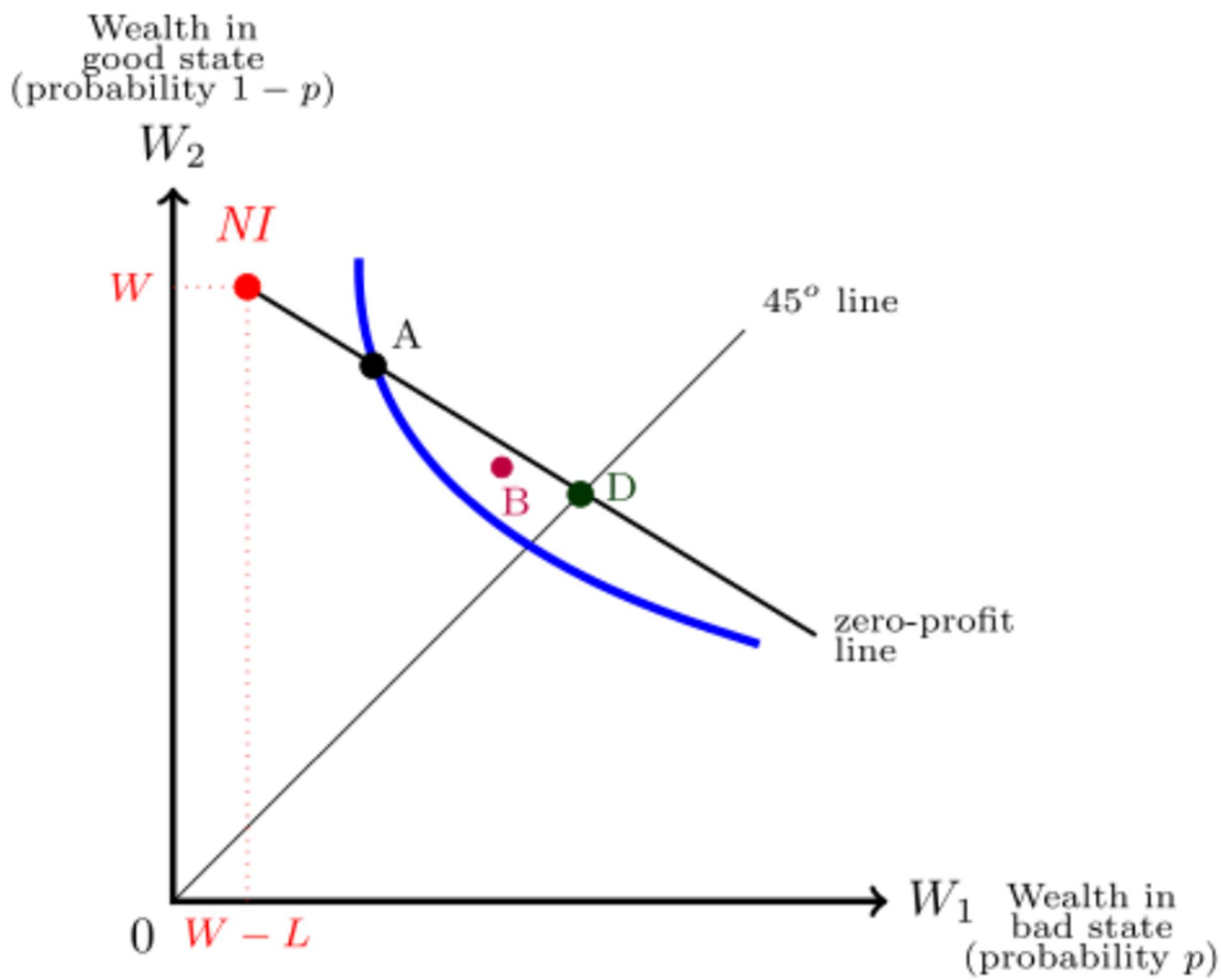
A contract that yields zero profit is called a **fair contract** and the zero profit line is called the **fair odds line**. Recall that the zero profit line is the straight line that goes through the No Insurance point and has slope $-\frac{p}{1-p}$.

Define an equilibrium in a competitive insurance industry as a situation where

(1) every firm makes zero profits and

(2) no firm (existing or new) can make positive profits by offering a new contract.

By the zero-profit condition (1), any equilibrium contract must be on the zero-profit line.

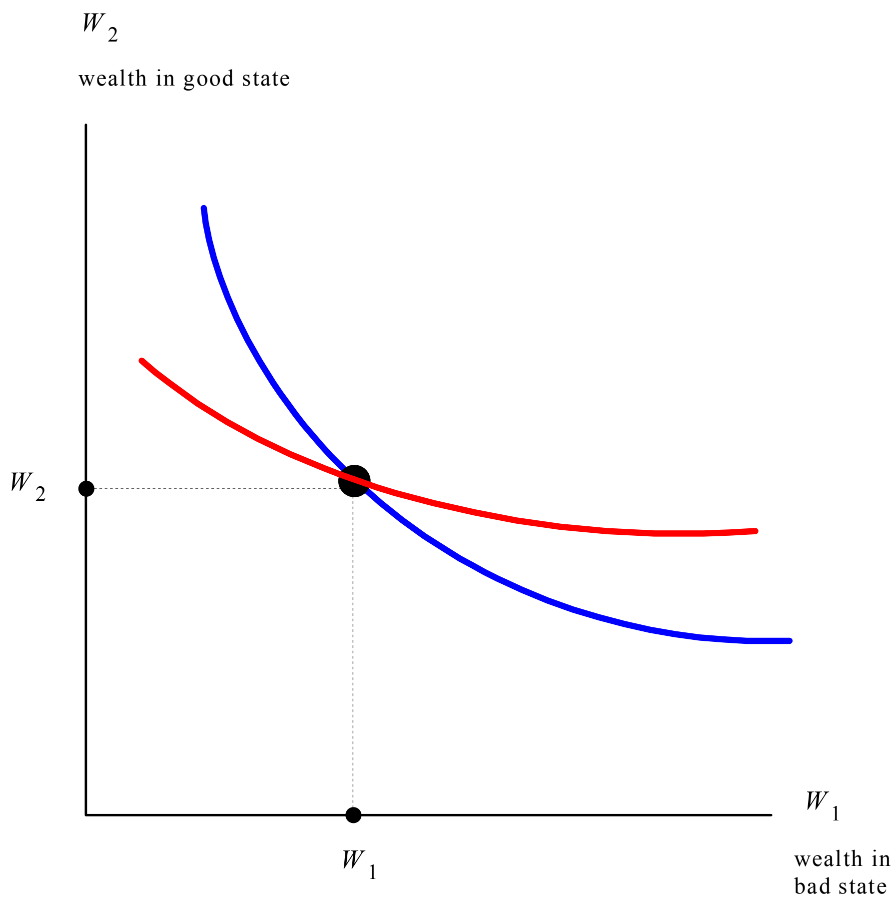


$$d_D = 0 \text{ and } h_D =$$

Adverse selection in insurance markets

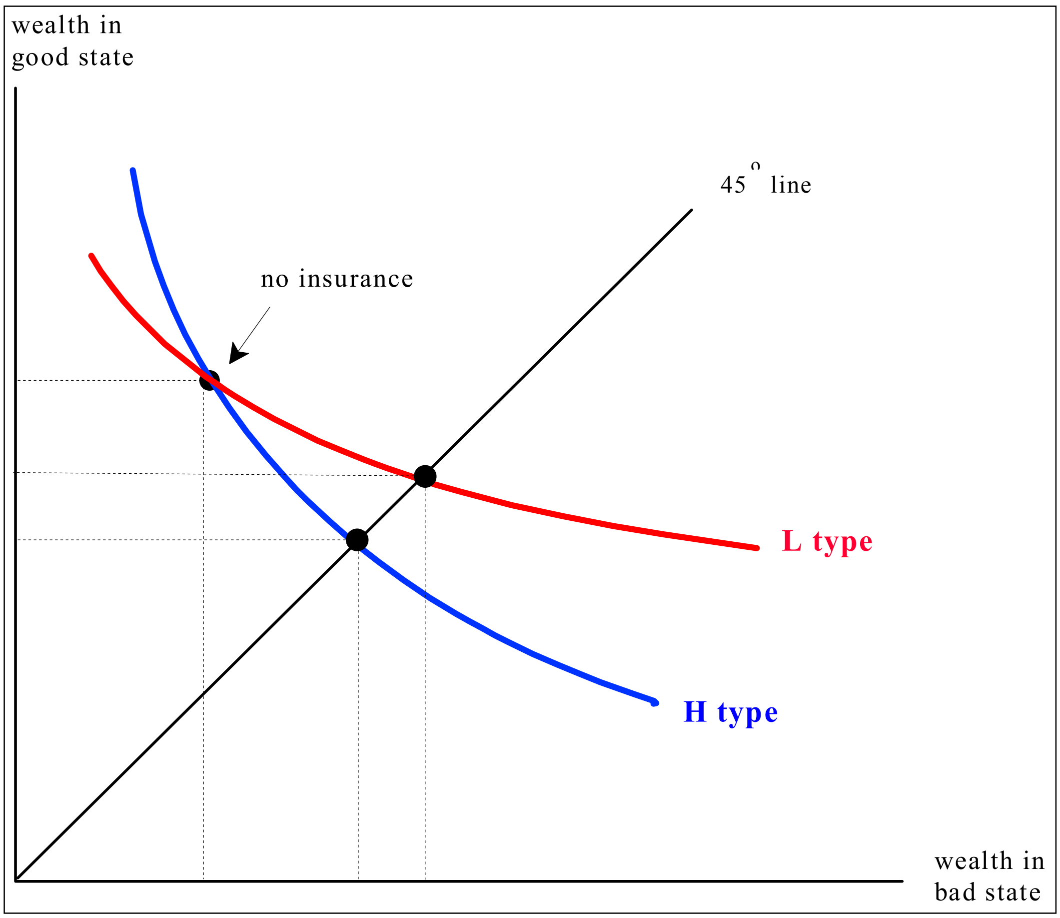
Two types of customers, H and L, identical in terms of initial wealth W, potential loss L and vNM utility-of-money function U, but with different probability of loss: $\boxed{p_H > p_L}$.

Slope of indifference curves at point (w_1, w_2)



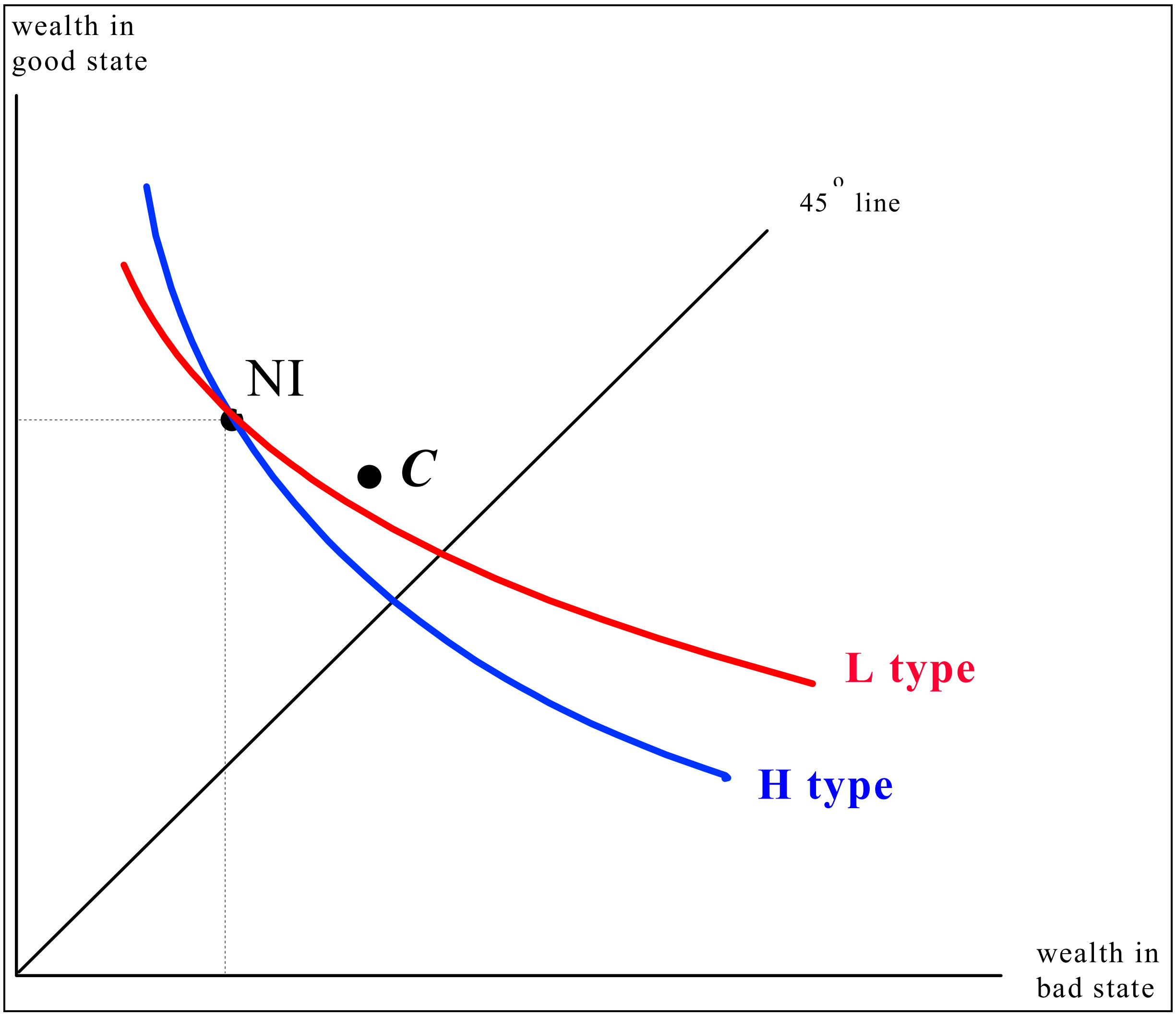
h_H^* maximum premium that the H people are willing to pay for full insurance

h_L^* maximum premium that the L people are willing to pay for full insurance:



Let q_H be the fraction of H types in the population $0 < q_H < 1$

$$\boxed{\text{If } \mathbb{E}[U_L(C)] \geq \mathbb{E}[U_L(NI)] \text{ then } \mathbb{E}[U_H(C)] \geq \mathbb{E}[U_H(NI)]}$$

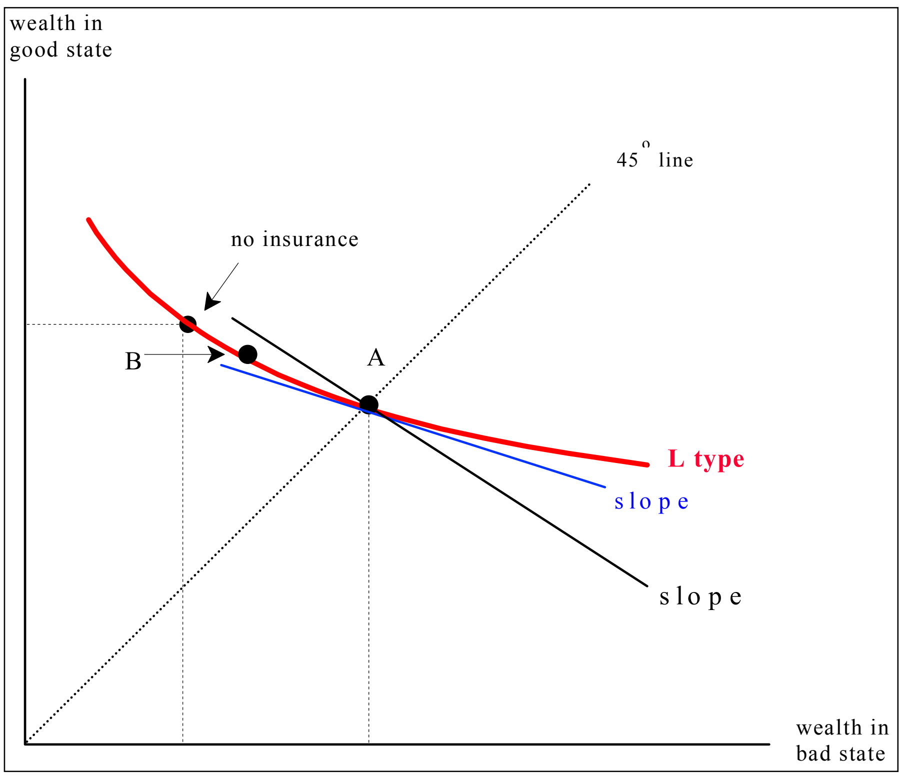


Case 1: MONOPOLY

OPTION 1. Offer only one contract, which is attractive only to the H type.

$C_1 = (\qquad , \qquad)$ Profits: $\pi_1^* =$

OPTION 2. Offer only one contract, which is attractive to both types. **Not optimal to offer full insurance**



Best contract under Option 2:

$\pi_2^* =$

OPTION 3: Offer two contracts,

$C_H = (h_H, d_H)$, targeted to the H type

$C_L = (h_L, d_L)$ targeted to the L type.

expected utility for L-type from C_L: $EU_L[C_L] =$

expected utility for L-type from C_H: $EU_L[C_H] =$

expected utility for H-type from C_L: $EU_H[C_L] =$

expected utility for H-type from C_H: $EU_H[C_H] =$

expected utility for L-type from NI : $EU_L[NI] =$

expected utility for L-type from NI : $EU_H[NI] =$

Monopolist's problem is to

$$\underset{h_H, d_H, h_L, d_L}{Max} \pi_3 = q_H N\left[h_H - p_H(L - d_H)\right] + (1 - q_H)N\left[h_L - p_L(L - d_L)\right]$$

subject to

(IR_L)

(IC_L)

(IR_H)

(IC_H)

(IR_H) follows from (IR_L) and (IC_H)

Thus, the problem can be reduced to

$$\underset{h_H, d_H, h_L, d_L}{Max} \ \pi_3 = q_H N\left[h_H - p_H(L - d_H)\right] + (1 - q_H)N\left[h_L - p_L(L - d_L)\right]$$

subject to

$(IR_L)\quad EU_L[C_L] \geq EU_L[NI]$

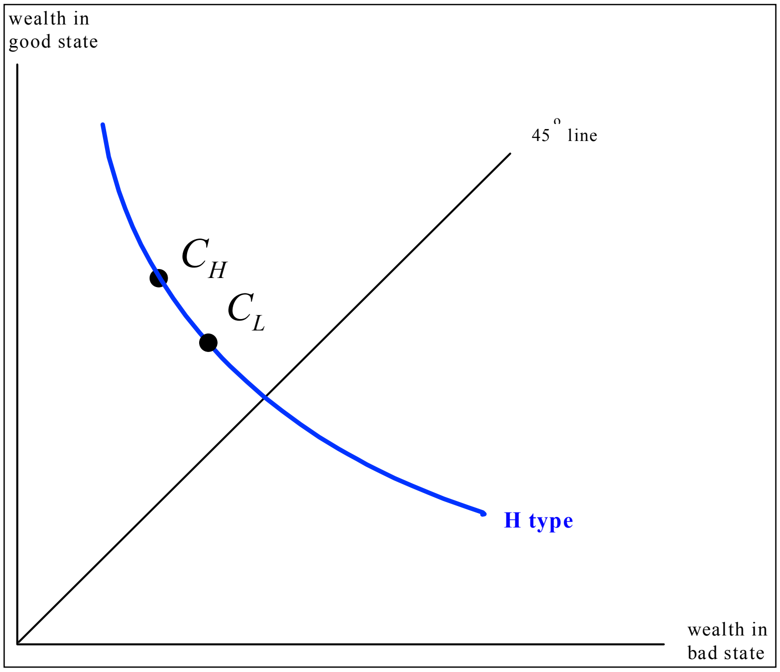
$(IC_L)\quad EU_L[C_L] \geq EU_L[C_H]$

$(IC_H)\quad EU_H[C_H] \geq EU_H[C_L]$

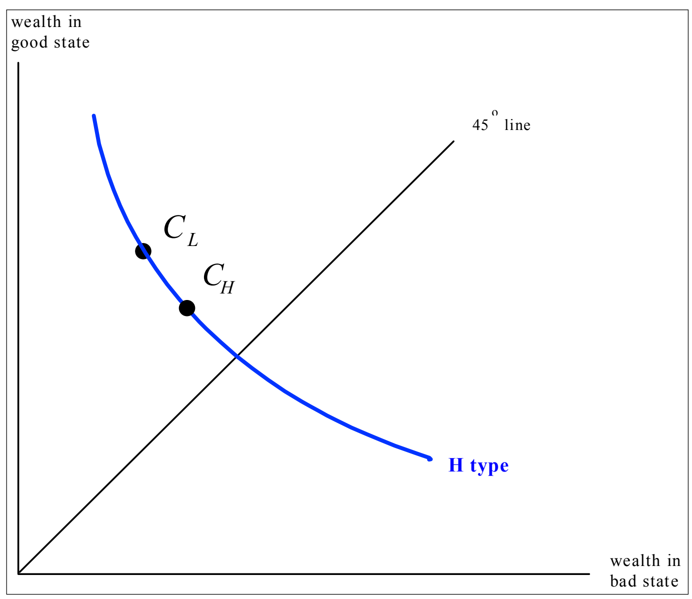
(IC_H) must be satisfied as an equality.

So C_H and C_L be on the same indifference curve for the H type.

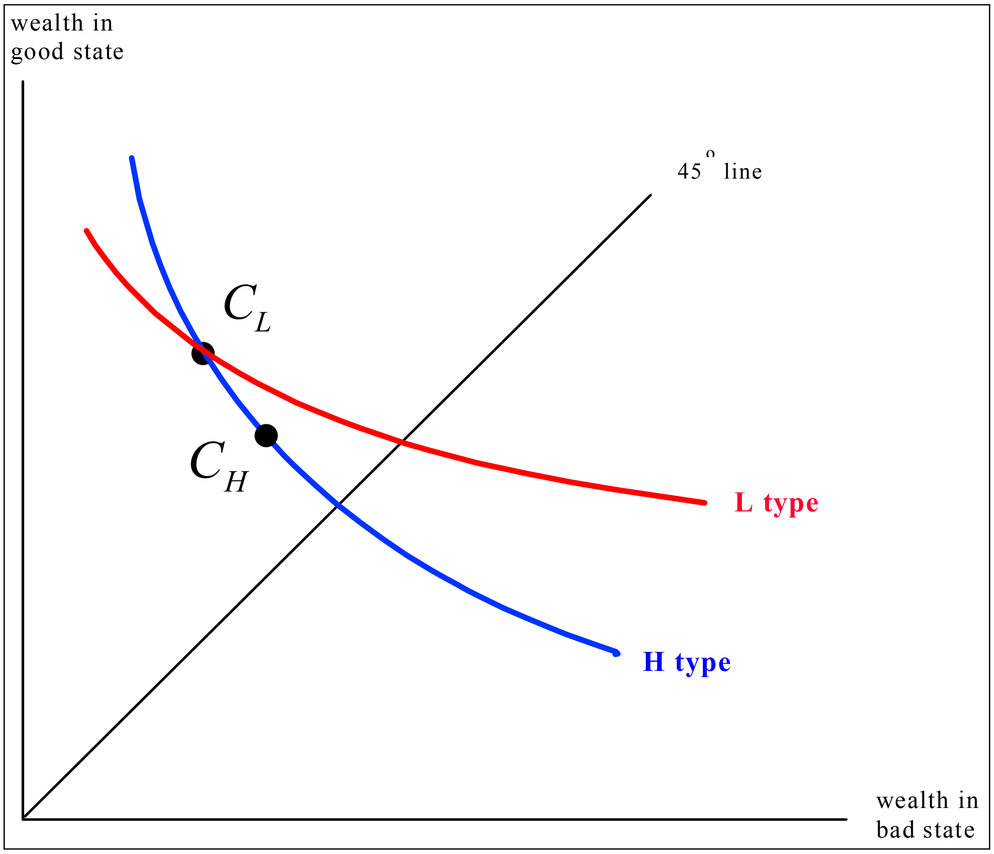
On this indifference curve, contract C_H cannot be above contract C_L



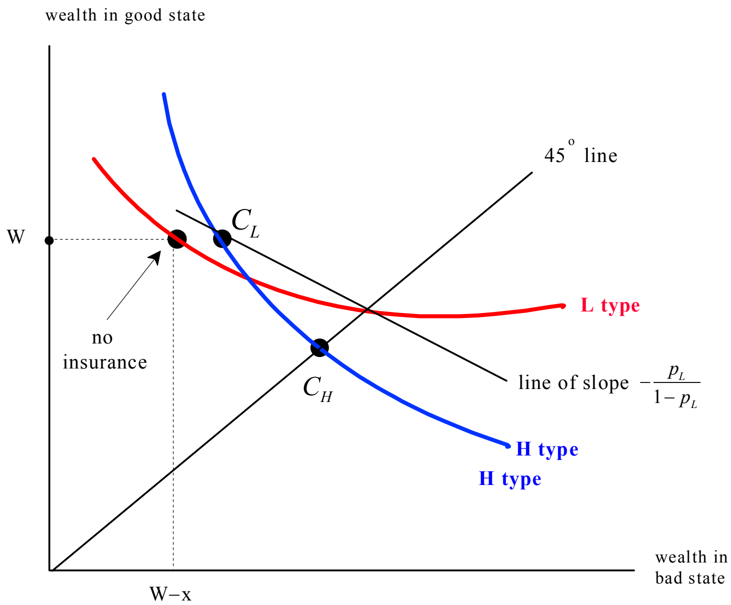
So it must be:

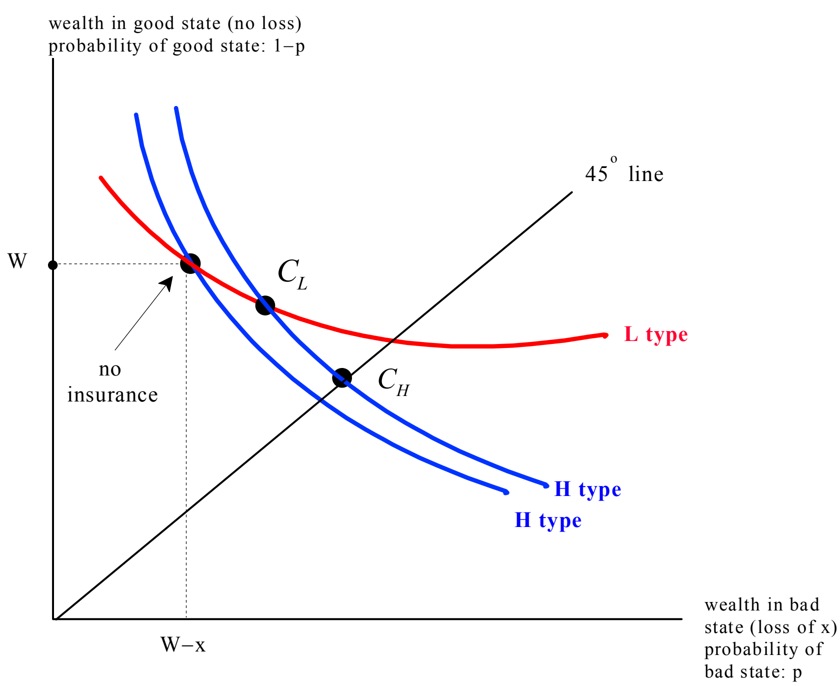


C_H must be a full insurance contract



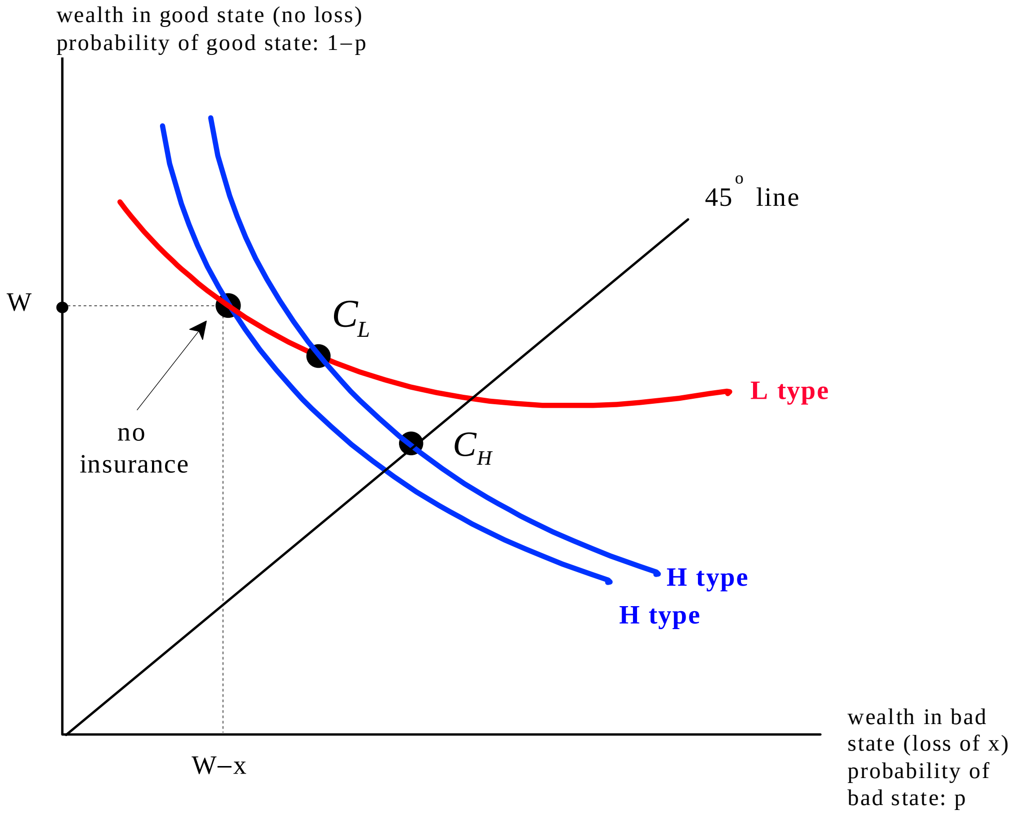
(IR_L) must be satisfied as an equality.



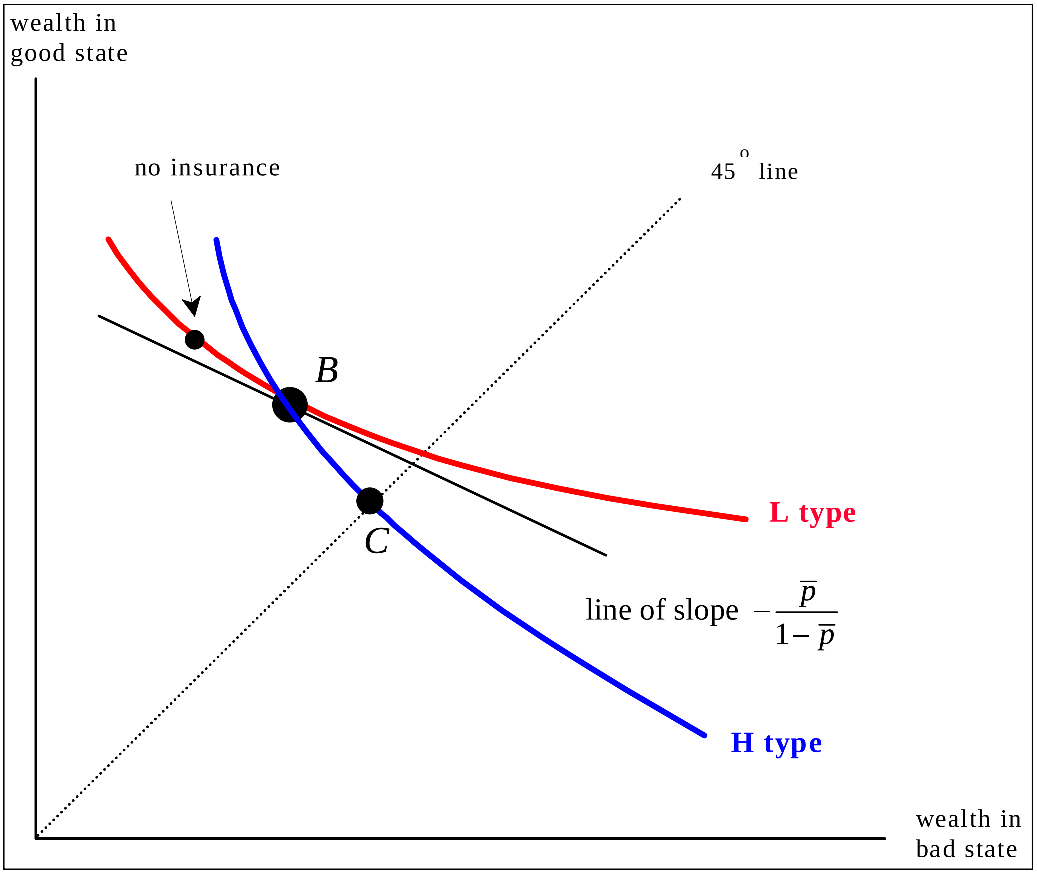


(IC_L) **is not binding: it is always satisfied as a strict inequality**.

Option 1 is a special case of Option 3



Option 3 yields higher profits than Option 2: $\pi_2^* < \pi_3^*$



In conclusion, the monopolist will always choose Option 3, although in some cases (namely when q_H is close to 1) **the outcome is the same as in Option 1.**

EXAMPLE. $W = 1{,}600$, $x = 700$, $p_H = \frac{1}{5}$, $p_L = \frac{1}{10}$, $U(m) = \sqrt{m}$.

h_H^* is given by the solution to

Thus under $\boxed{\textbf{Option 1}}$ profits are:

Now **Option 3**. Let $h_H \in [79,\ 156]$ be the premium for the full-insurance contract targeted to the *H* type To find c_L solve:

We can solve the two equations in terms of h_H:

$$h_L(h_H) = h_H + 156\sqrt{1,600 - h_H} - 6,084$$
$$d_L(h_H) = 80h_H + 5,460\sqrt{1,600 - h_H} - 219,260$$

Then the monopolist will choose h_H to maximize

$$\pi_3 =$$

This function is strictly concave and $\left.\frac{d\pi_3}{dh_H}\right|_{h_h=79} = q_H N > 0$ and

$\left.\frac{d\pi_3}{dh_H}\right|_{h_h=156} = \frac{47}{38}q_H - \frac{9}{38}$. This is negative if and only if $q_H < \frac{9}{47}$. Thus,

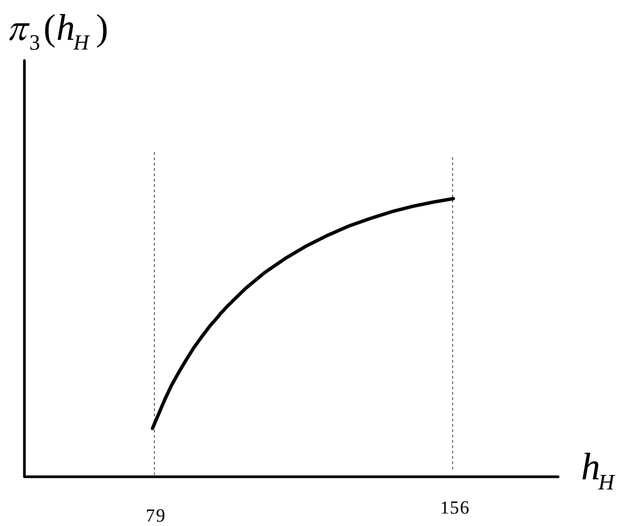


Figure 23a

$\boxed{q_H \geq \frac{9}{47}}$: the optimal solution is to offer only the full insurance contract at premium $h_H^* = 156$ (Option 1)

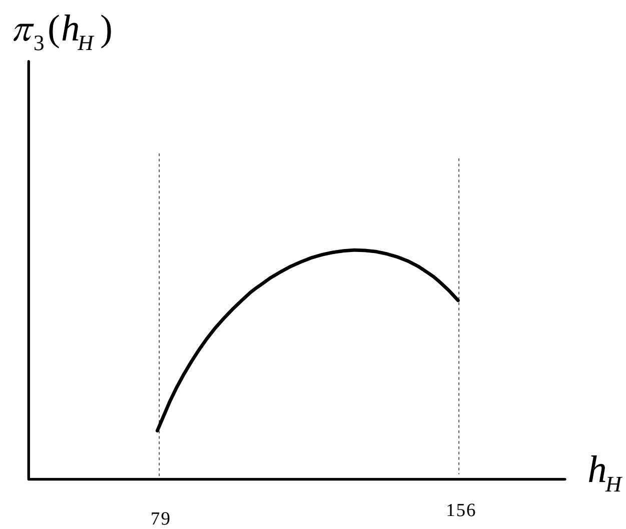


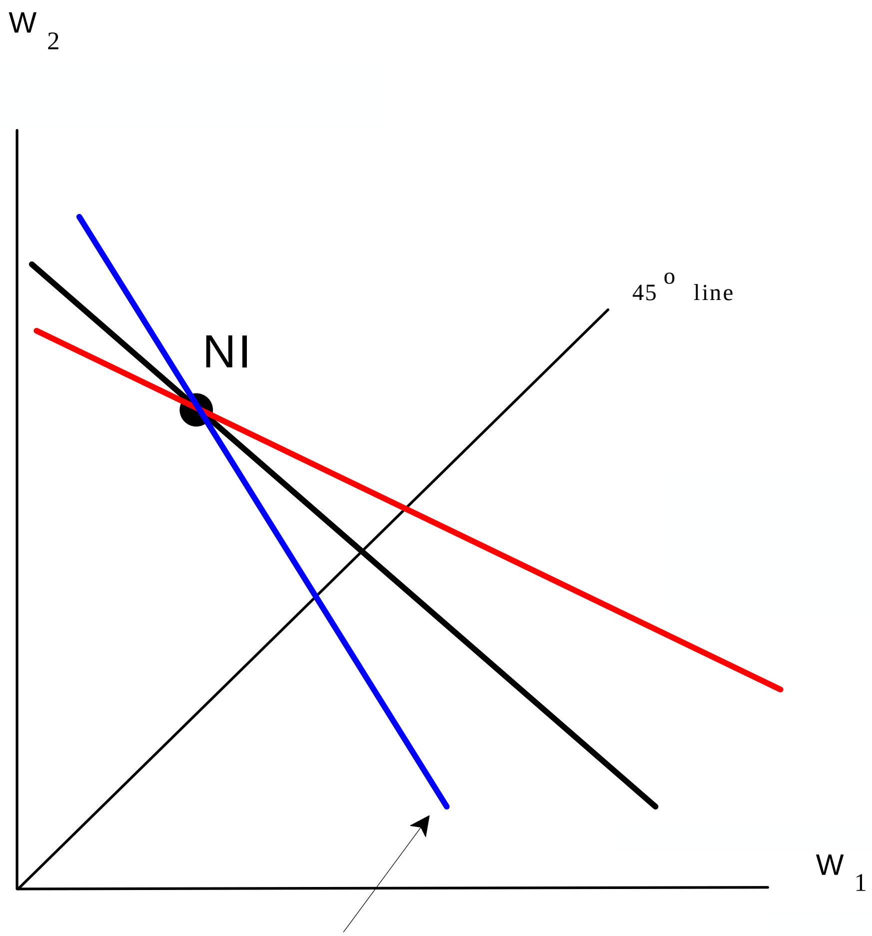
Figure 23b

$\boxed{q_H < \frac{9}{47}}$: the optimal solution is to offer two contracts (Option 3)

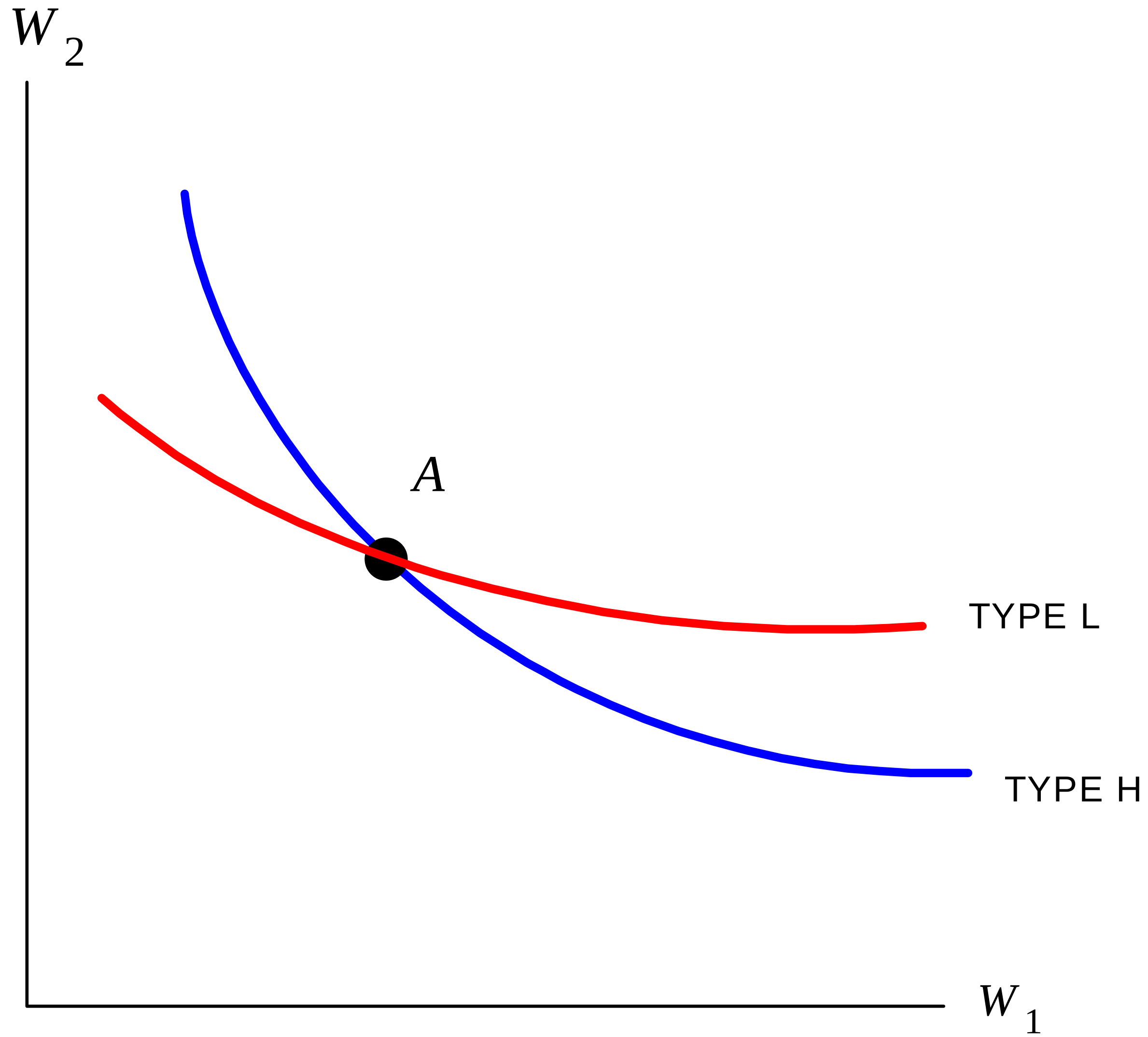
COMPETITIVE INDUSTRY with free entry

Equilibrium: (1) every firm makes zero profits and (2) no firm could make positive profits by introducing a new contract.

Three zero-profit lines:

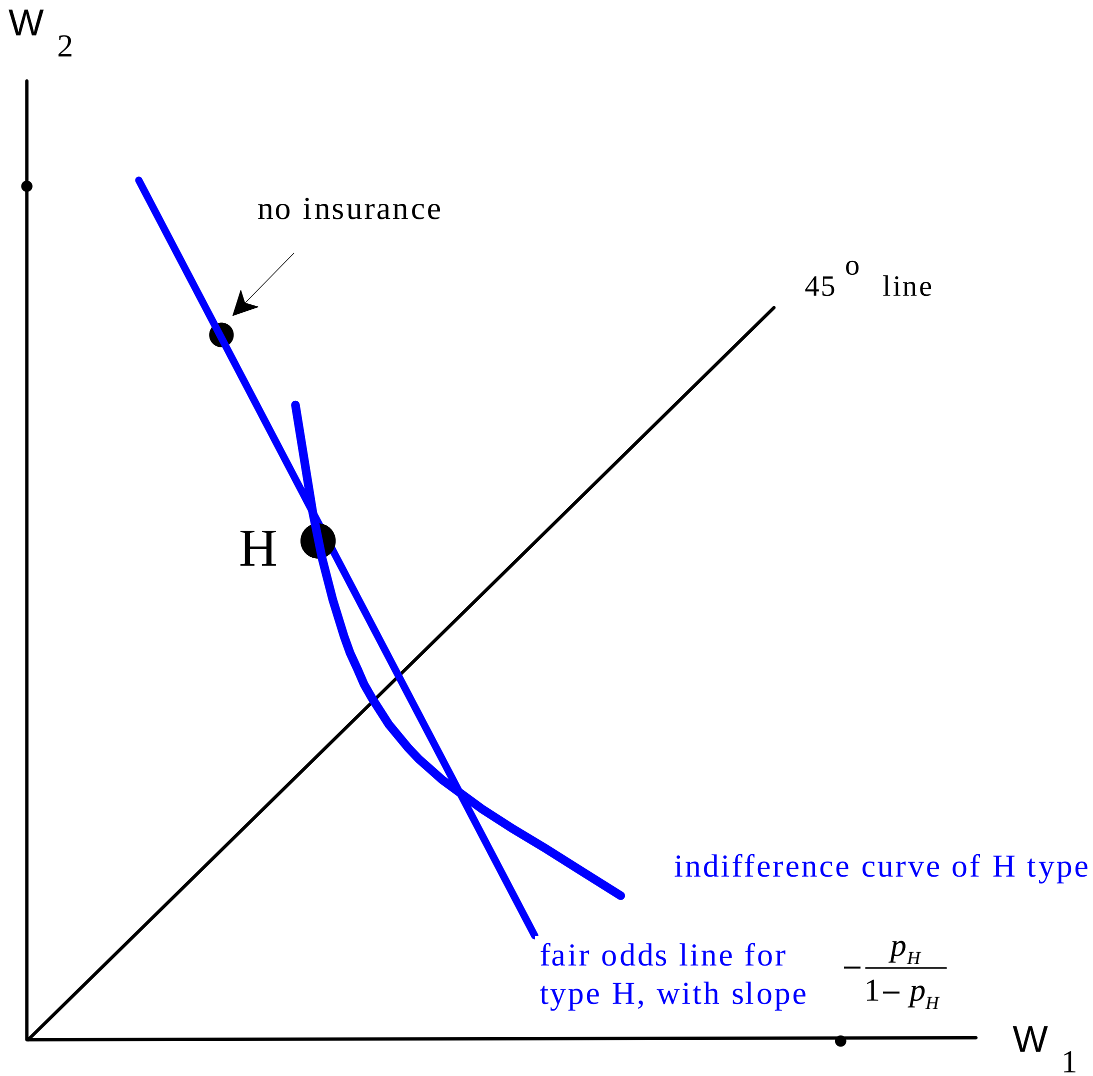


Remark 1: there cannot be a single-contract equilibrium serving both types.

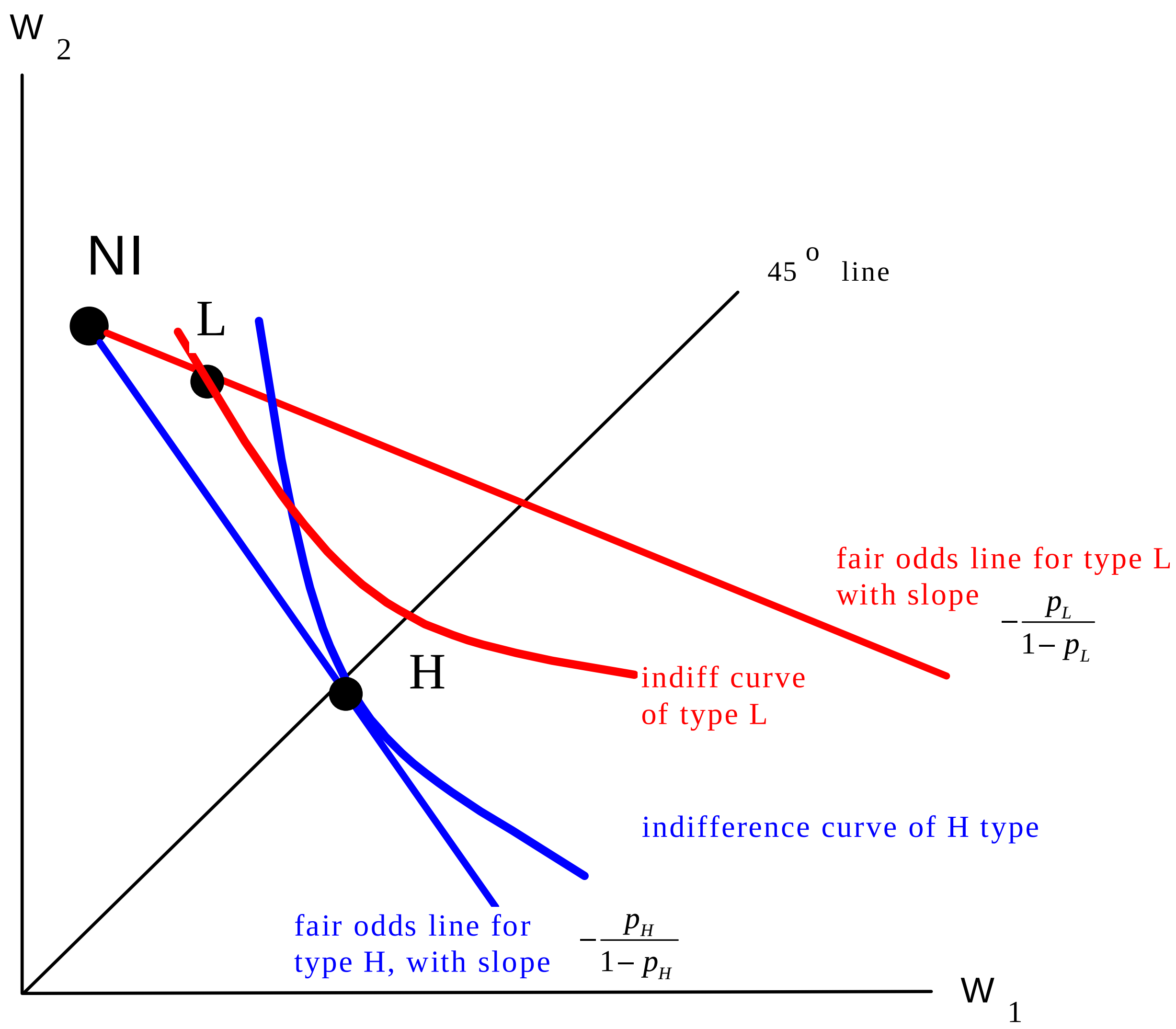

W_2
A
TYPE L
TYPE H
W_1

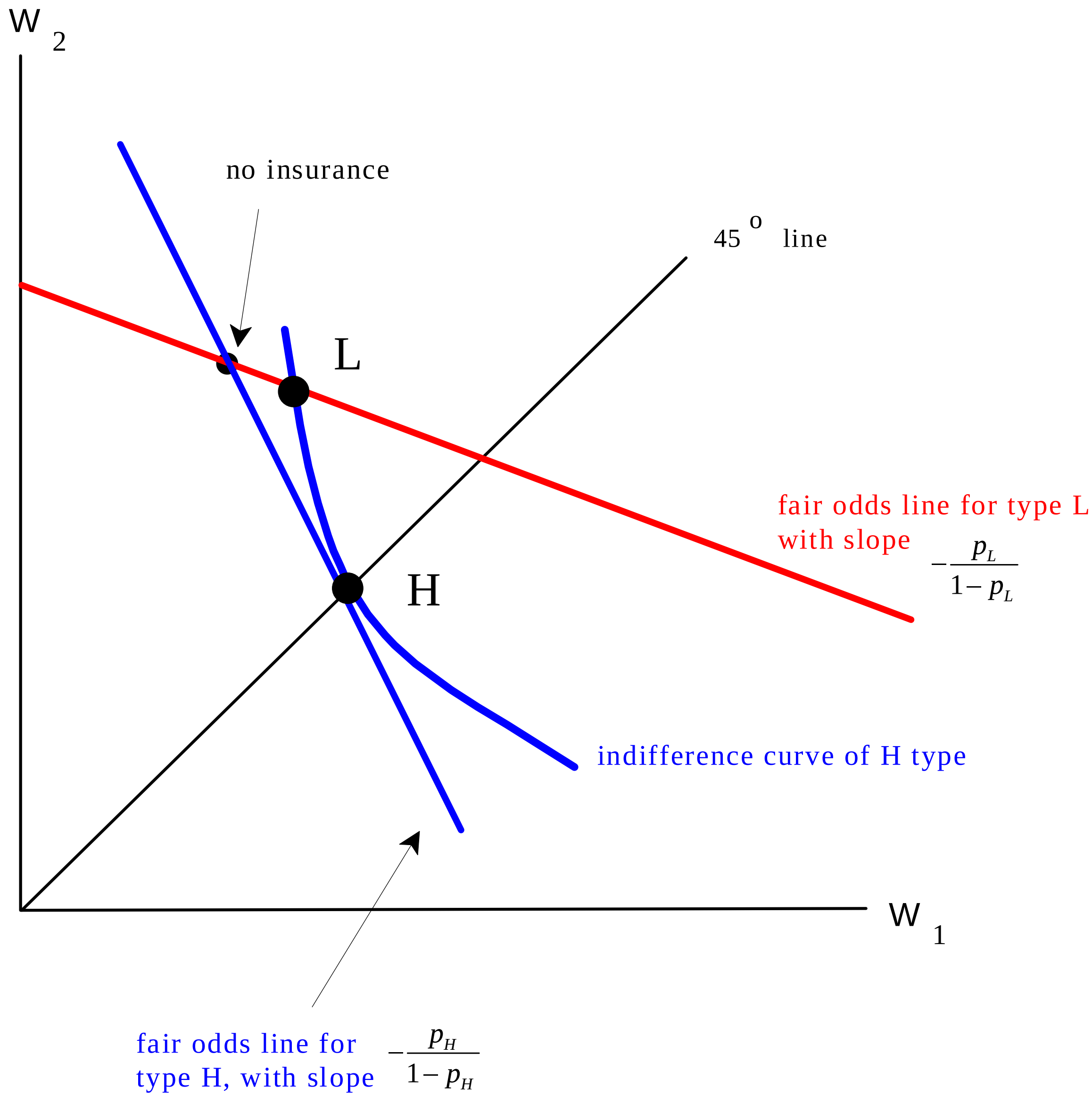
If there is a zero-profit equilibrium it must be an equilibrium with two contracts: the *L* types buy one and the *H* types buy the other

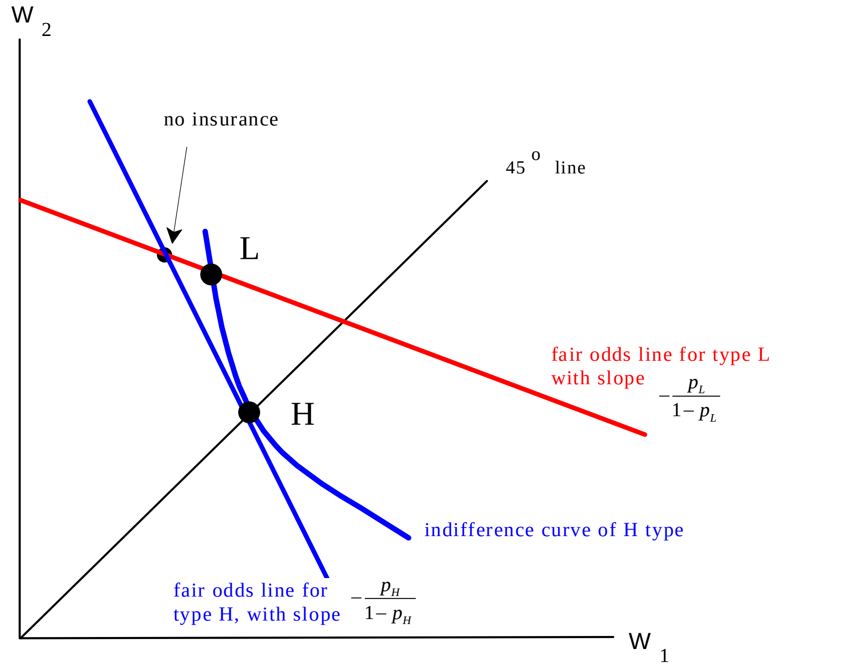
The contract bought by the H types must be a full-insurance contract:



What about the L contract?




W 2
no insurance
45 o line
L
H
fair odds line for type L
with slope $-\frac{p_L}{1-p_L}$
indifference curve of H type
W 1
fair odds line for
type H, with slope $-\frac{p_H}{1-p_H}$

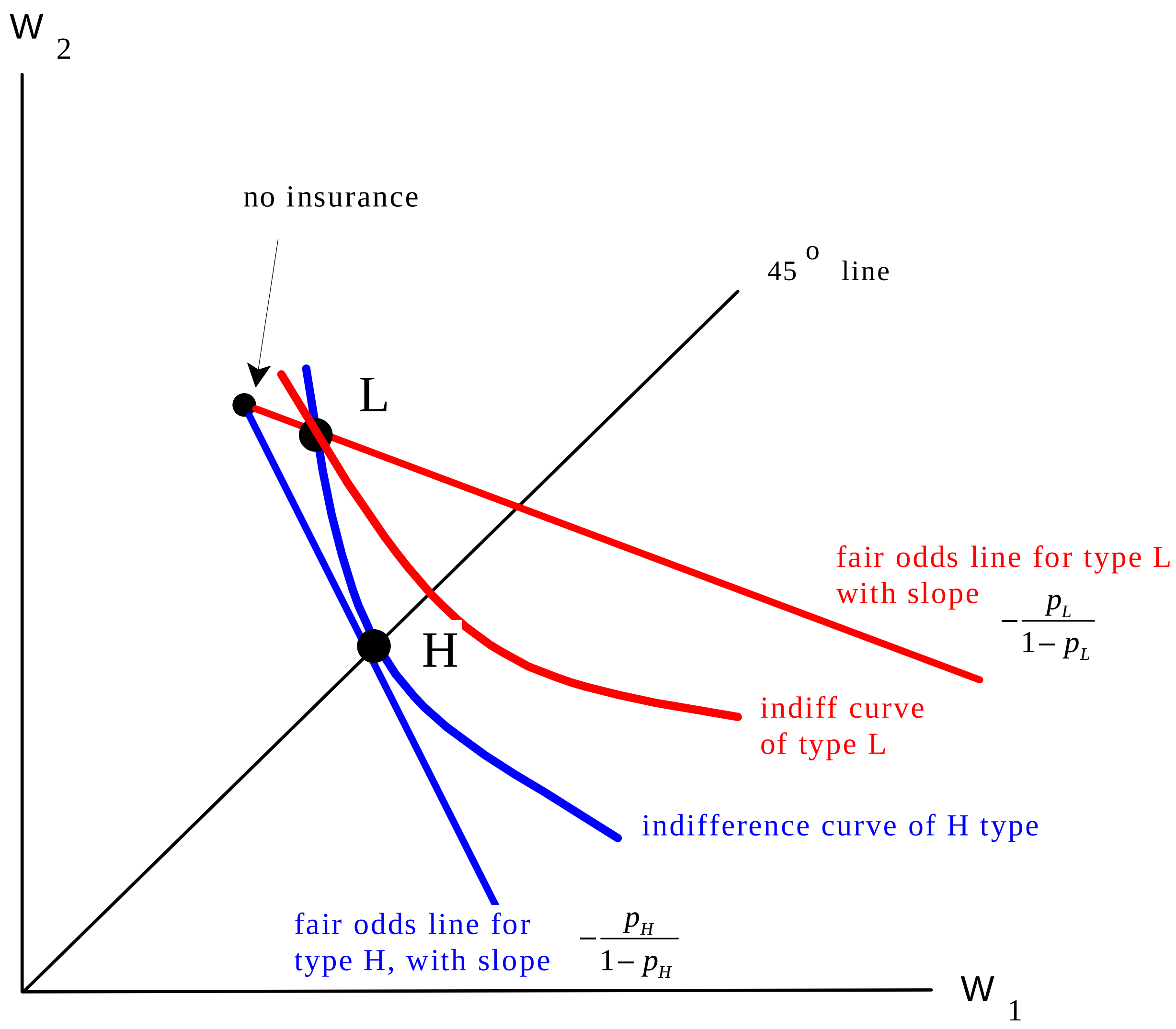


Example:

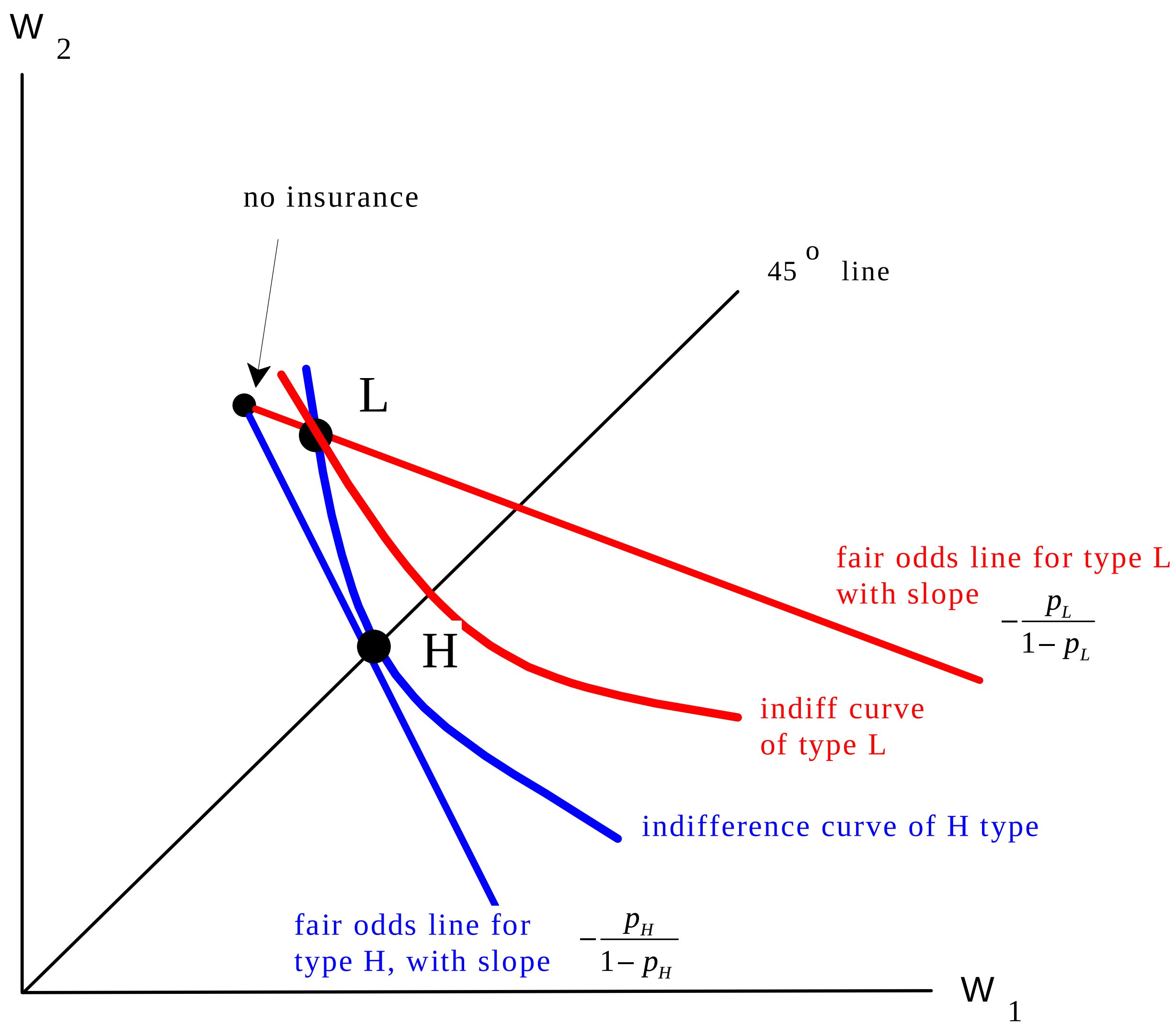
$$W = 625,\ loss = 225,$$

$$U(\$m) = \sqrt{m},$$

$$p_H = \frac{1}{10},\ \ p_L = \frac{1}{20},$$



W_2
no insurance
45^o line
L
H
fair odds line for type L
with slope $-\frac{p_L}{1-p_L}$
indiff curve
of type L
indifference curve of H type
fair odds line for
type H, with slope $-\frac{p_H}{1-p_H}$
W_1


W 2
no insurance
45 o line
L
H
fair odds line for type L
with slope
$-\frac{p_L}{1-p_L}$
indiff curve
of type L
indifference curve of H type
fair odds line for
type H, with slope
$-\frac{p_H}{1-p_H}$
W 1