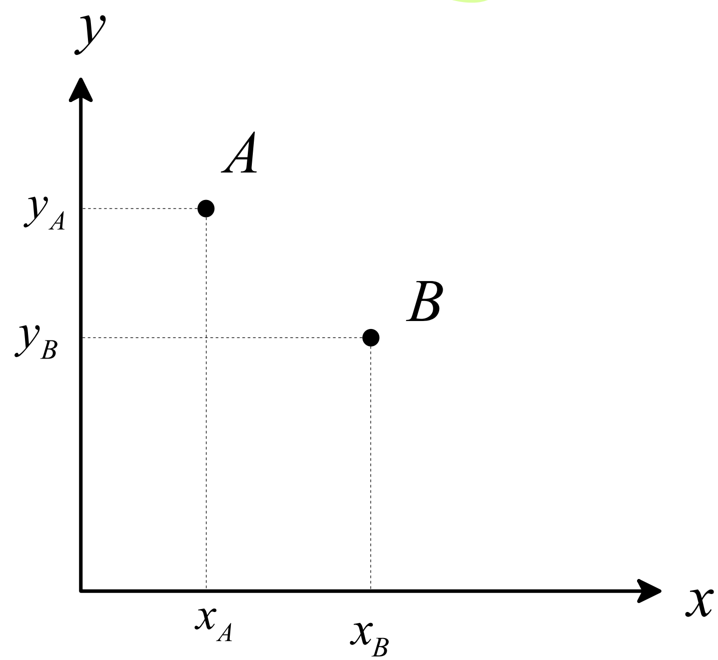
BINARY LOTTERIES

Lotteries of the form $\begin{pmatrix} \$x & \$y \\ p & 1-p \end{pmatrix}$ with p fixed and x and y allowed to vary.



$W_0 - (W_0 - W_2)$

$1-p$

$U(\$m)$

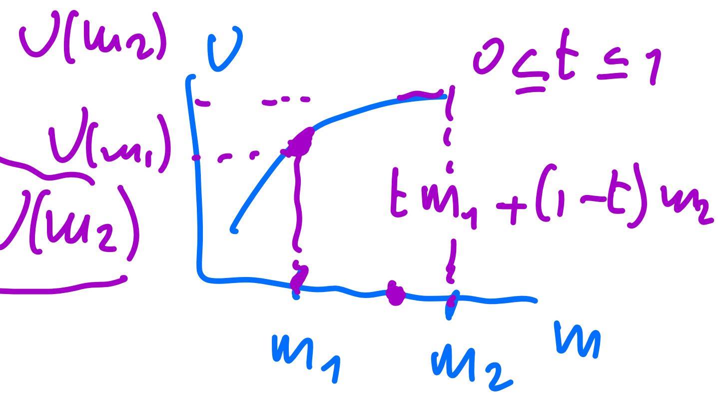
$E[U(A)] = E[U(B)]$

draw indifference curve through A

$E[U(L)]$

Risk averse if

We want to draw indifference curves in this diagram.

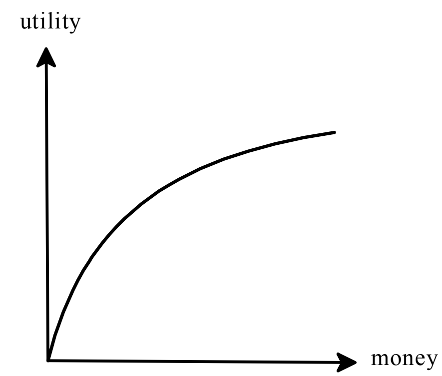


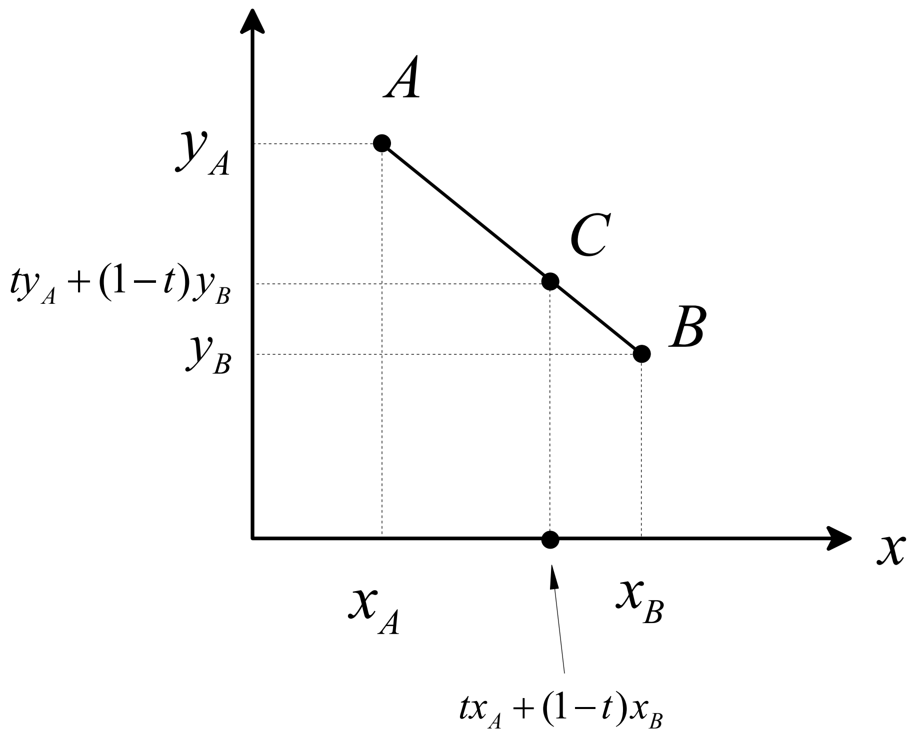
$U(tm_1 + (1-t)m_2) > tU(m_1) + (1-t)U(m_2)$

$E[L]$ $\quad L = \begin{pmatrix} m_1 & m_2 \\ t & 1-t \end{pmatrix}$

Case 2: risk-averse agent

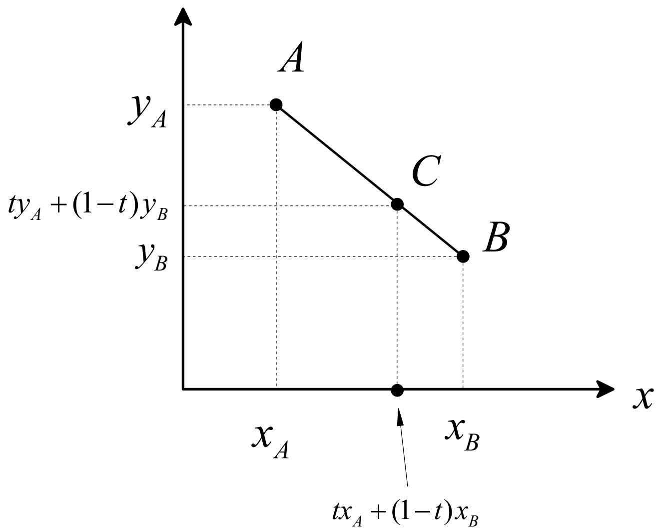
$U(m)$ is strictly concave:





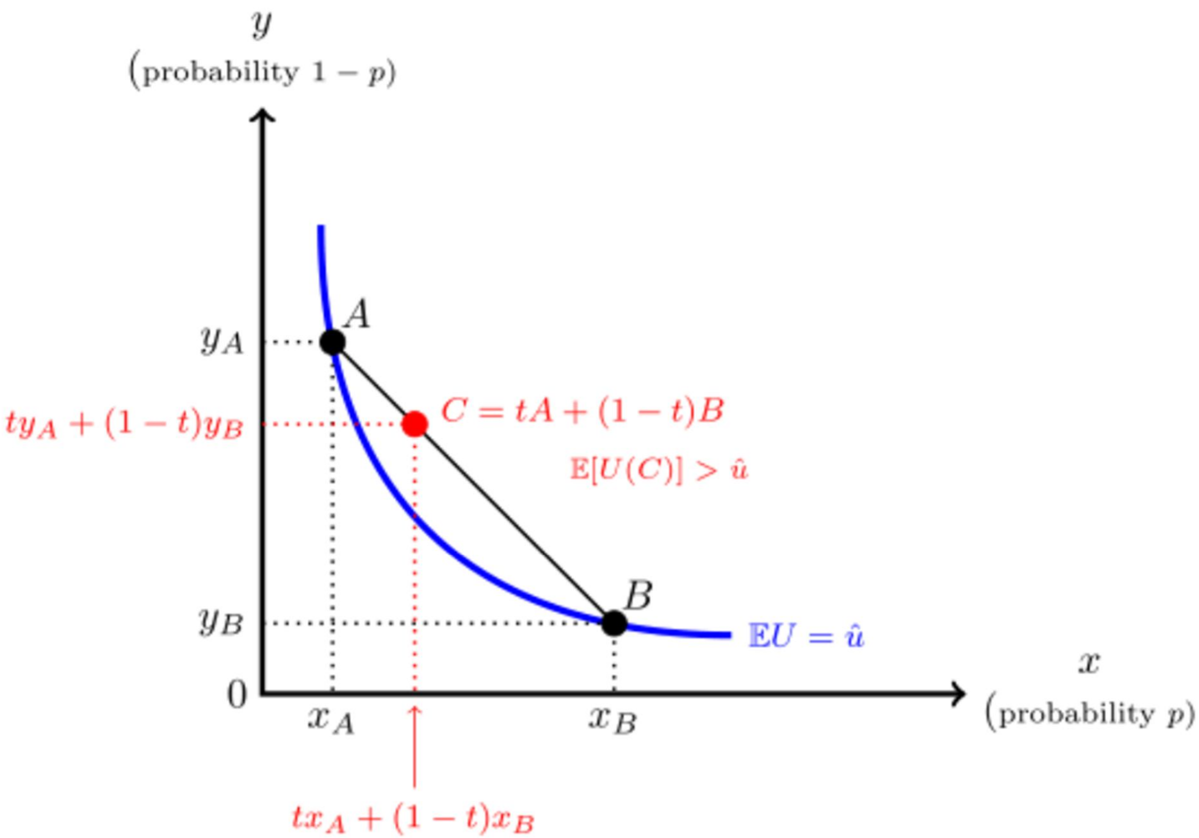
Suppose that $\mathbb{E}[U(A)] = \mathbb{E}[U(B)]$

Want to show that
$\mathbb{E}[U(C)] > \mathbb{E}[U(A)] (= \mathbb{E}[U(B)])$



$\mathbb{E}[U(C)] =$

The indifference curve must lie below the straight-line segment joining A and B.


y
(probability $1-p$)
A
y_A
$ty_A + (1-t)y_B$
$C = tA + (1-t)B$
$\mathbb{E}[U(C)] > \hat{u}$
B
y_B
$\mathbb{E}U = \hat{u}$
x
(probability p)
0
x_A
x_B
$tx_A + (1-t)x_B$

Slope of indifference curve

Let A and B be two points that lie on the same indifference curve: $\mathbb{E}[U(A)] = \mathbb{E}[U(B)]$,

(*)

- Since x_B is close to x_A, $U(x_B) \simeq$
- Since y_B is close to y_A, $U(y_B) \simeq$

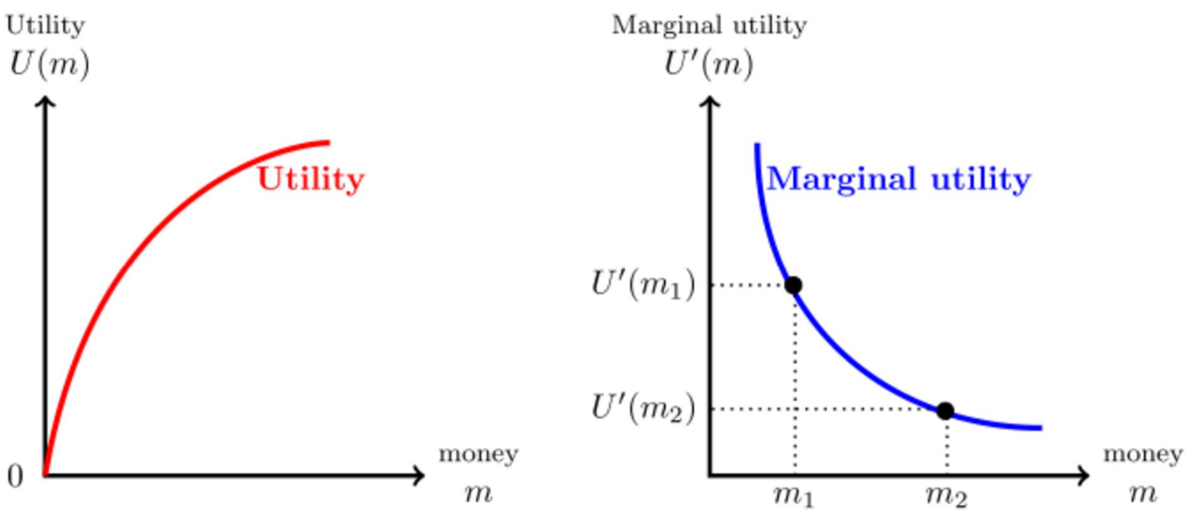
Thus the RHS of (*) can be written as

So (*) becomes

that is, which can be written as

Comparing the slope at a point with the ratio $\frac{p}{1-p}$

Look at the case of risk aversion but the other cases are similar.

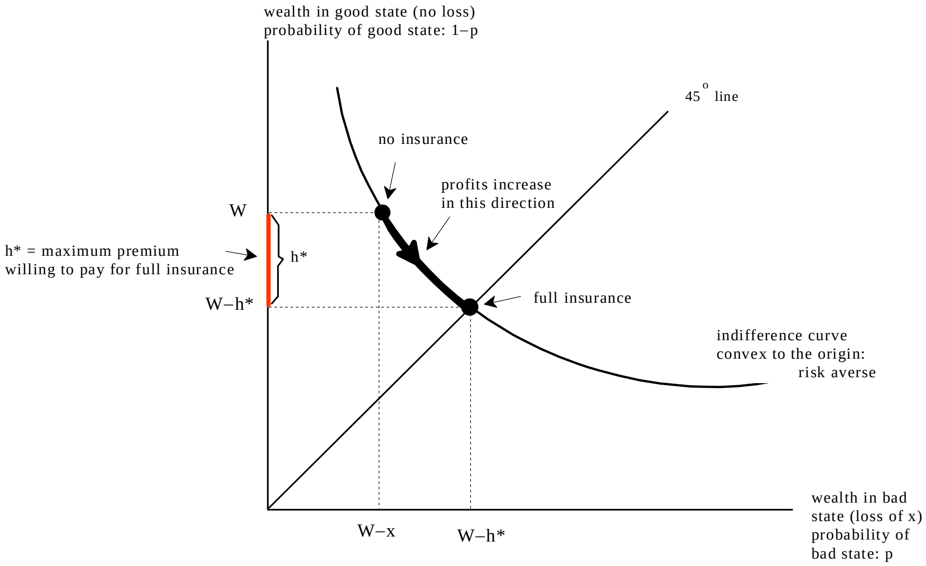


- at a point **above** the 45° line, where $x < y$,

- at a point **on** the 45° line, where $x = y$,

- at a point **below** the 45° line, where $x > y$,

Case 1: THE INSURANCE INDUSTRY IS A MONOPOLY



2. Suppose the insurance industry is perfectly competitive

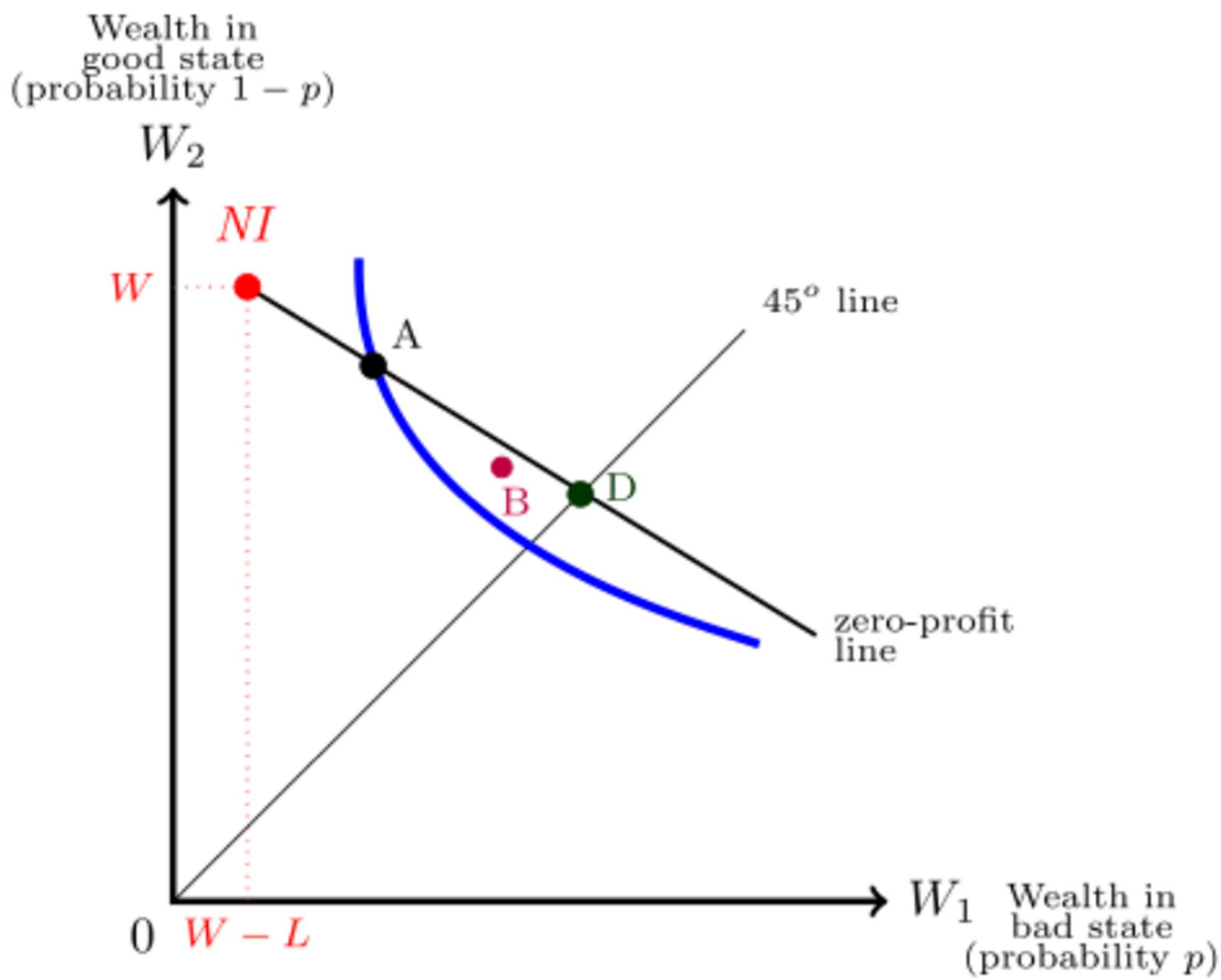
A contract that yields zero profit is called a **fair contract** and the zero profit line is called the **fair odds line**. Recall that the zero profit line is the straight line that goes through the No Insurance point and has slope $-\frac{p}{1-p}$.

Define an equilibrium in a competitive insurance industry as a situation where

(1) every firm makes zero profits and

(2) no firm (existing or new) can make positive profits by offering a new contract.

By the zero-profit condition (1), any equilibrium contract must be on the zero-profit line.

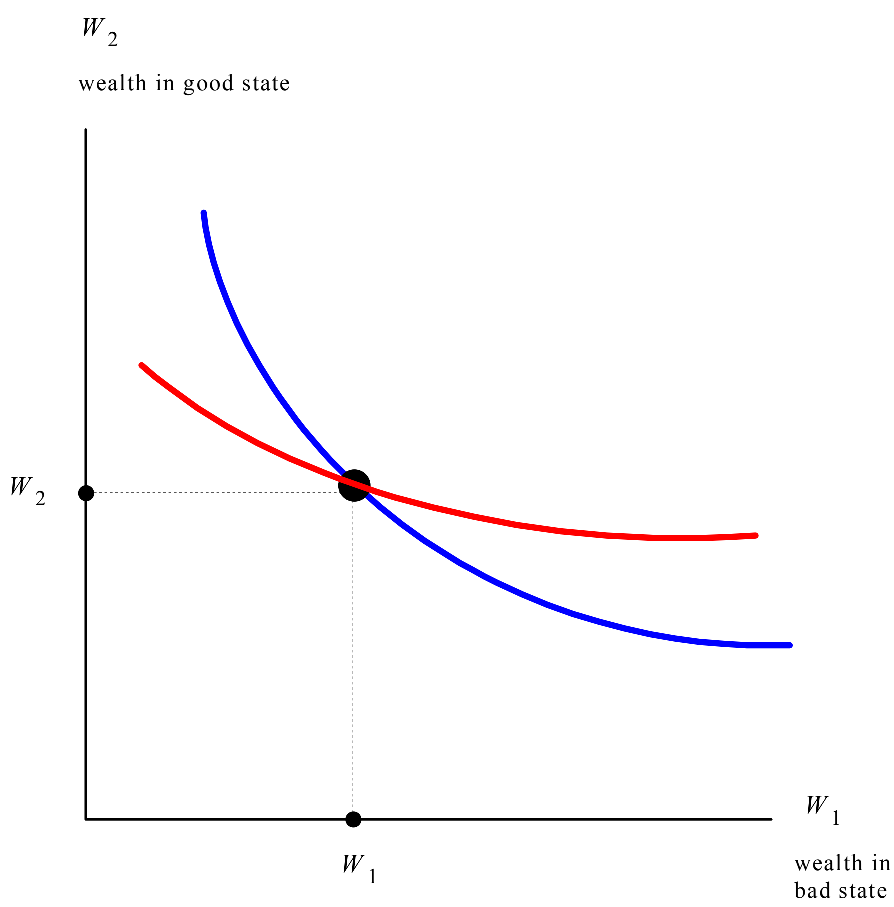

Wealth in
good state
(probability $1-p$)
W_2
NI
W
A
45^o line
B
D
zero-profit
line
0
$W-L$
W_1
Wealth in
bad state
(probability p)

$d_D = 0$ and $h_D =$

Adverse selection in insurance markets

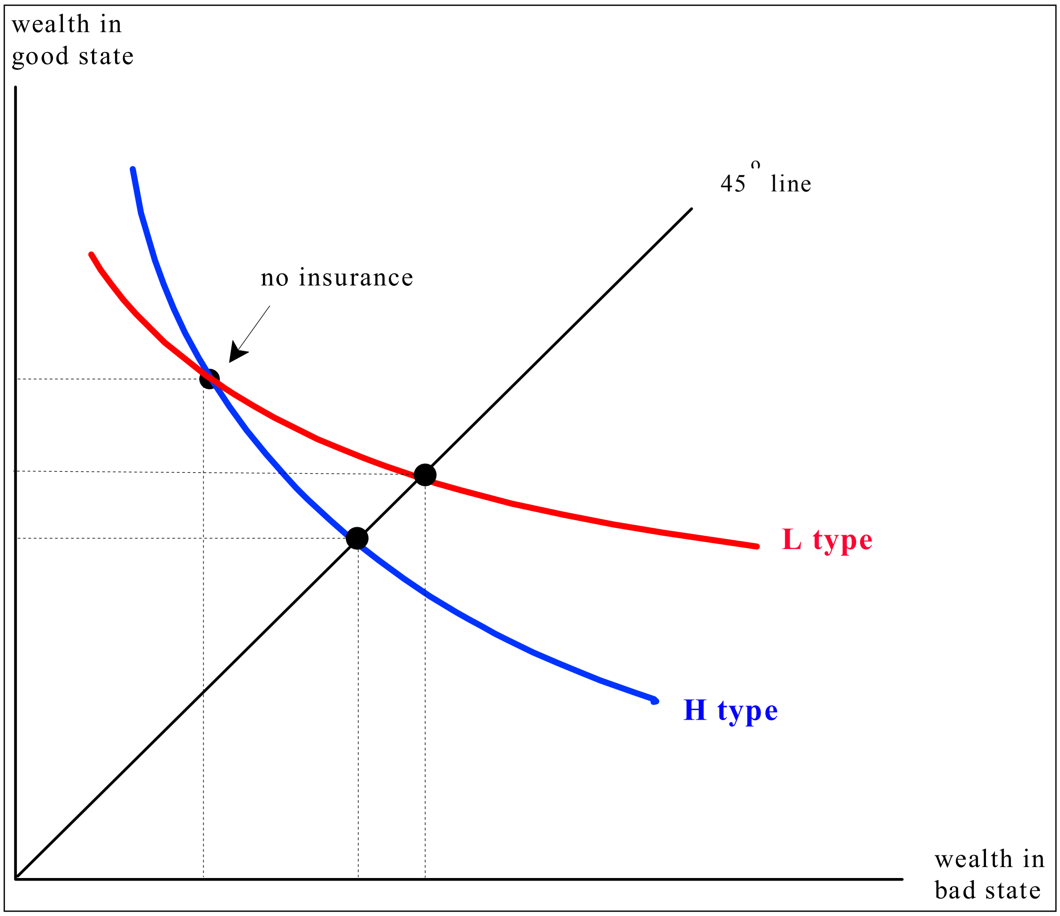
Two types of customers, *H* and *L,* identical in terms of initial wealth *W*, potential loss *L* and vNM utility-of-money function *U*, but with different probability of loss: $\boxed{p_H > p_L}$.

Slope of indifference curves at point (w_1, w_2)



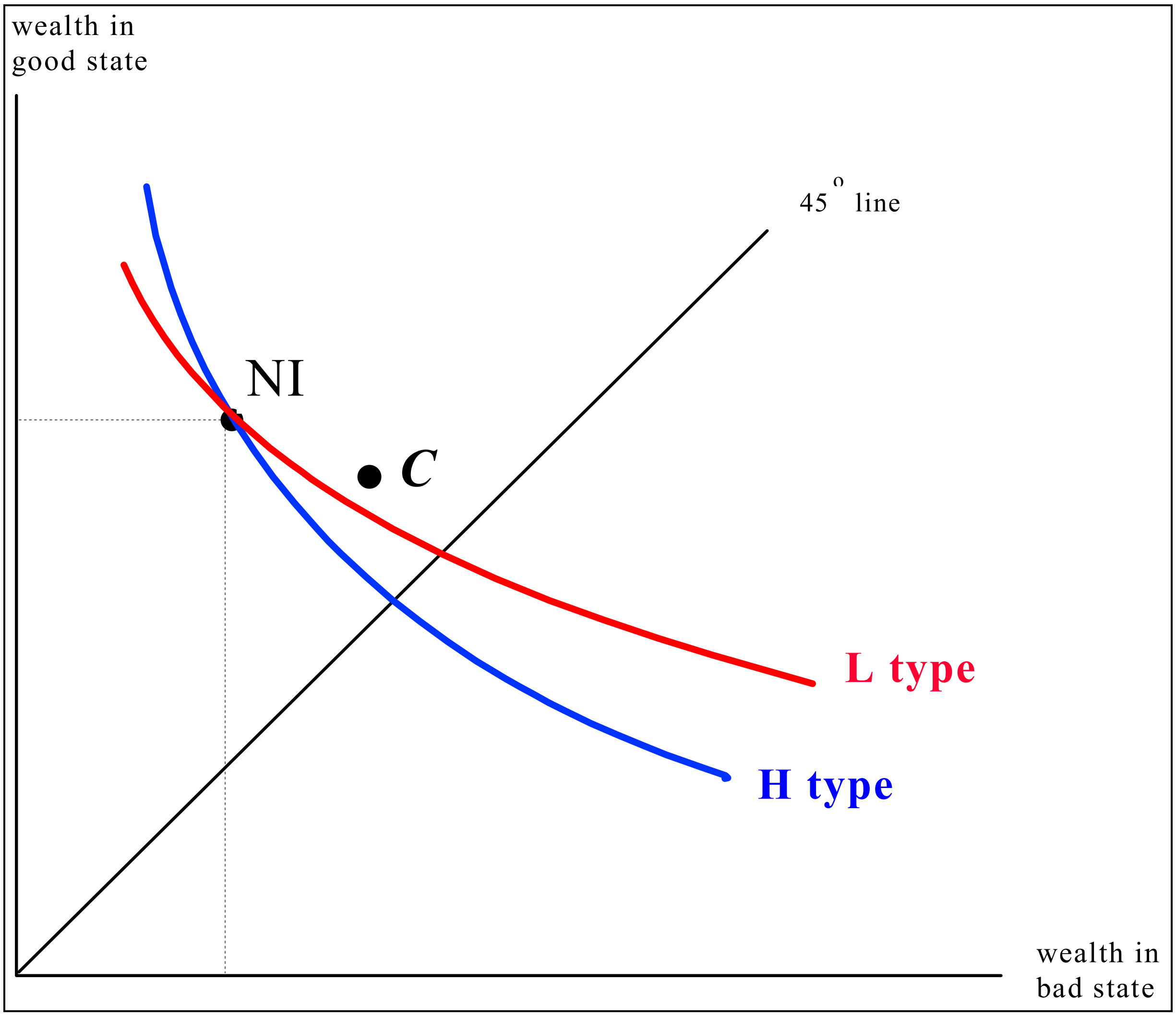
h_H^* maximum premium that the *H* people are willing to pay for full insurance

h_L^* maximum premium that the *L* people are willing to pay for full insurance:



Let q_H be the fraction of *H* types in the population $0 < q_H < 1$

$$\boxed{\text{If } \mathbb{E}[U_L(C)] \geq \mathbb{E}[U_L(NI)] \text{ then } \mathbb{E}[U_H(C)] \geq \mathbb{E}[U_H(NI)]}$$

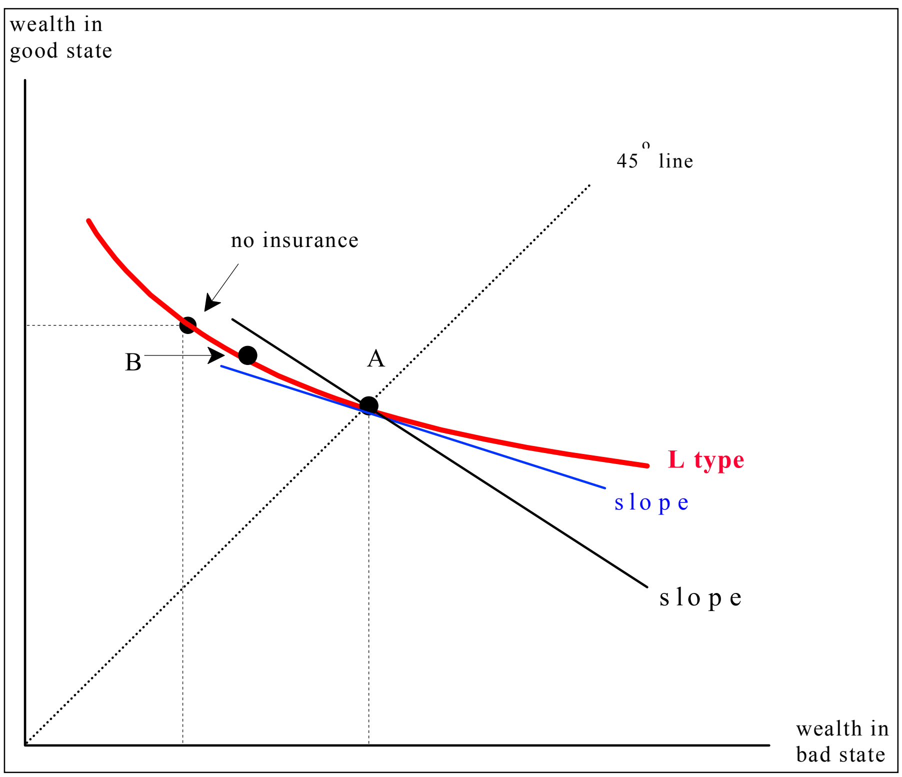


Case 1: **MONOPOLY**

OPTION 1. Offer only one contract, which is attractive only to the H type.

$C_1 = (\quad , \quad)$ Profits: $\pi_1^* =$

OPTION 2. Offer only one contract, which is attractive to both types. **Not optimal to offer full insurance**



Best contract under Option 2:

$\pi_2^* =$

OPTION 3: Offer two contracts,

$C_H = (h_H, d_H)$, targeted to the *H* type

$C_L = (h_L, d_L)$ targeted to the *L* type.

expected utility for L-type from C_L: $EU_L[C_L] =$

expected utility for L-type from C_H: $EU_L[C_H] =$

expected utility for H-type from C_L: $EU_H[C_L] =$

expected utility for H-type from C_H: $EU_H[C_H] =$

expected utility for L-type from NI : $EU_L[NI] =$

expected utility for L-type from NI : $EU_H[NI] =$

Monopolist's problem is to

$$\underset{h_H, d_H, h_L, d_L}{Max} \pi_3 = q_H N\left[h_H - p_H(L-d_H)\right] + (1-q_H)N\left[h_L - p_L(L-d_L)\right]$$

subject to

(IR_L)

(IC_L)

(IR_H)

(IC_H)

(IR_H) follows from (IR_L) and (IC_H)

Thus, the problem can be reduced to

$$\underset{h_H, d_H, h_L, d_L}{Max}\ \pi_3 = q_H N\left[h_H - p_H(L-d_H)\right] + (1-q_H)N\left[h_L - p_L(L-d_L)\right]$$

subject to

$(IR_L)\quad EU_L[C_L] \geq EU_L[NI]$

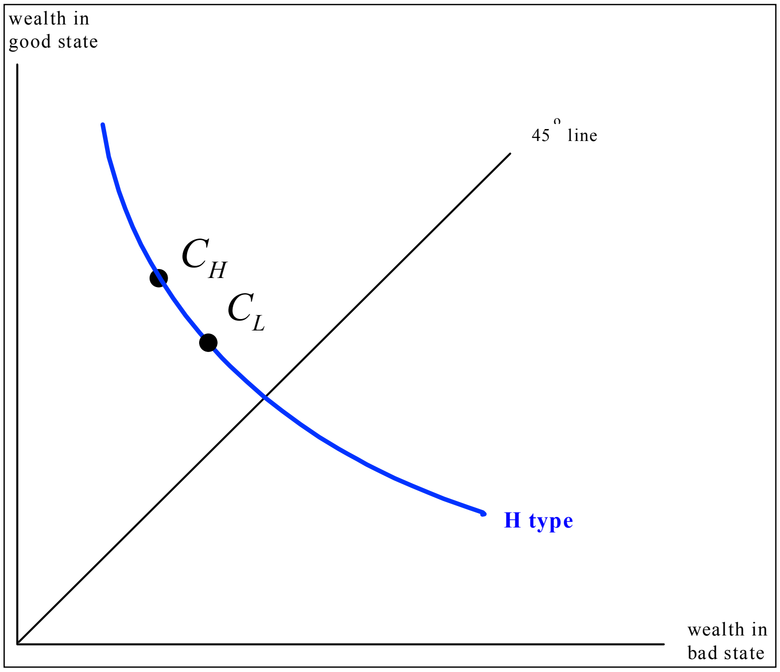
$(IC_L)\quad EU_L[C_L] \geq EU_L[C_H]$

$(IC_H)\quad EU_H[C_H] \geq EU_H[C_L]$

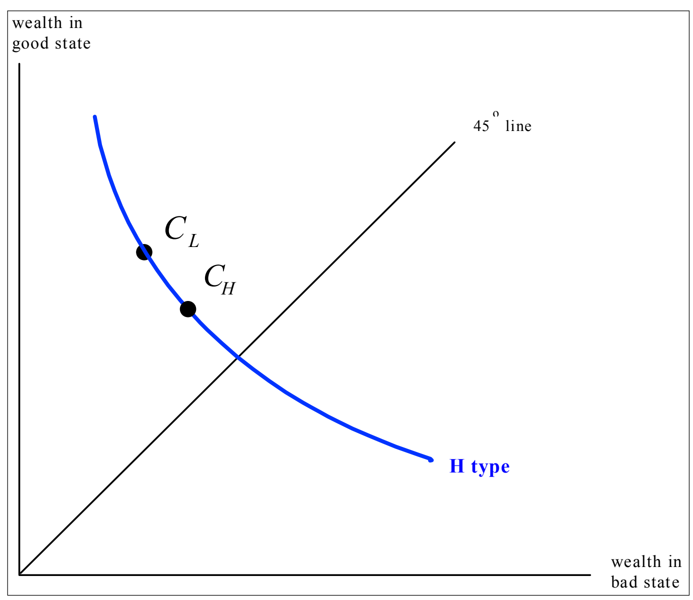
(IC_H) must be satisfied as an equality.

So C_H and C_L be on the same indifference curve for the H type.

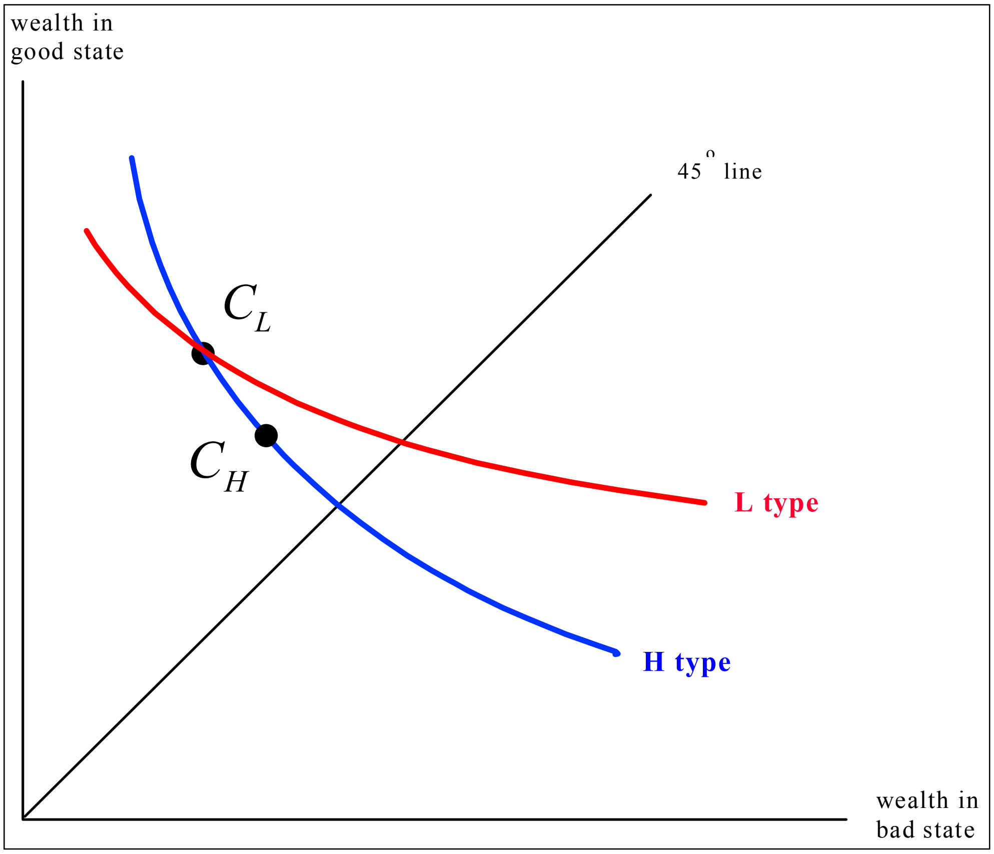
On this indifference curve, contract C_H cannot be above contract C_L



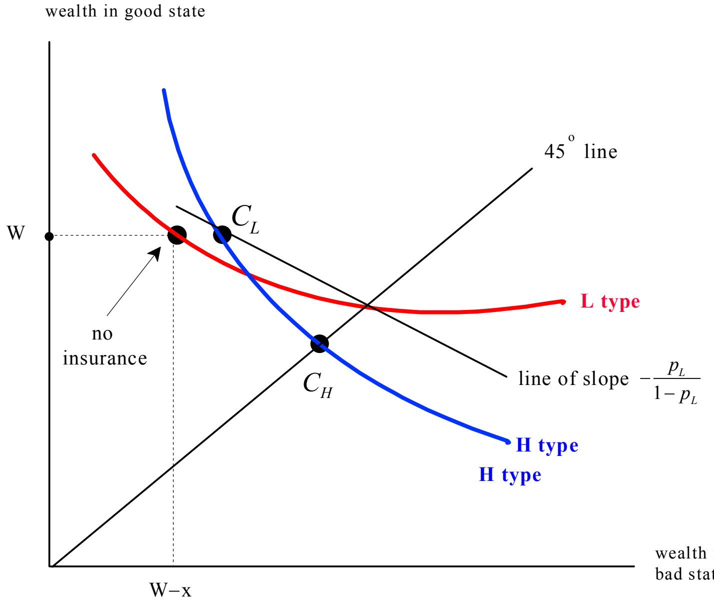
So it must be:

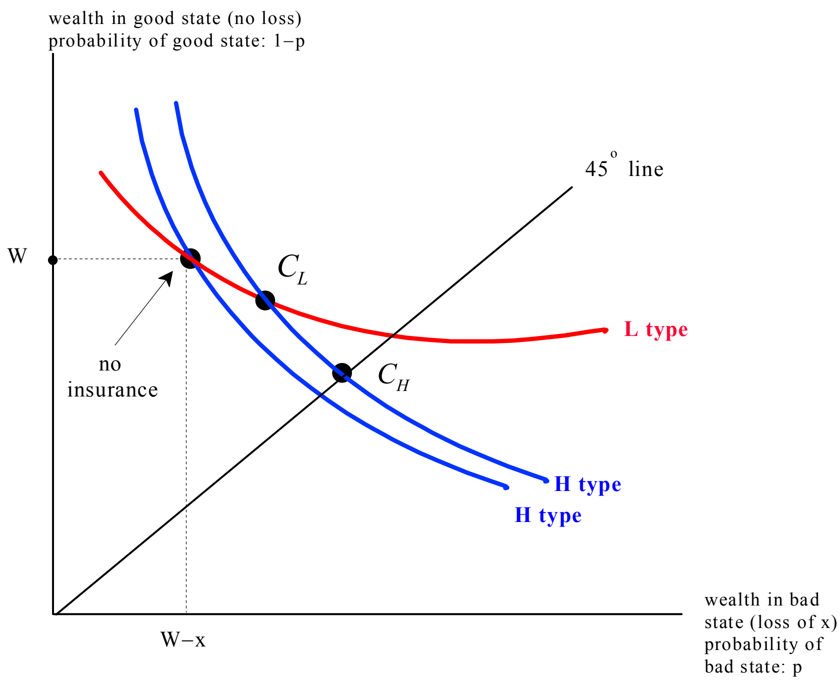


C_H must be a full insurance contract



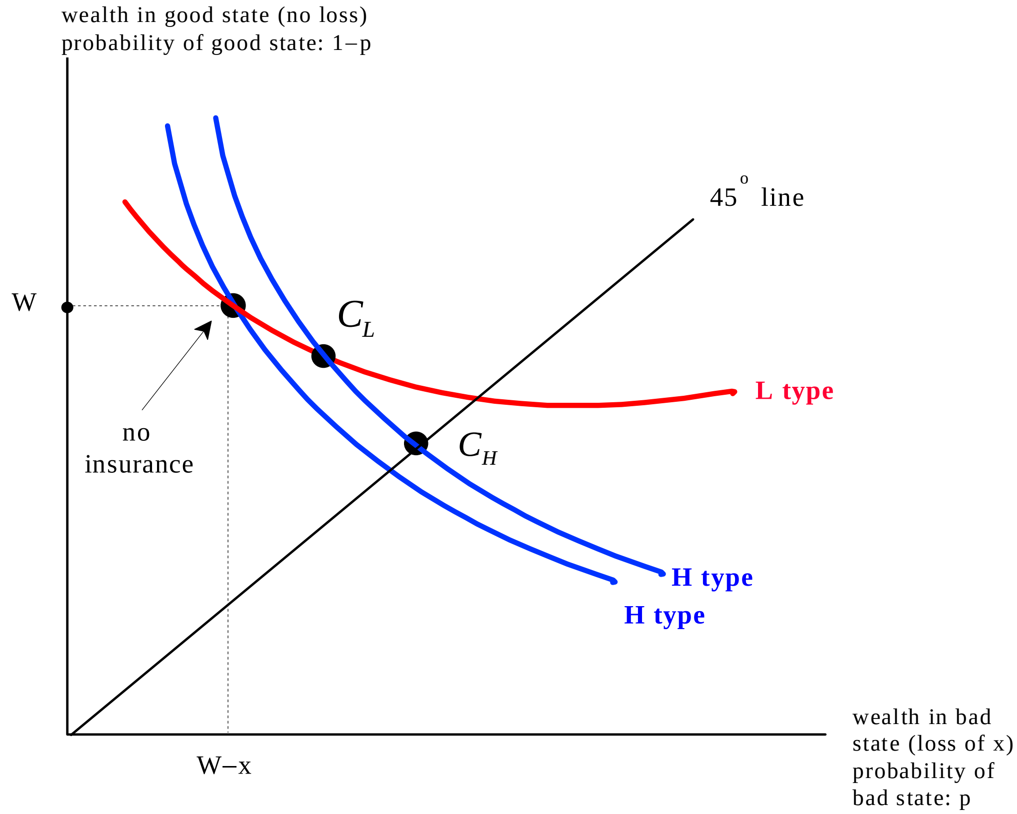
(IR_L) must be satisfied as an equality.



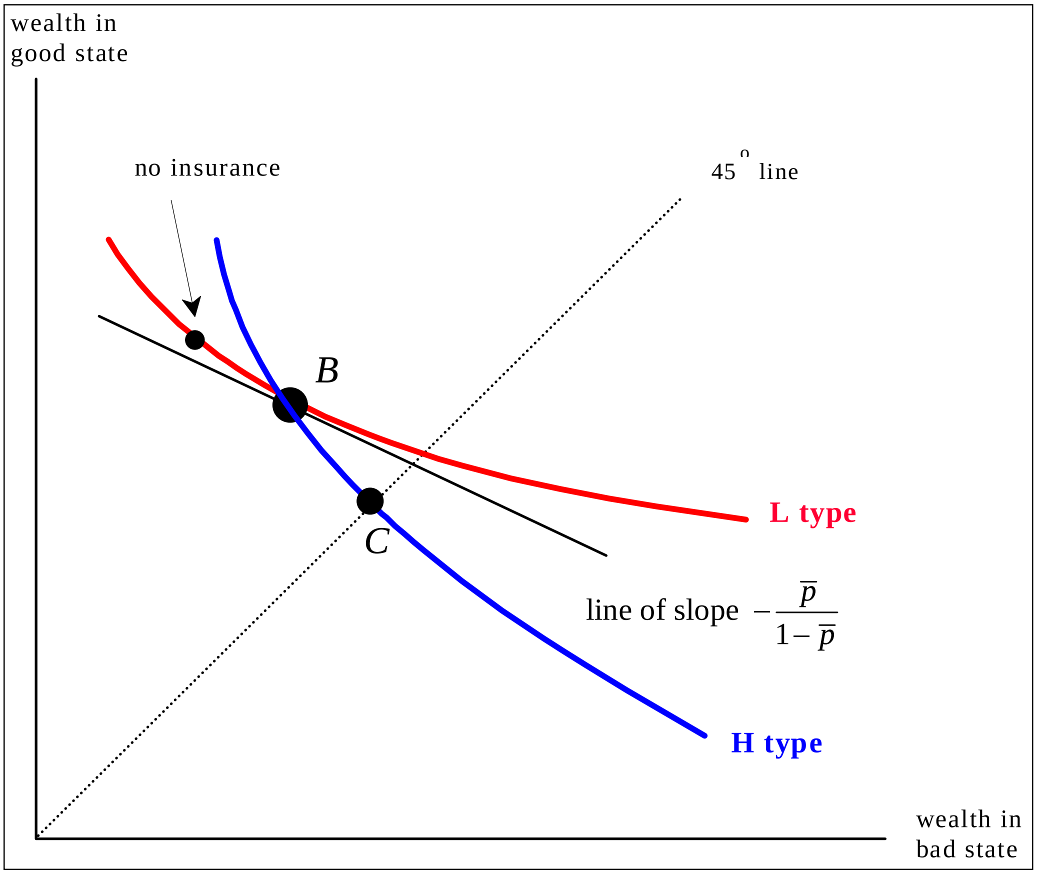


(IC_L) **is not binding: it is always satisfied as a strict inequality.**

Option 1 is a special case of Option 3



Option 3 yields higher profits than Option 2: $\pi_2^* < \pi_3^*$



In conclusion, the monopolist will always choose Option 3, although in some cases (namely when q_H is close to 1) **the outcome is the same as in Option 1.**

EXAMPLE. W = 1,600, x = 700, $p_H = \frac{1}{5}$, $p_L = \frac{1}{10}$, $U(m) = \sqrt{m}$.

h_H^* is given by the solution to

Thus under **Option 1** profits are:

Now **Option 3**. Let $h_H \in [79, 156]$ be the premium for the full-insurance contract targeted to the *H* type To find c_L solve:

We can solve the two equations in terms of h_H:

$$h_L(h_H) = h_H + 156\sqrt{1,600 - h_H} - 6,084$$
$$d_L(h_H) = 80h_H + 5,460\sqrt{1,600 - h_H} - 219,260$$

Then the monopolist will choose h_H to maximize

$\pi_3 =$

This function is strictly concave and $\left.\frac{d\pi_3}{dh_H}\right|_{h_h=79} = q_H N > 0$ and

$\left.\frac{d\pi_3}{dh_H}\right|_{h_h=156} = \frac{47}{38}q_H - \frac{9}{38}$. This is negative if and only if $q_H < \frac{9}{47}$. Thus,

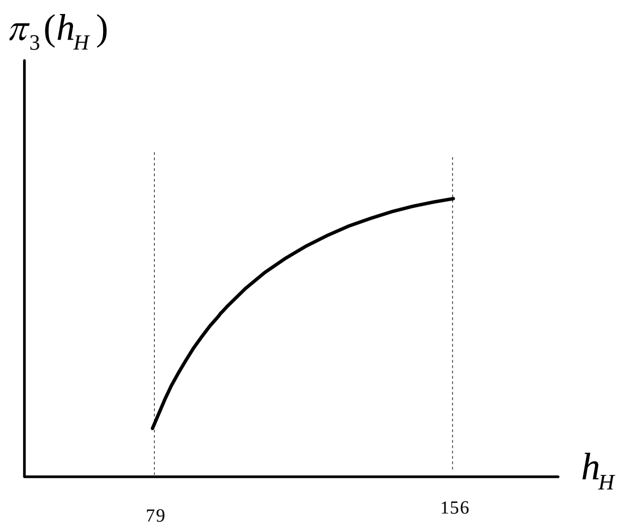


Figure 23a

$q_H \geq \frac{9}{47}$: the optimal solution is to offer only the full insurance contract at premium $h_H^* = 156$ (Option 1)

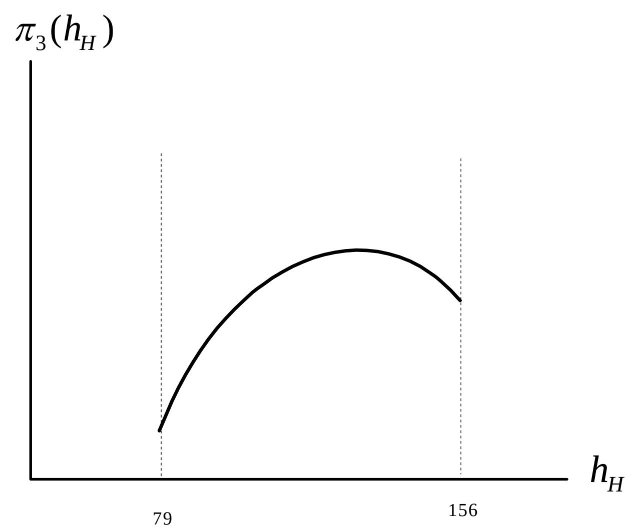


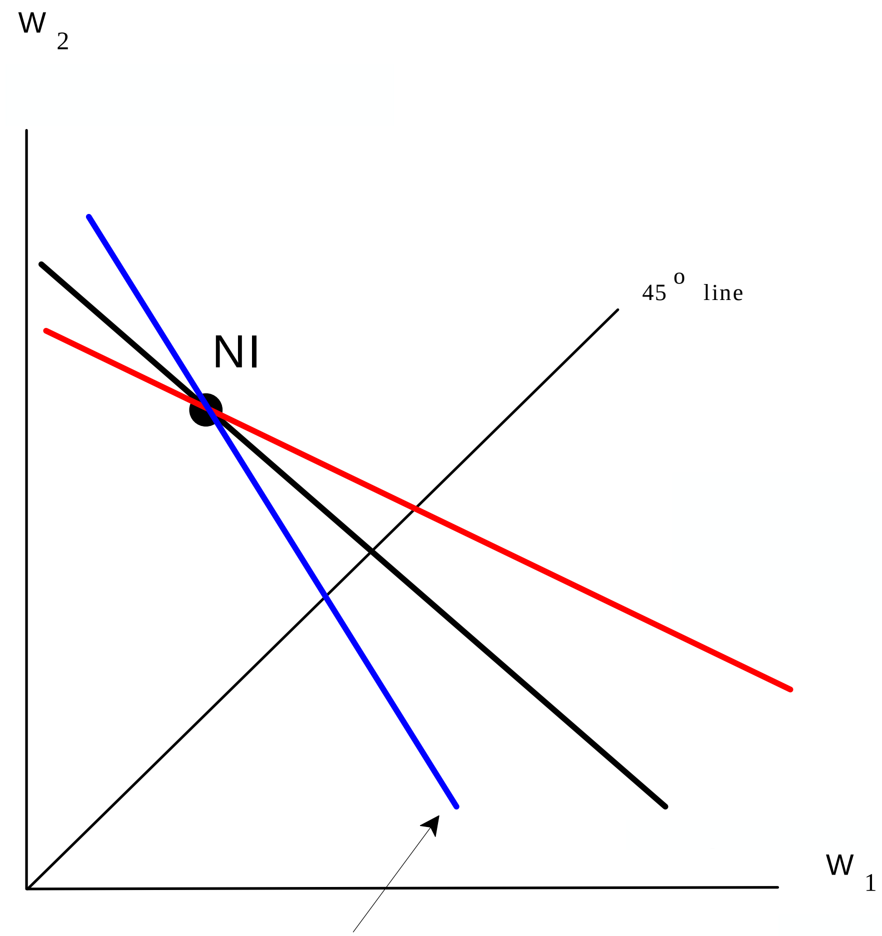
Figure 23b

$q_H < \frac{9}{47}$: the optimal solution is to offer two contracts (Option 3)

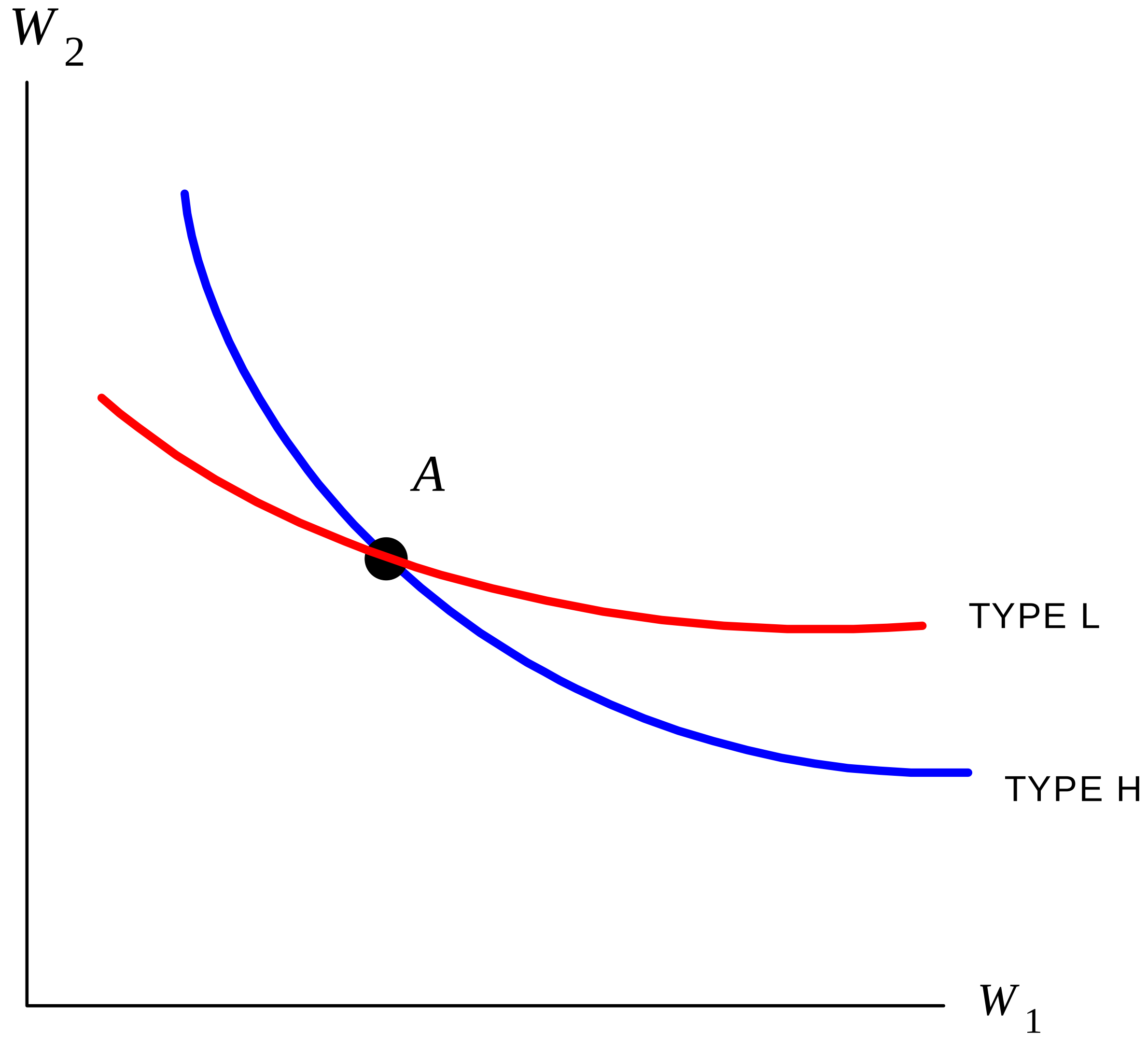
COMPETITIVE INDUSTRY with free entry

Equilibrium: (1) every firm makes zero profits and (2) no firm could make positive profits by introducing a new contract.

Three zero-profit lines:

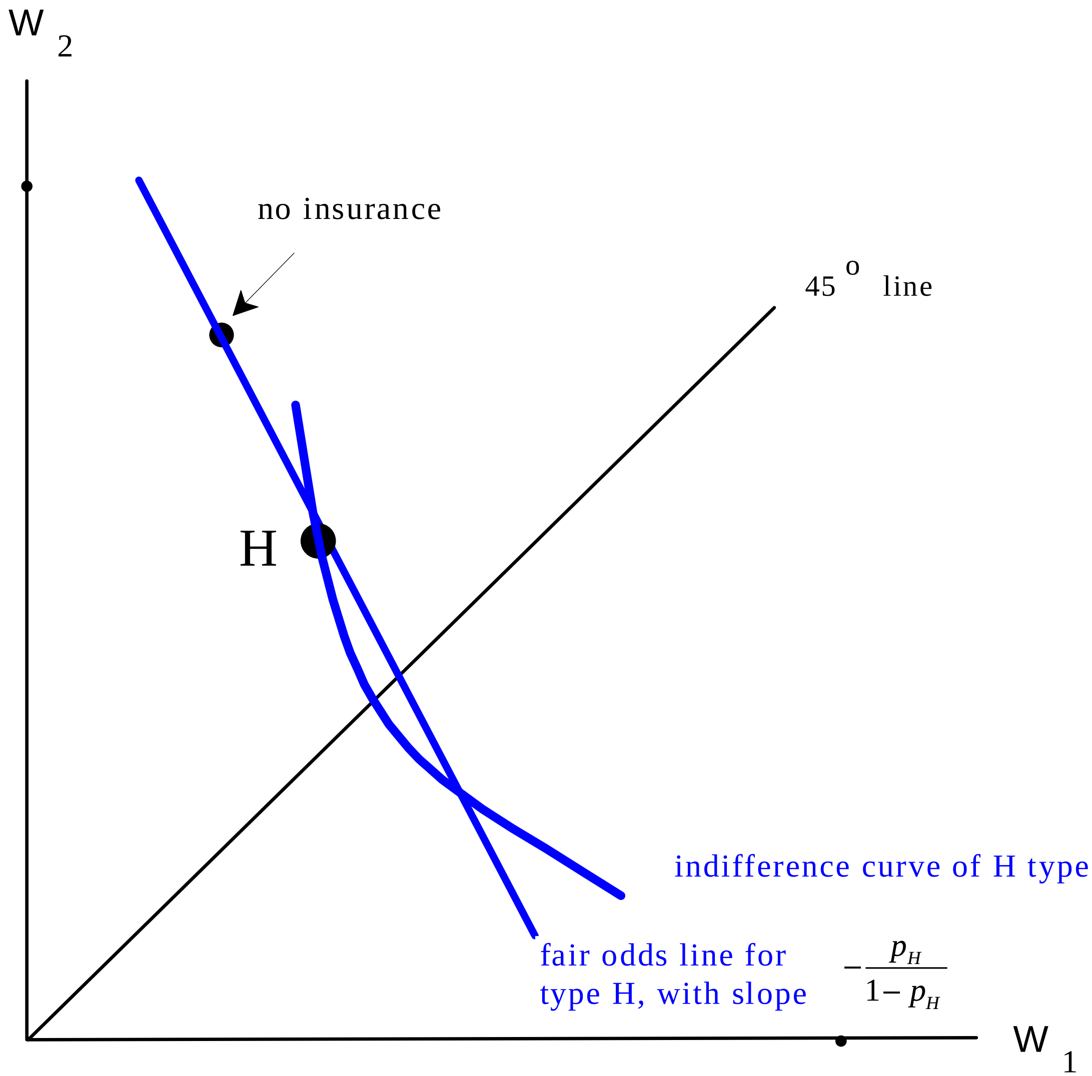


Remark 1: there cannot be a single-contract equilibrium serving both types.

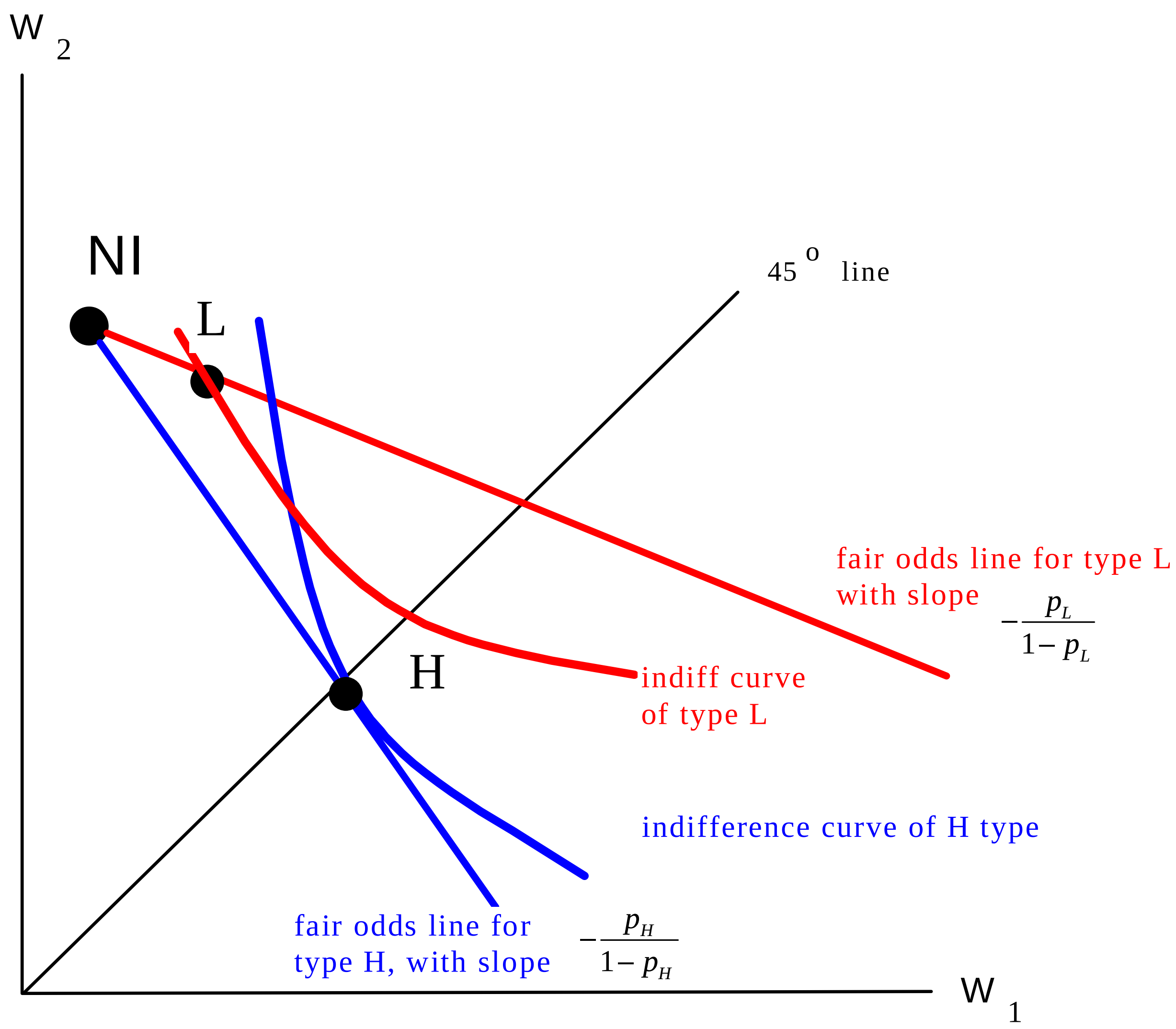

W_2
A
TYPE L
TYPE H
W_1

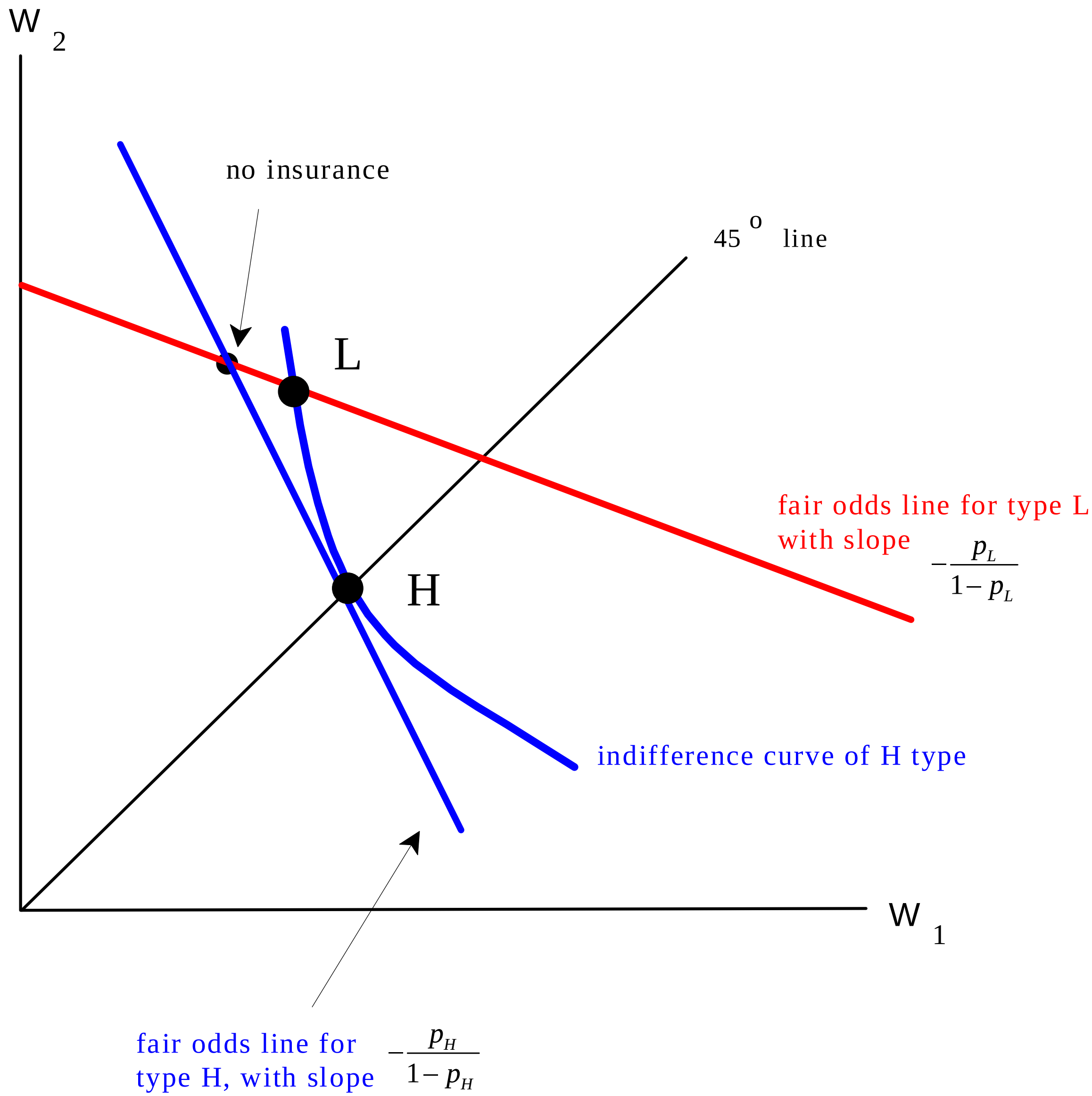
If there is a zero-profit equilibrium it must be an equilibrium with two contracts: the *L* types buy one and the *H* types buy the other

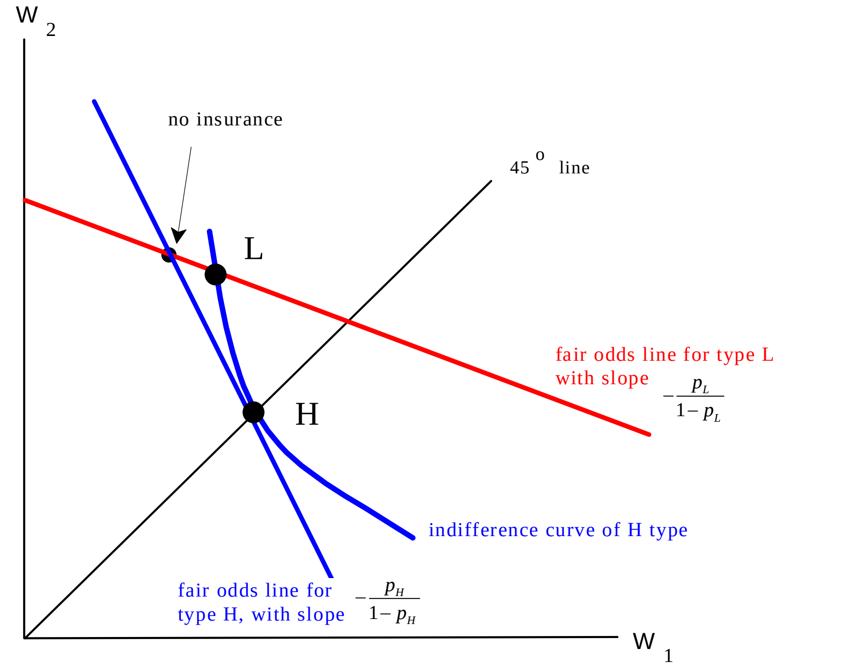
The contract bought by the H types must be a full-insurance contract:



What about the L contract?




W 2
no insurance
45 o line
L
H
fair odds line for type L
with slope $-\frac{p_L}{1-p_L}$
indifference curve of H type
W 1
fair odds line for
type H, with slope $-\frac{p_H}{1-p_H}$

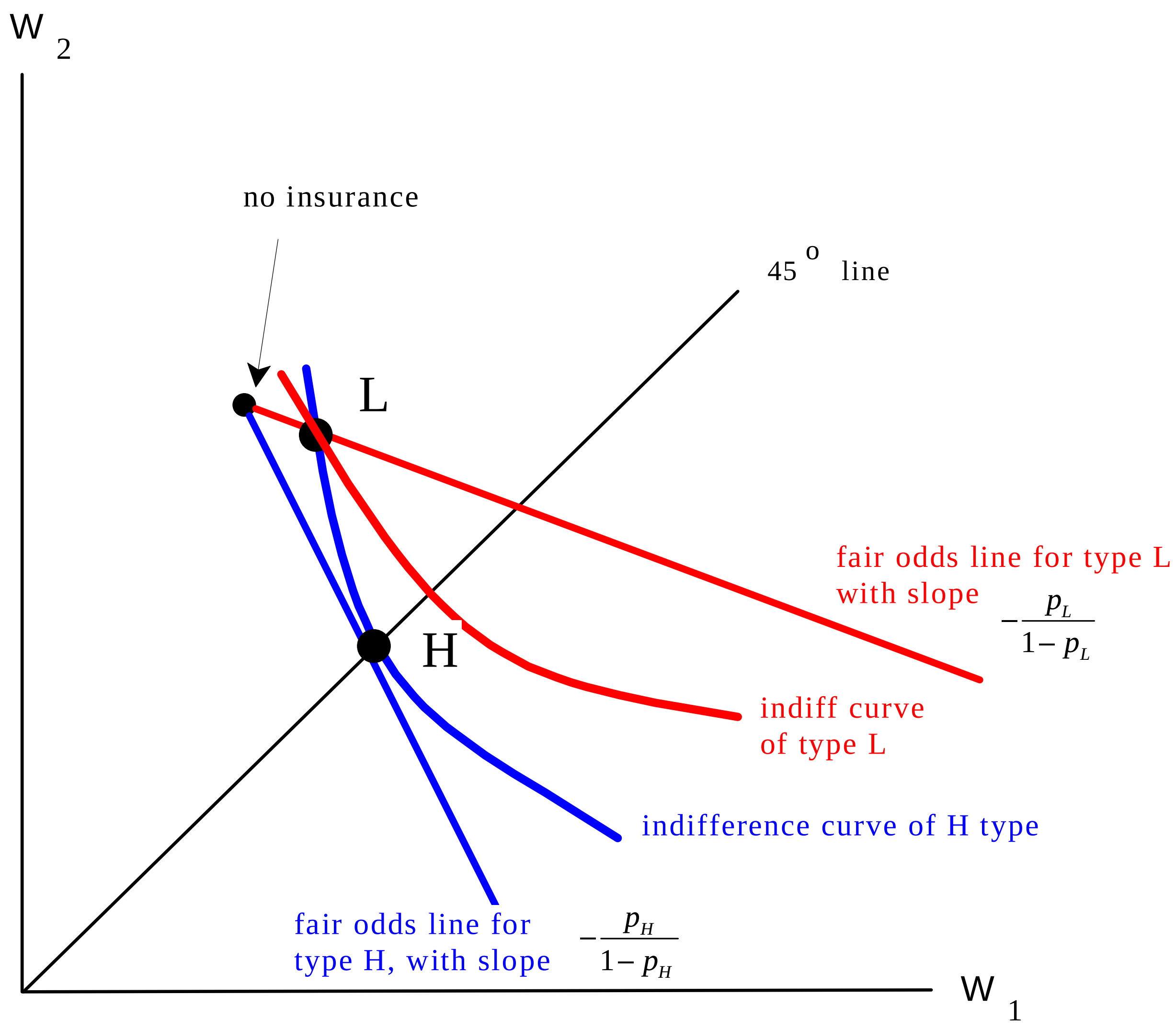


Example:

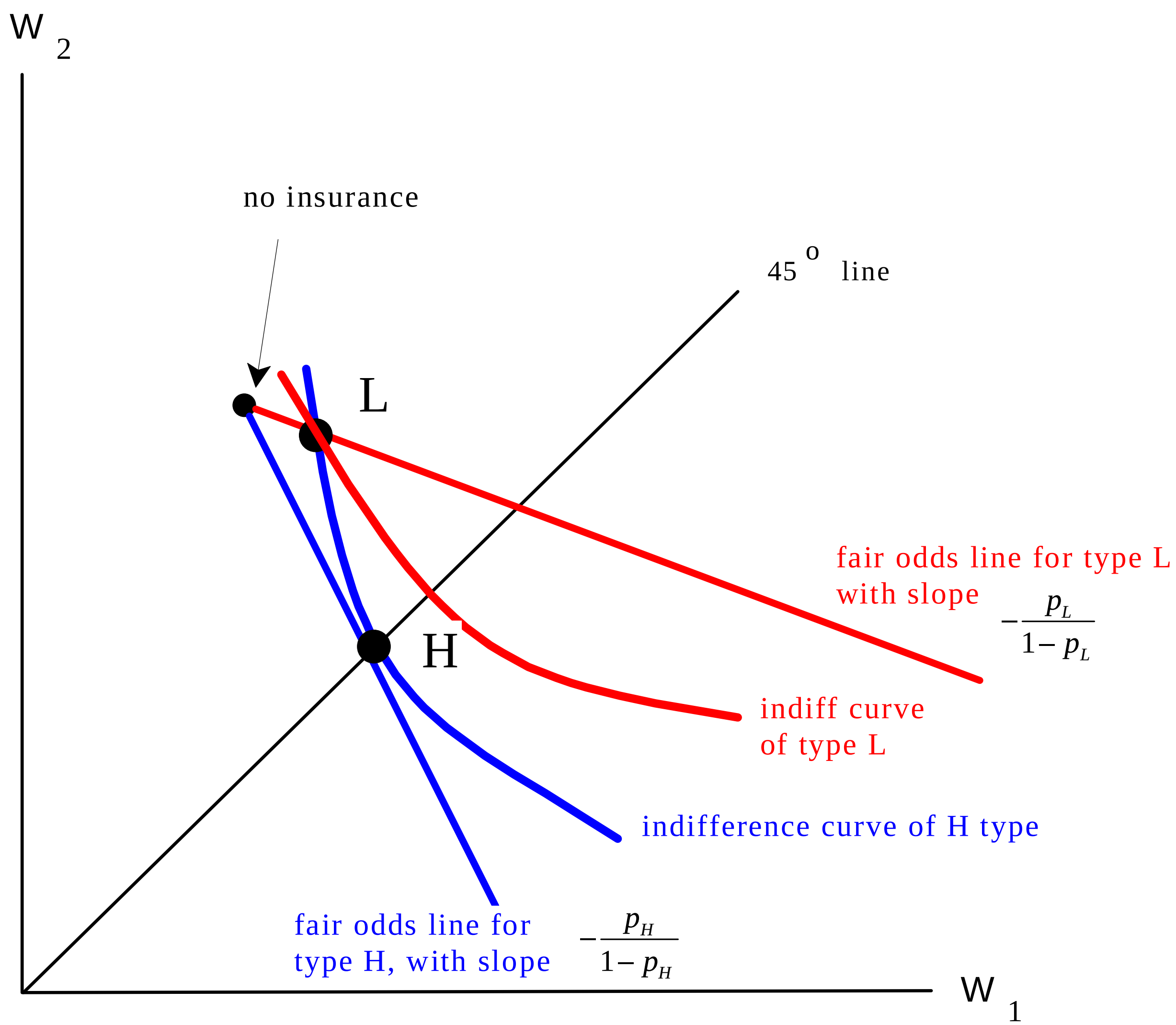
$$W = 625,\ loss = 225,$$

$$U(\$m) = \sqrt{m},$$

$$p_H = \frac{1}{10},\ \ p_L = \frac{1}{20},$$



W 2
no insurance
45 o line
L
H
fair odds line for type L
with slope
$-\frac{p_L}{1-p_L}$
indiff curve
of type L
indifference curve of H type
fair odds line for
type H, with slope
$-\frac{p_H}{1-p_H}$
W 1



W 2
no insurance
45 o line
L
H
fair odds line for type L
with slope
$-\frac{p_L}{1-p_L}$
indiff curve
of type L
indifference curve of H type
fair odds line for
type H, with slope
$-\frac{p_H}{1-p_H}$
W 1