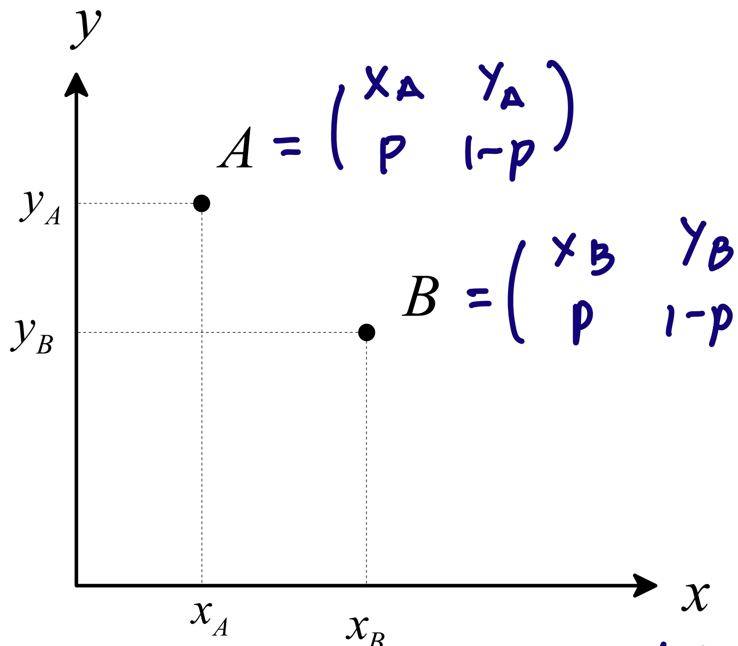
$W_0 - (W_0 - W_1)$

$1-p$

# BINARY LOTTERIES

Lotteries of the form $\begin{pmatrix} \$x & \$y \\ p & 1-p \end{pmatrix}$ with $p$ fixed and $x$ and $y$ allowed to vary.

$(1-p)$




$(p)$

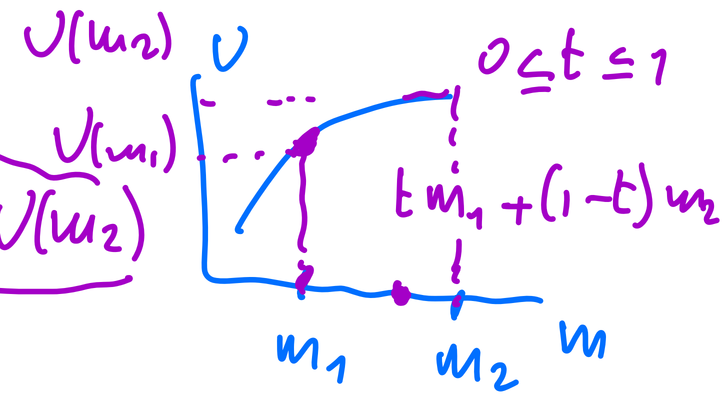
$U(\$m)$

$E[U(A)] = E[U(B)]$

draw indifference curve through A

$E[U(L)]$

Risk averse if



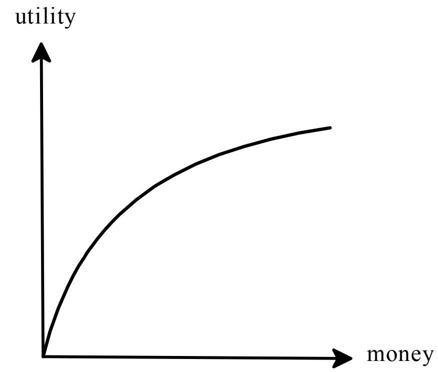

We want to draw indifference curves in this diagram.

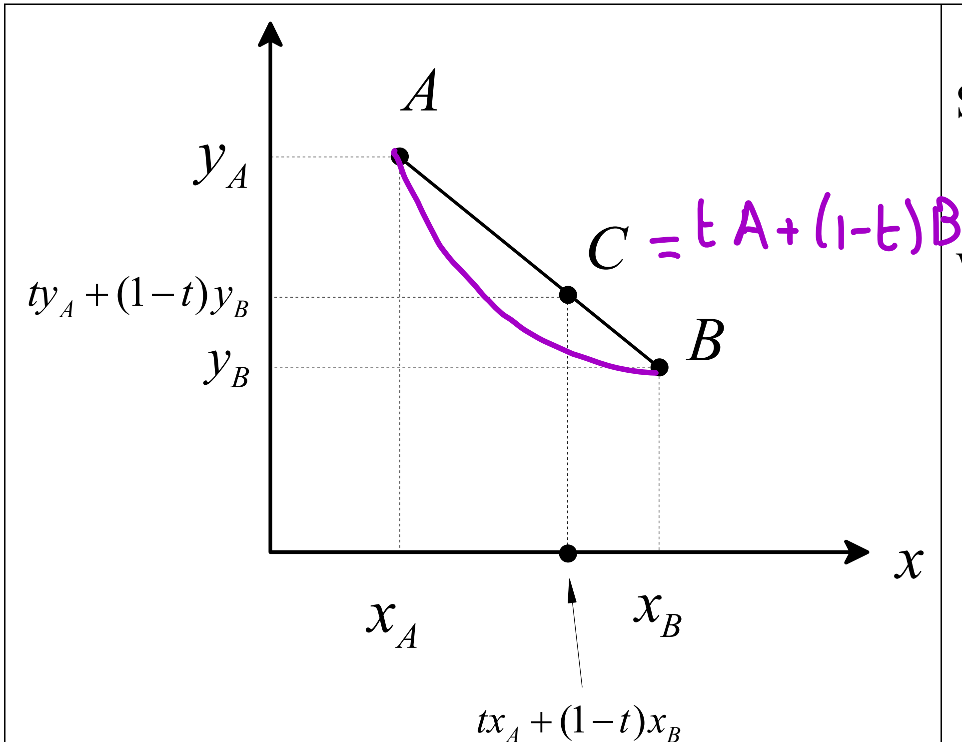
$$U(\underbrace{tm_1 + (1-t)m_2}) > \underbrace{tU(m_1) + (1-t)U(m_2)}$$

$E[L] \quad L = \begin{pmatrix} m_1 & m_2 \\ t & 1-t \end{pmatrix}$

# Case 2: risk-averse agent

$U(m)$ is strictly concave:

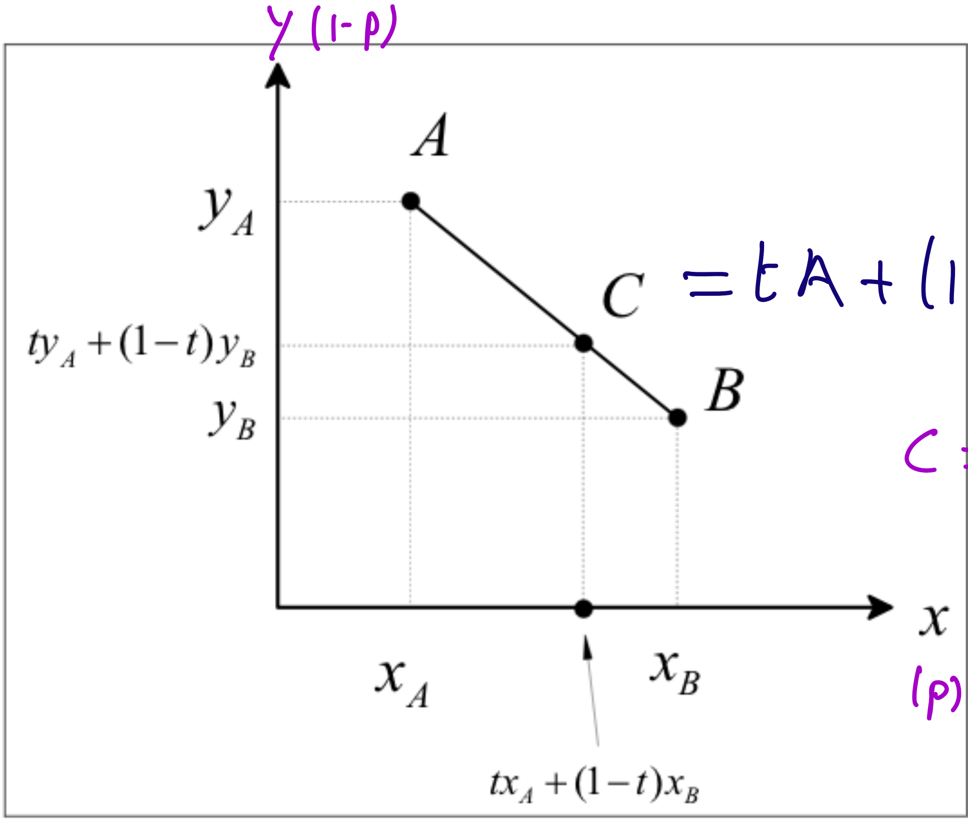



Suppose that $\mathbb{E}[U(A)] = \mathbb{E}[U(B)]$

Want to show that
$\mathbb{E}[U(C)] > \mathbb{E}[U(A)] (= \mathbb{E}[U(B)])$

$C = \left(tx_A + (1-t)x_B,\ ty_A + (1-t)y_B\right)$




$$C = tA + (1-t)B \qquad 0<t<1$$

$$C = \begin{pmatrix} tx_A+(1-t)x_B & ty_A+(1-t)y_B \\ p & 1-p \end{pmatrix}$$

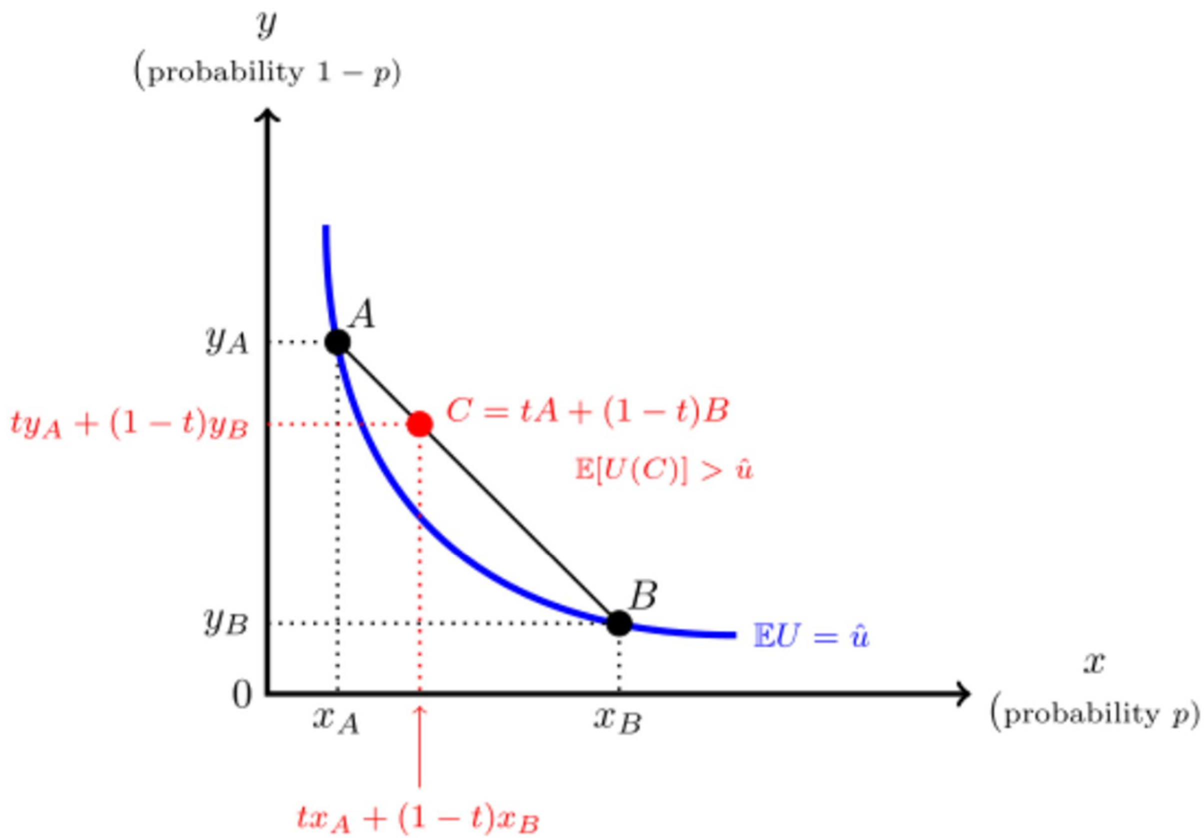
$$E[U(C)] = p\,\underbrace{U(tx_A+(1-t)x_B)}_{\geq\ tU(x_A)+(1-t)U(x_B)} + (1-p)\,\underbrace{U(ty_A+(1-t)y_B)}_{\geq\ tU(y_A)+(1-t)U(y_B)}$$

by concavity of U

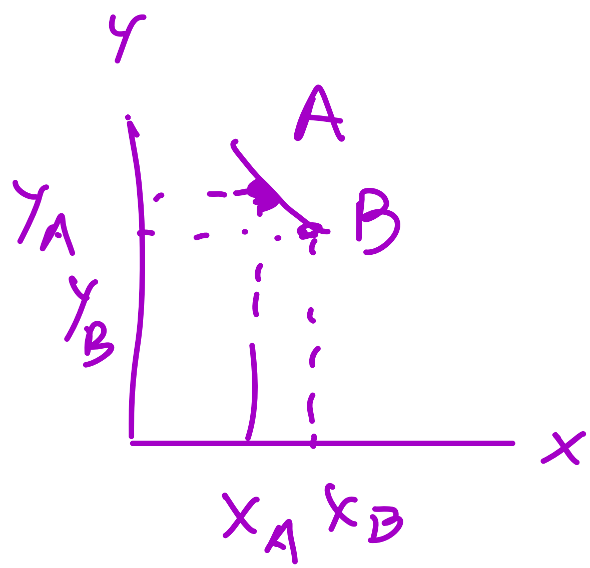
$$> p[tU(x_A)+(1-t)U(x_B)] + (1-p)[tU(y_A)+(1-t)U(y_B)]$$

$$= \underbrace{ptU(x_A)+(1-p)tU(y_A)}_{t\,EU(A)} + \underbrace{p(1-t)U(x_B)+(1-p)(1-t)U(y_B)}_{+(1-t)\,EU(B)}$$

$$= t\,EU(A) + (1-t)\,EU(A) = EU(A)$$



$y$
(probability $1-p$)
$y_A$
$ty_A + (1-t)y_B$
$y_B$
0
$A$
$C = tA + (1-t)B$
$\mathbb{E}[U(C)] > \hat{u}$
$B$
$\mathbb{E}U = \hat{u}$
$x$
(probability $p$)
$x_A$
$x_B$
$tx_A + (1-t)x_B$

y
A
$y_A$
B
$y_B$
x
$x_A$
$x_B$

# Slope of indifference curve

Let $A$ and $B$ be two points that lie on the same indifference curve: $\mathbb{E}[U(A)] = \mathbb{E}[U(B)]$, (*)

- Since $x_B$ is close to $x_A$, $U(x_B) \simeq U(x_A) + U'(x_A)(x_B - x_A)$
- Since $y_B$ is close to $y_A$, $U(y_B) \simeq U(y_A) + U'(y_A)(y_B - y_A)$

Thus the RHS of (*) can be written as

$$\cancel{pU(x_A) + (1-p)U(y_A)} = pU(x_B) + (1-p)U(y_B) \simeq$$

$$p\left[U(x_A) + U'(x_A)(x_B - x_A)\right] +$$

So (*) becomes

$$(1-p)\left[U(y_A) + U'(y_A)(y_B - y_A)\right]$$

that is,

$$= pU(x_A) + (1-p)U(y_A) +$$ which can be written as

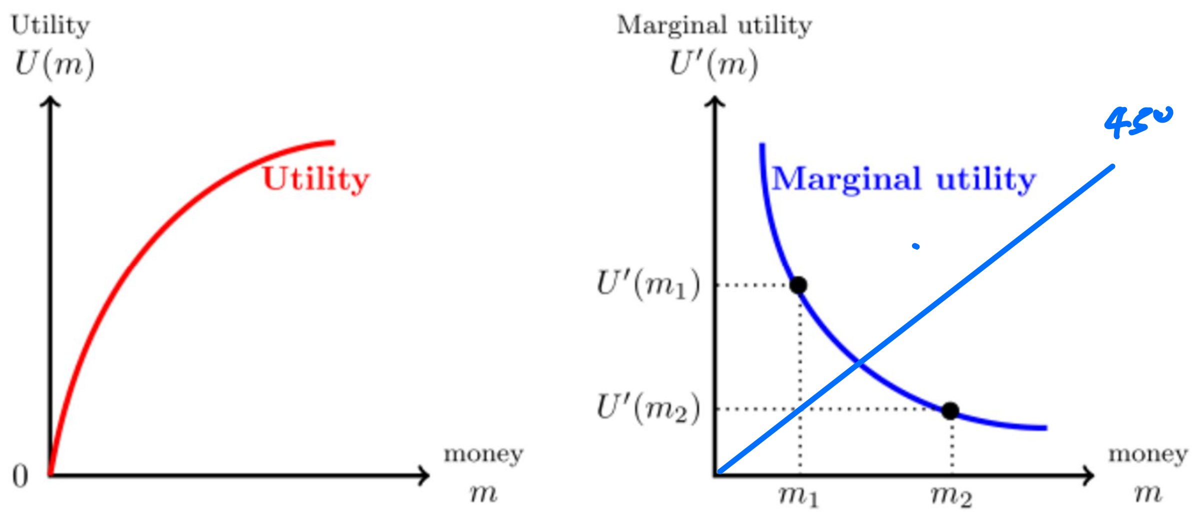
$$pU'(x_A)(x_B - x_A) + (1-p)U'(y_A)(y_B - y_A) = 0$$

$$\frac{\overbrace{y_B - y_A}^{\text{rise}}}{\underbrace{x_B - x_A}_{\text{run}}} = \underbrace{-\frac{p}{1-p}\,\frac{U'(x_A)}{U'(y_A)}}$$

Slope of ind. curve through A at A

## Comparing the slope at a point with the ratio $\dfrac{p}{1-p}$

Look at the case of risk aversion but the other cases are similar.




$m_2 > m_1$

$U(m_2) > U(m_1)$

$U'(m_2) < U'(m_1)$

- at a point **above** the 45° line, where $x < y$, $U'(x) > U'(y)$ $\quad \dfrac{U'(x)}{U'(y)} > 1$

- at a point **on** the 45° line, where $x = y$, $U'(x) = U'(y)$ $\quad \dfrac{U'(x)}{U'(y)} = 1$

- at a point **below** the 45° line, where $x > y$, $U'(x) < U'(y)$ $\quad \dfrac{U'(x)}{U'(y)} < 1$

Slope of ind. curve at point $(x, y)$ $\quad -\dfrac{p}{1-p}\dfrac{U'(x)}{U'(y)}$

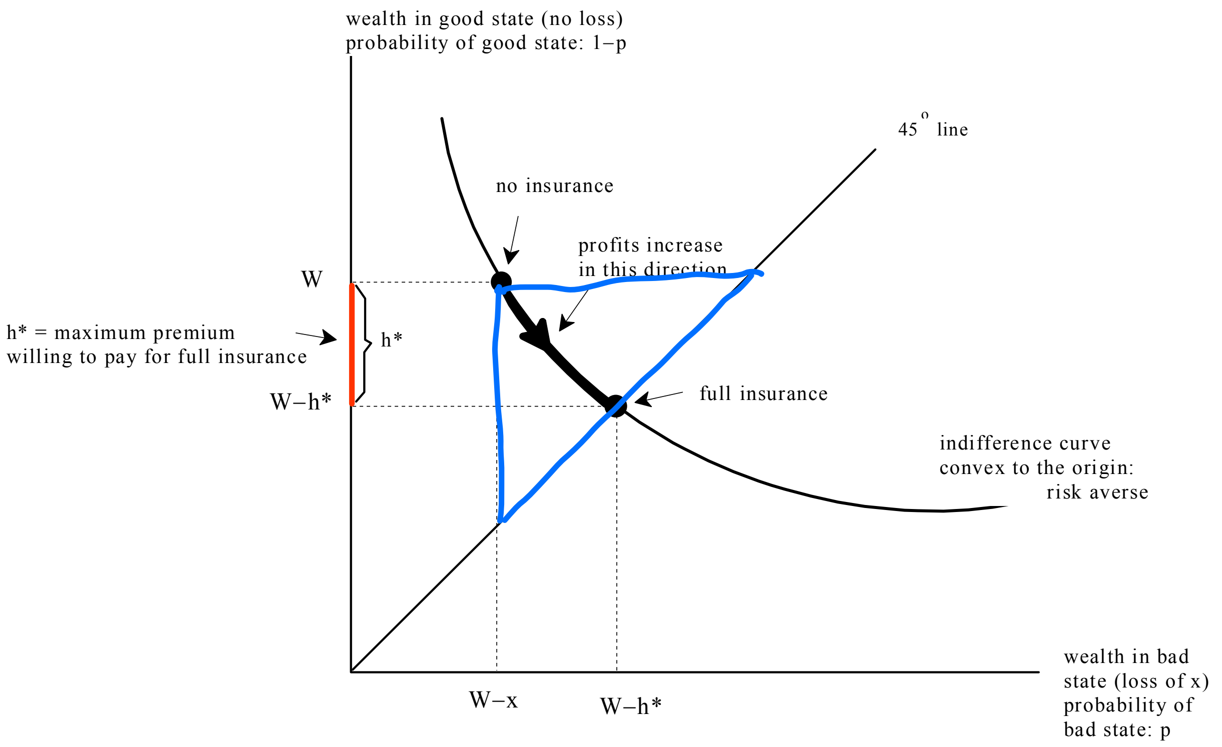
in absolute value slope of ind curve at $(x,y)$ $> \dfrac{p}{1-p}$ above 45°

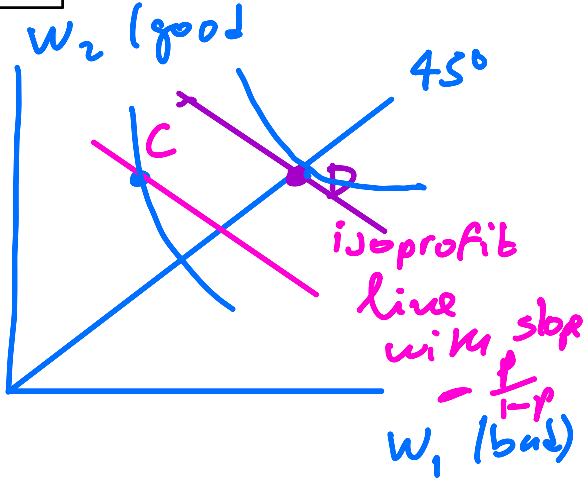
$= \dfrac{p}{1-p}$ on 45°

$1-p$

$< \frac{p}{1-p}$ below 45°

# Case 1: THE INSURANCE INDUSTRY IS A MONOPOLY

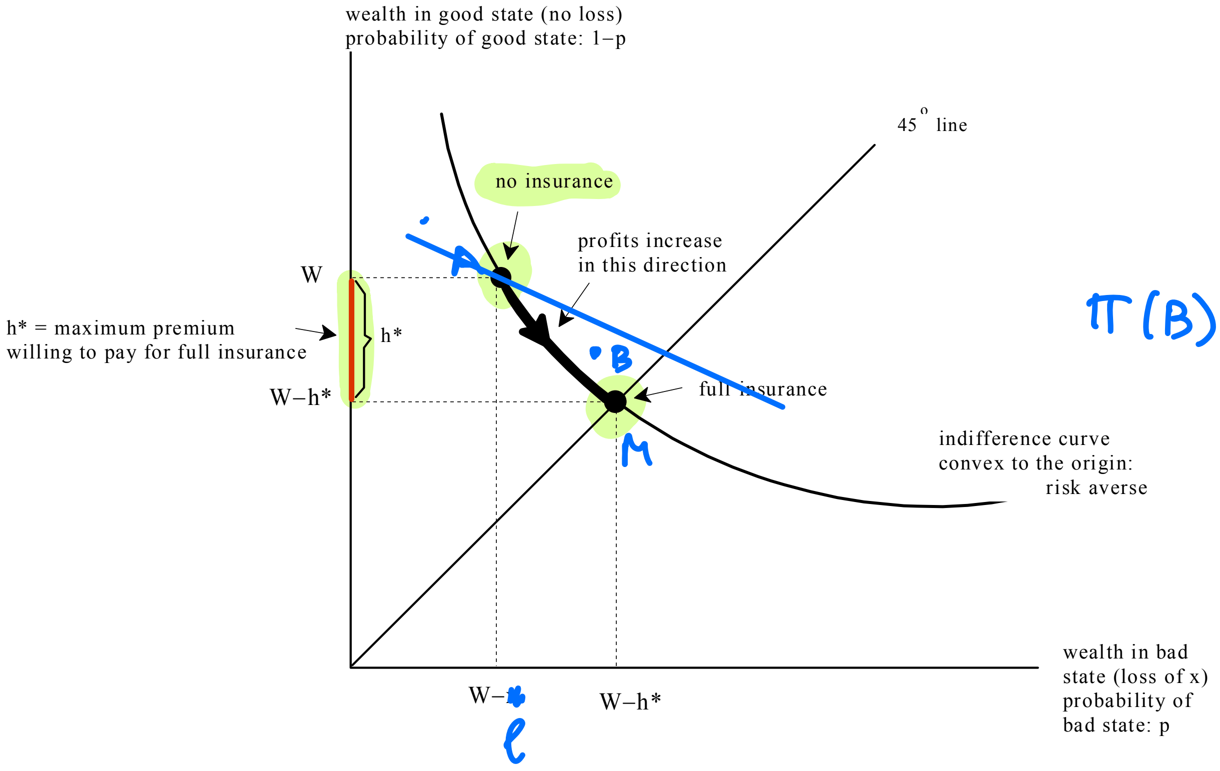


$M = (h^*, d^* = 0)$

$pU(W-\ell) + (1-p)U(W) = U(W-h^*)$

# Case 1: THE INSURANCE INDUSTRY IS A MONOPOLY




$\pi < 10$

$\pi > 10$

$\pi = 10$

$\pi(B) > \pi(A)$

$EU(B) > EU(A)$

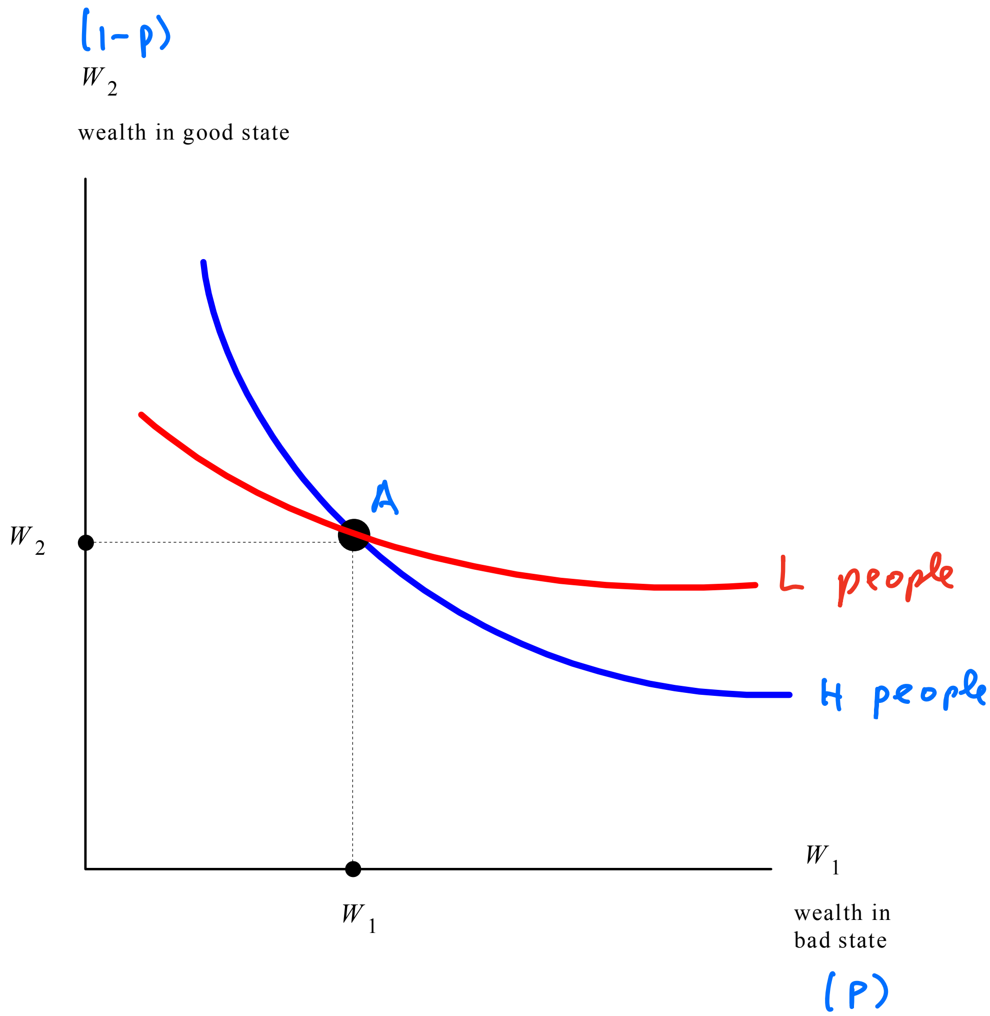
# Adverse selection in insurance markets

Two types of customers, $H$ and $L$, identical in terms of initial wealth $W$, potential loss $L$ and vNM utility-of-money function $U$, but with different probability of loss: $\boxed{p_H > p_L}$.

Slope of indifference curves at point $(w_1, w_2)$ in general : $-\dfrac{p}{1-p}\dfrac{U'(w_1)}{U'(w_2)}$

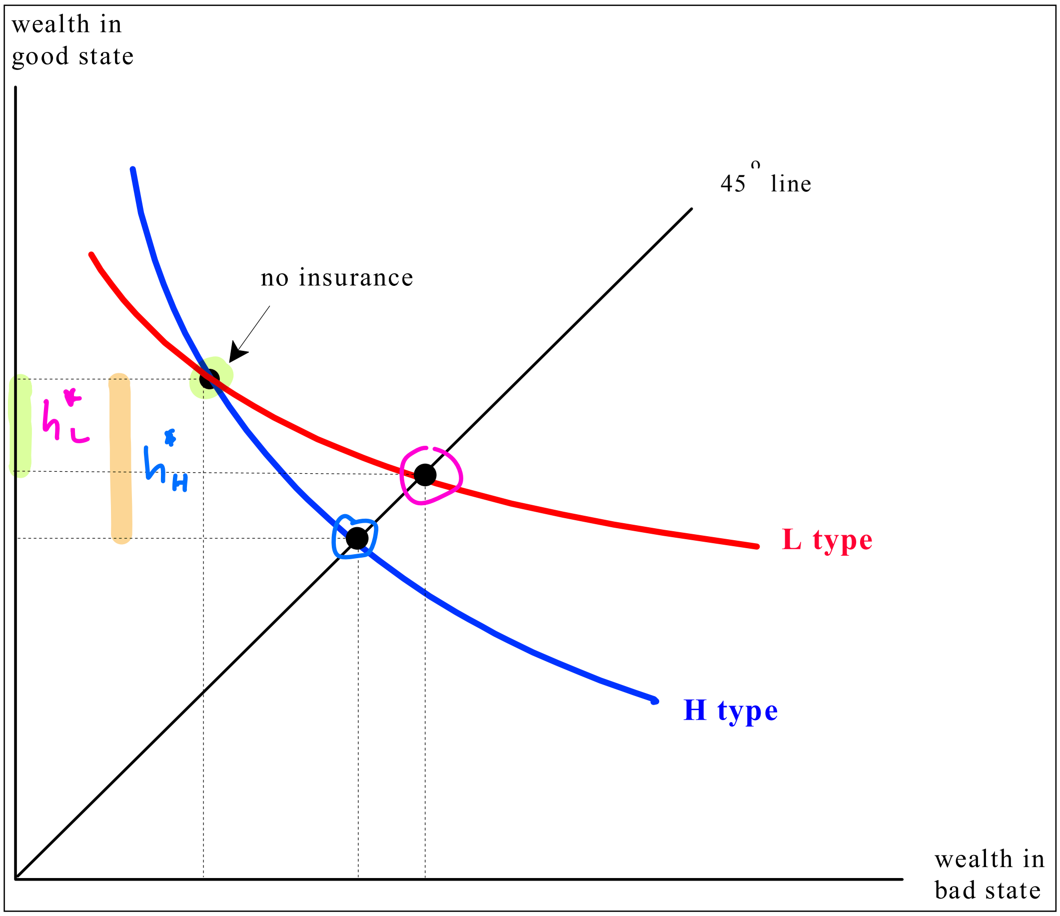
$$\frac{p_L}{1-p_L} < \frac{p_L}{1-p_H} < \frac{p_H}{1-p_H}$$

$$\frac{p_L}{1-p_L} < \frac{p_H}{1-p_H}$$



$h_H^*$ maximum premium that the $H$ people are willing to pay for full insurance

$h_L^*$ maximum premium that the $L$ people are willing to pay for full insurance:




N = total number of potential customers

Let $q_H$ be the fraction of $H$ types in the population $0 < q_H < 1$

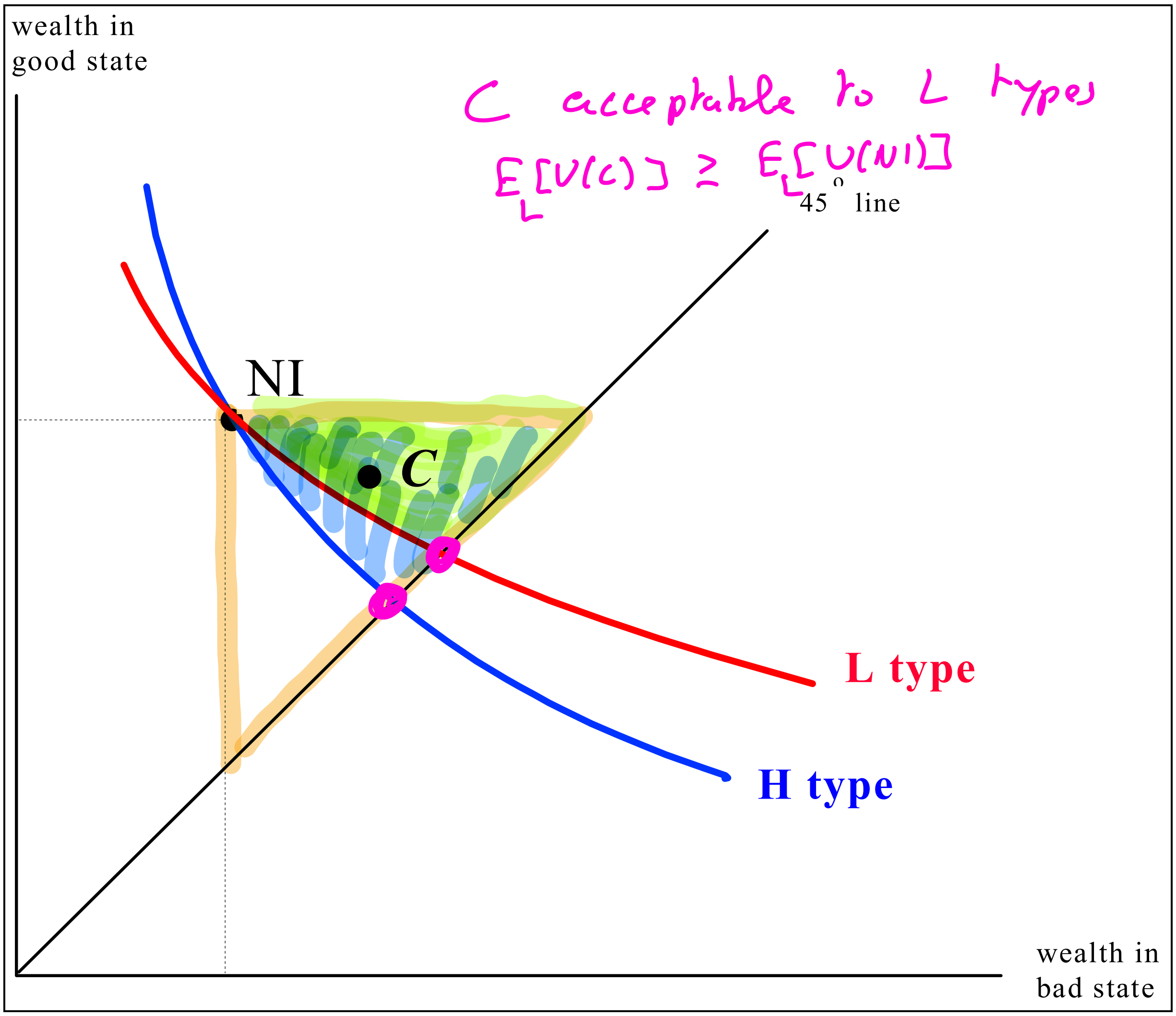
$N_H$ = number of H types = $q_H N$

$N_L$ = " L " = $(1-q_H)N$

$$\boxed{\text{If } \mathbb{E}[U_L(C)] \geq \mathbb{E}[U_L(NI)] \text{ then } \mathbb{E}[U_H(C)] \geq \mathbb{E}[U_H(NI)]}$$

$>$

think of NI as $C = (h=0,\ d=\ell)$

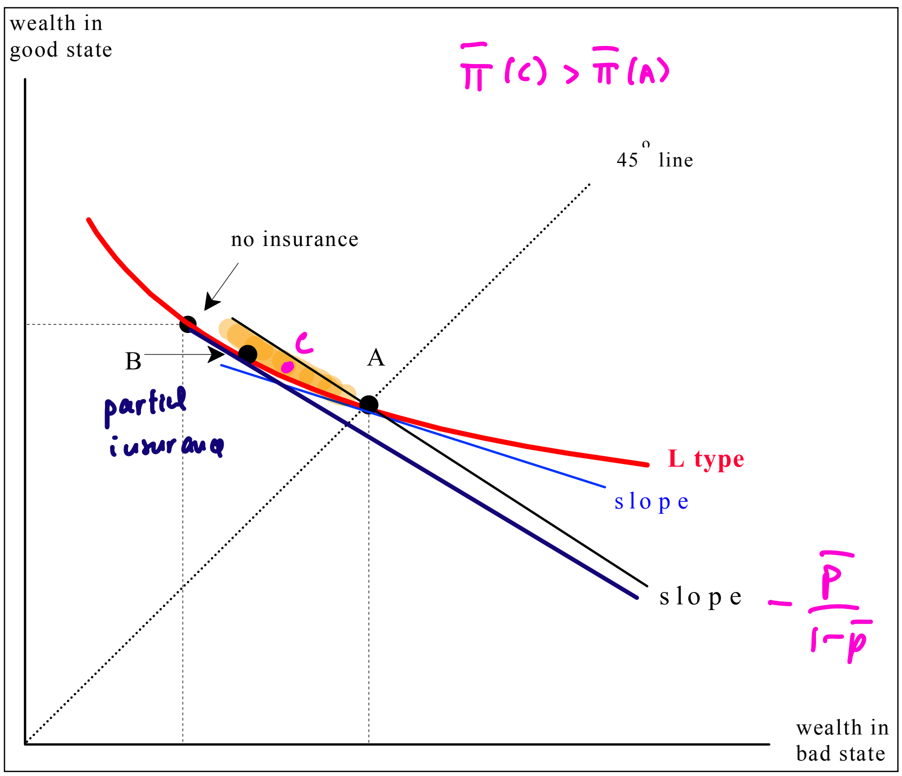


# Case 1: **MONOPOLY**

**OPTION 1.** Offer only one contract, which is attractive only to the H type.

$C_1 = ( h_H^* \ , d = 0 )$ Profits: $\pi_1^* = q_H N [ h_H^* - p_H \ell ]$

**OPTION 2.** Offer only one contract, which is attractive to both types. **Not optimal to offer full insurance**




Slope of isoprofit line

$$-\frac{\bar{p}}{1-\bar{p}}$$

$$\bar{p} = q_H p_H + (1-q_H) p_L$$

$$\underbrace{p_L < \bar{p}}\ < p_H$$

$$\frac{p_L}{1-p_L} < \frac{\bar{p}}{1-\bar{p}}$$

slope of red ind. curve at any point on 45°

Best contract under Option 2:

$\pi_2^* = N [ h_B - \bar{p} (\ell - d_B) ]$

**OPTION 3:** Offer two contracts,

$C_H = (h_H, d_H)$, targeted to the $H$ type

$C_L = (h_L, d_L)$ targeted to the $L$ type.

expected utility for L-type from $C_L$: $EU_L[C_L] =$ $p_L U(W - h_L - d_L) + (1 - p_L) U(W - h_L)$

expected utility for L-type from $C_H$: $EU_L[C_H] =$ $p_L U(W - h_H - d_H) + (1 - p_L) U(W - h_H)$

expected utility for H-type from $C_L$: $EU_H[C_L] =$ $p_H U(W - h_L - d_L) + (1 - p_H) U(W - h_L)$

expected utility for H-type from $C_H$: $EU_H[C_H] =$ $p_H U(W - h_H - d_H) + (1 - p_H) U(W - h_H)$

expected utility for L-type from $NI$: $EU_L[NI] =$ $p_L U(W - \ell) + (1 - p_L) U(W)$

expected utility for L-type from $NI$: $EU_H[NI] =$ $p_H U(W - \ell) + (1 - p_H) U(W)$

IR = individual rationality
IC = incentive compatibility

Monopolist's problem is to

$$\underset{h_H, d_H, h_L, d_L}{Max}\ \pi_3 = q_H N\left[h_H - p_H(\ell - d_H)\right] + (1-q_H)N\left[h_L - p_L(\ell - d_L)\right]$$

subject to

$(IR_L)$ $EU_L[C_L] \geq EU_L(NI)$ $C_L$ is acceptable to L $\Rightarrow$ $C_L$ is acceptable to H

$(IC_L)$ $EU_L[C_L] \geq EU_L[C_H]$

$(IR_H)$ $EU_H[C_H] \geq EU_H[NI]$ $EU_H[C_L] \geq EU_H[NI]$

$(IC_H)$ $EU_H[C_H] \geq EU_H[C_L]$

$(IR_H)$ follows from $(IR_L)$ and $(IC_H)$

$\Pi_3$ is increasing in $h_H$

Thus, the problem can be reduced to

$$\underset{h_H, d_H, h_L, d_L}{Max} \; \pi_3 = q_H N\left[h_H - p_H(L-d_H)\right] + (1-q_H)N\left[h_L - p_L(L-d_L)\right]$$

subject to

=

$(IR_L) \quad EU_L[C_L] \geq EU_L[NI]$ independent of $h_H$

$(IC_L) \quad EU_L[C_L] \geq EU_L[C_H]$ decreases with $h_H$

$(IC_H) \quad EU_H[C_H] \geq EU_H[C_L]$

independent of $h_H$

Suppose $C_H$ and $C_L$ are such that $EU_H(C_H) > EU_H(C_L)$ increase $h_H$

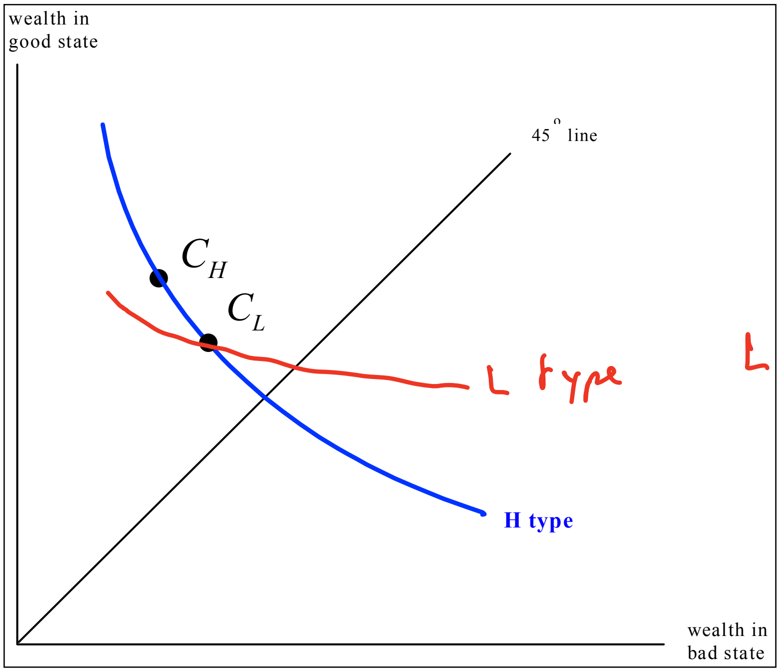
$(IC_H)$ must be satisfied as an equality.

replace with =

H people are indifferent between $C_H$ and $C_L$

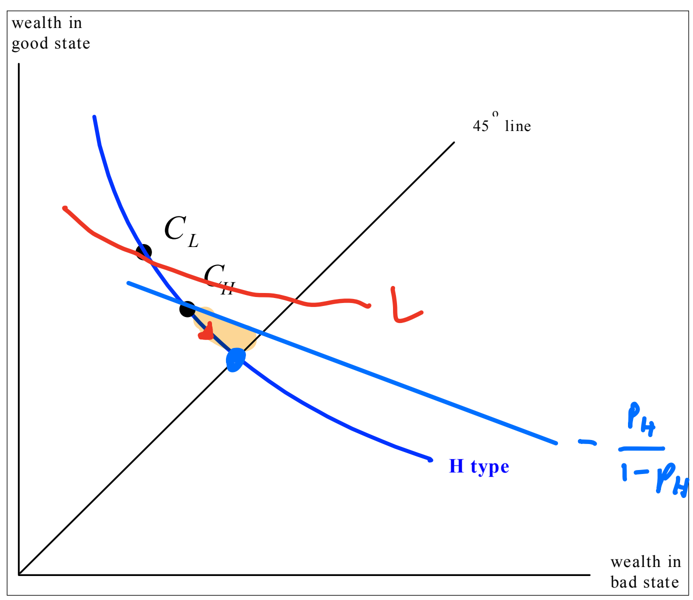
So $C_H$ and $C_L$ be on the same indifference curve for the H type.

**On this indifference curve, contract $C_H$ cannot be above contract $C_L$**



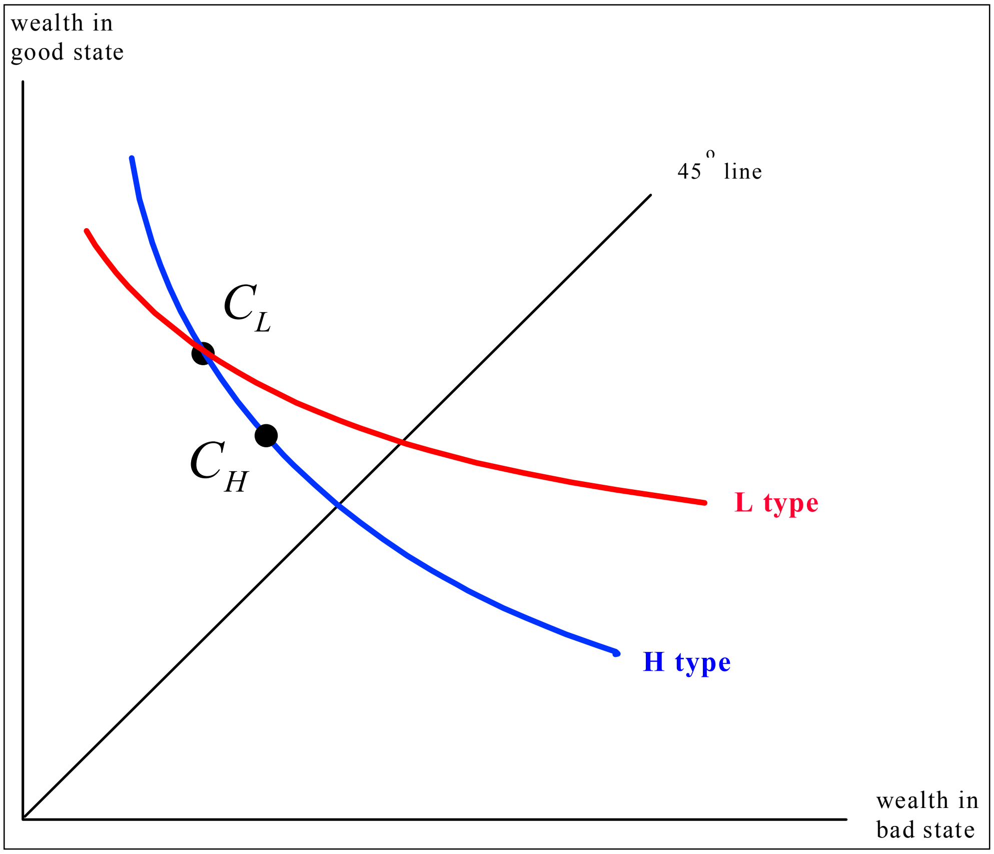

L people would prefer $C_H$ to $C_L$, violating $IC_L$

**So it must be:**

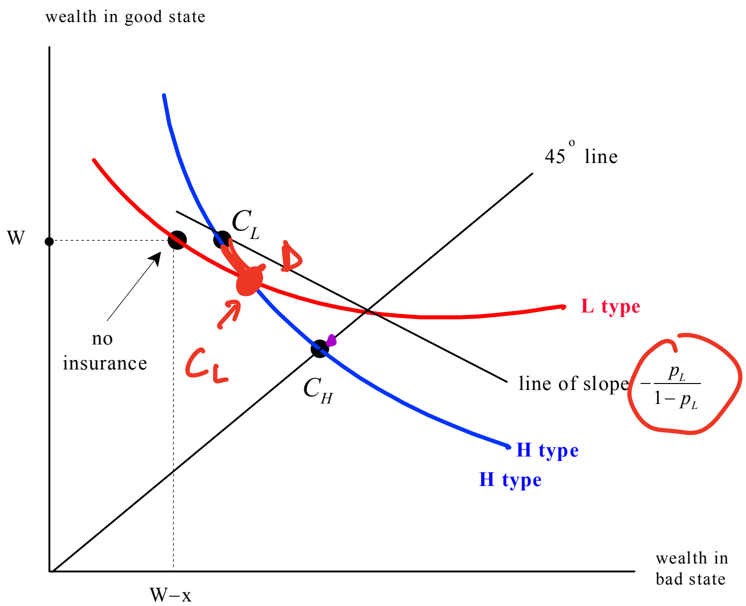


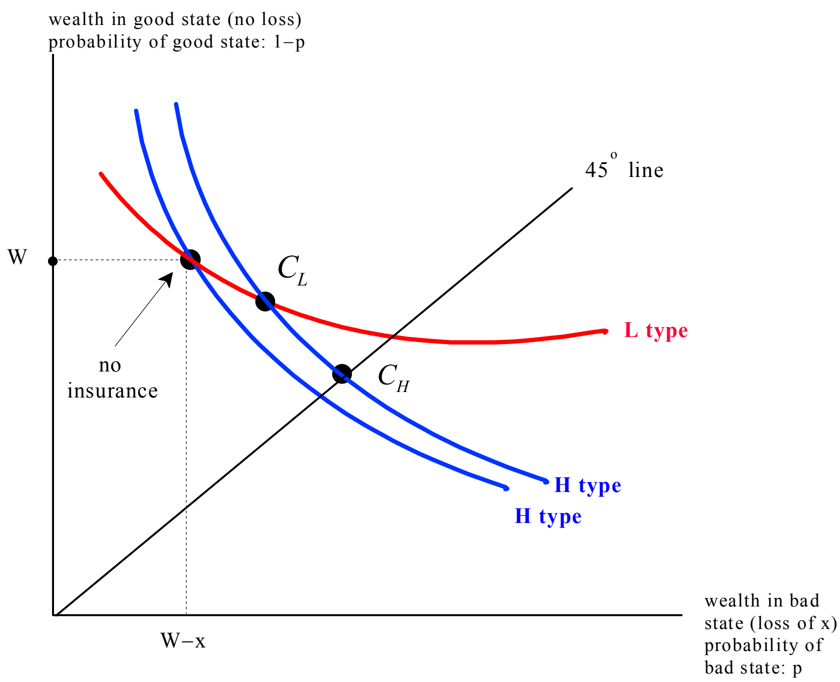

$C_H = (h_H, d_H = 0)$

$C_H$ must be a full insurance contract



$(IR_L)$ must be satisfied as an equality.




$(IC_L)$ **is not binding: it is always satisfied as a strict inequality.**