Akerlof: market for second-hand cars

$P(D \mid \{D,E,F\}) = \frac{4}{8}$

Quality	A	B	C	D	E	F	
Number of cars	10	20	10	40	30	10	Total: 120
fraction	$p_A = \frac{10}{120} = \frac{1}{12}$	$p_B = \frac{2}{12}$	$p_C = \frac{1}{12}$	$p_D = \frac{4}{12}$	$p_E = \frac{3}{12}$	$p_F = \frac{1}{12}$	
Buyer's value	\$6,000	\$5,000	\$4,000	\$3,000	\$2,000	\$1,000	
Seller's value	\$5,400	\$4,500	\$3,600	\$2,700	\$1,800	\$900	

$$M = \begin{pmatrix} \$6,000 & \$5,000 & \$4,000 & \$3,000 & \$2,000 & \$1,000 \\ \frac{1}{12} & \frac{2}{12} & \frac{1}{12} & \frac{4}{12} & \frac{3}{12} & \frac{1}{12} \end{pmatrix}$$

$E[M] = 3,250$

$P = 3,100$

$$L = \begin{pmatrix} \$3000 & \$2000 & \$1,000 \\ \frac{4}{8} & \frac{3}{8} & \frac{1}{8} \end{pmatrix}$$

$E[L] = 2,375$

$P = 2,120$

$$H = \begin{pmatrix} \$2,000 & \$1,000 \\ \frac{3}{4} & \frac{1}{4} \end{pmatrix}$$

$E[H] = 1,750$

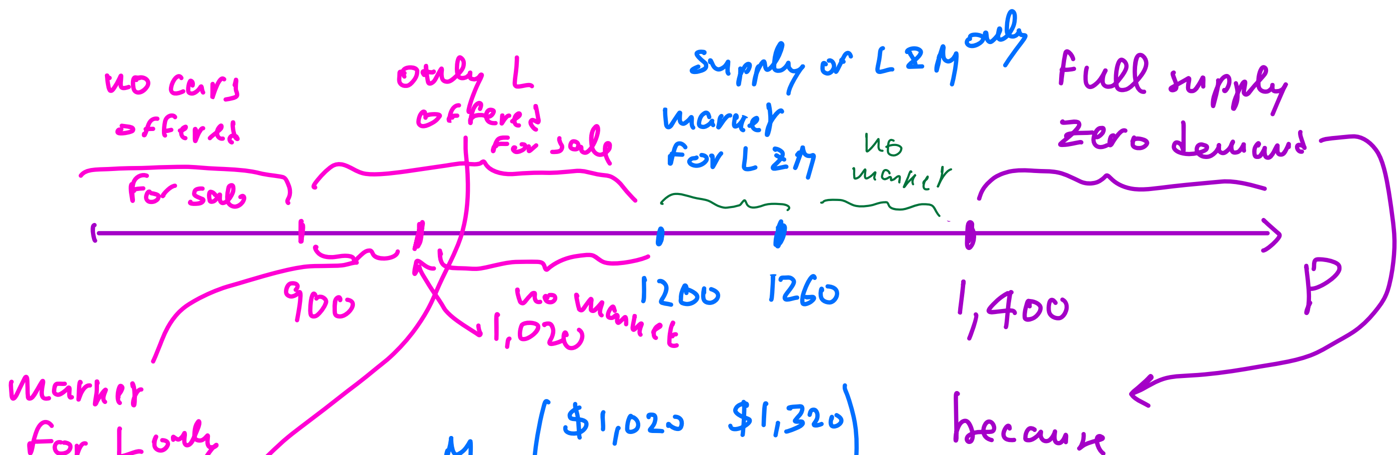
$P < 1,750$

$$\begin{pmatrix} \$1,000 \\ 1 \end{pmatrix}$$

$900 < P < 1,000$ There is a market but only for lowest quality F

Quality	L	M	H
probability	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$
seller's value	900	1,200	1,400
buyer's value	1,020	1,320	1,500

For every price p determine if there is a second-hand market.



$$M = \begin{pmatrix} \$1{,}020 & \$1{,}320 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$$

$$E[M] = 1{,}260$$

$$L = \begin{pmatrix} \$1{,}020 & \$1320 & \$1500 \\ \frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{pmatrix}$$

$$E[L] = 1{,}300$$

$$\begin{pmatrix} \$1{,}020 \\ 1 \end{pmatrix}$$

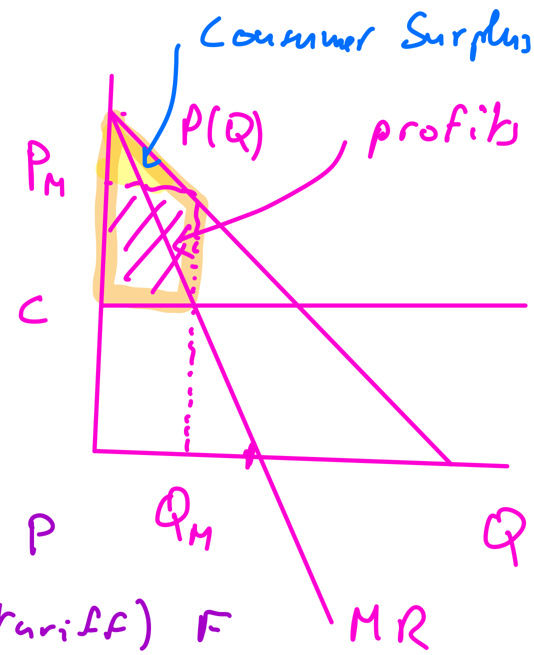
MONOPOLY and BUNDLING

A. ONE TYPE of customer

All consumers are identical with the same inverse demand function $P = P(Q)$. Monopolist's cost function: $C = cQ$ with $0 < c < P(0)$.

Naïve theory: $\pi(Q) = R(Q) - cQ = QP(Q) - cQ$

$$\frac{d\pi}{dQ} = \frac{dR}{dQ} - \frac{dC}{dQ} = 0$$



Bundling.

Two-part tariff : price per unit p
fixed change (tariff) F

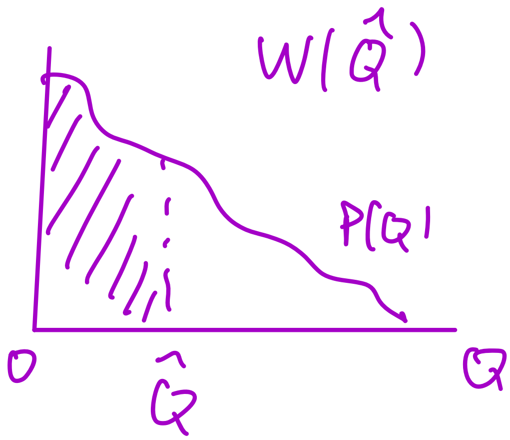
Bundling : (V, Q)

$$\underset{V,Q}{\text{Max}} \quad \pi = N(V - cQ)$$

s.t. $V \le W(Q)$

$$W(Q) = \int_0^Q P(x)\,dx$$

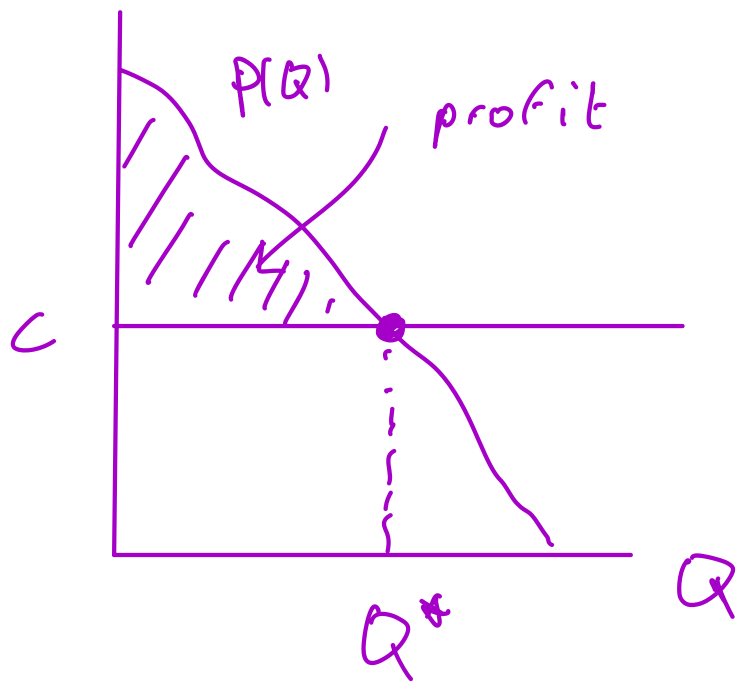
willingness to pay for Q units



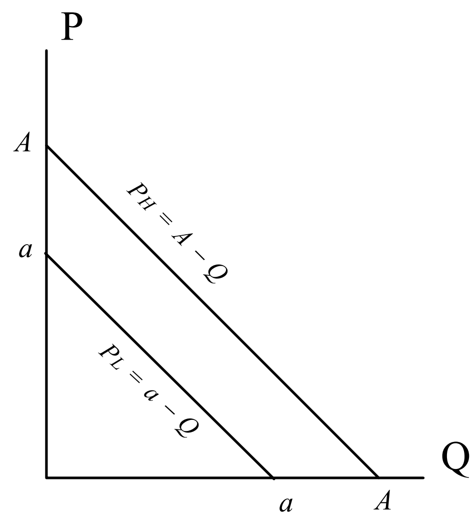
$$\underset{Q}{\text{Max}} \quad \pi = N(W(Q) - cQ)$$

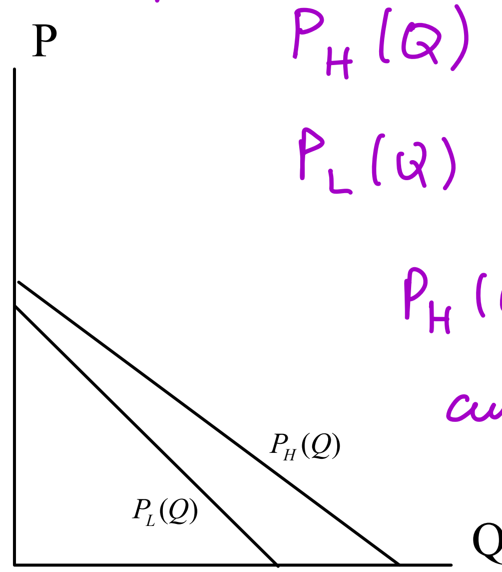
$$\frac{d\pi}{dQ} = 0 \qquad N\Big(\underbrace{\frac{dW}{dQ}}_{P(Q)} - c\Big) = 0$$

profit maximizing output is given by solution to $P(Q) = c$



B. TWO TYPES of customers L , H





$P_H(Q)$

$P_L(Q)$

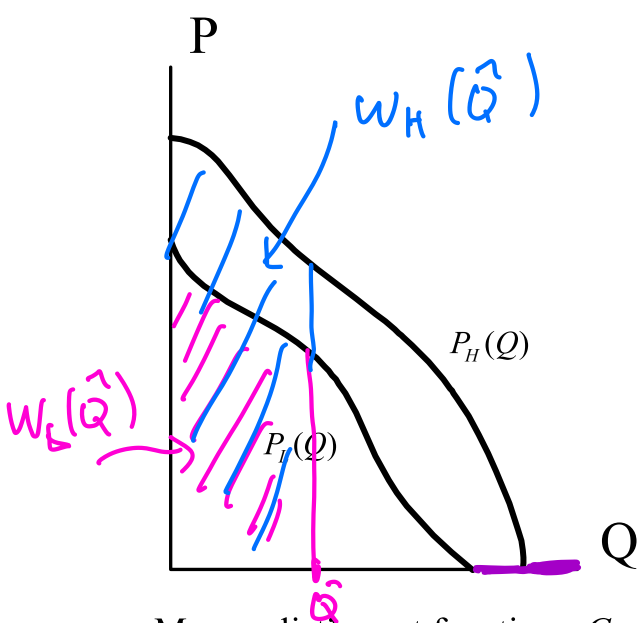
$P_H(Q) \geq P_L(Q)$

and if

$P_H(Q) > 0$

then

$P_H(Q) > P_L(Q)$



$$W_H(Q) = \int_0^Q P_H(x)\,dx$$

$$W_L(Q) = \int_0^Q P_L(x)\,dx$$

$$\Downarrow$$

$W_H(Q) > W_L(Q)$

$\forall Q$

Monopolist's cost function: $C = cQ$ with $0 < c < P_L(0)$.

q_H = fraction of H types

BUNDLING continued

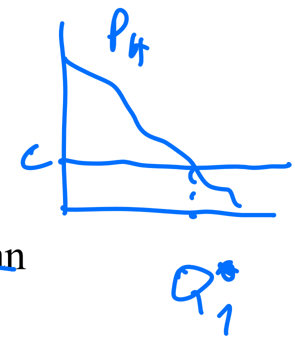
$\boxed{\textbf{OPTION 1}}$. Offer only one type of package (Q,V) which will be bought only by type H, because $V > W_L(Q)$ but $V \le W_H(Q)$.

$$\max_{V,Q} \Pi_1 = q_H N[V - cQ] \quad \text{s.t.} \quad V = W_H(Q)$$

~~It can be seen as included in Option 3~~:

$$\max_{Q} \Pi_1 = q_H N[W_H(Q) - cQ]$$

$$\frac{d\Pi_1}{dQ} = 0 \qquad Q = Q_1^*$$



Sufficient condition for Option 3 (two contracts) to be better than Option 1

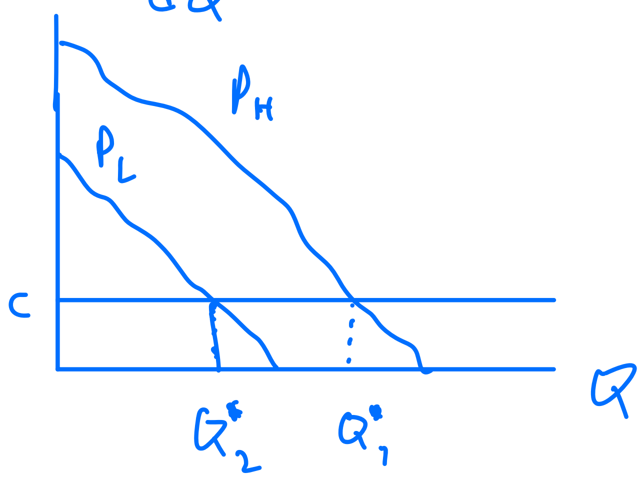
OPTION 2: Offer only one package (Q,V) which will be bought by both types, because $V \leq W_L(Q)$ (which implies that $V \leq W_H(Q)$ since $W_L(Q) \leq W_H(Q)$).

$$\max_{V,Q} \Pi_2 = N[V - cQ] \quad \text{s.t.} \quad \underbrace{V \leq W_L(Q)}\ (< W_H(Q))$$

~~Option 2 is always inferior to Option 3~~:

$$\max_{Q} \Pi_2 = N[W_L(Q) - cQ]$$

$$\frac{d\Pi_2}{dQ} = N[P_L(Q) - c] = 0$$



OPTION 3

Offer (V_H, Q_H) and (V_L, Q_L)

$$\max_{V_H, Q_H, V_L, Q_L} \quad \Pi_3 = q_H N\left[V_H - cQ_H\right] + (1-q_H) N\left[V_L - cQ_L\right]$$

IR_L $\quad V_L \le W_L(Q_L)$

IC_L $\quad W_L(Q_L) - V_L \ge W_L(Q_H) - V_H$

IR_H $\quad V_H \le W_H(Q_H)$ $\qquad$ REDUNDANT

IC_H $\quad W_H(Q_H) - V_H \ge \underbrace{W_H(Q_L) - V_L}_{+}$

Follows from IR_L and IC_H

IR_L $\qquad W_L(Q_L) - V_L \ge 0 \qquad \forall Q \;\; W_H(Q) > W_L(Q)$

$$\Downarrow$$

$W_H(Q_L) - V_L > 0$ i.e. RHS of $IC_H > 0$

so $W_H(Q_H) - V_H > 0$

$V_H < W_H(Q_H)$

$$\max_{V_H, Q_H, V_L, Q_L} \quad \Pi_3 = q_H N[V_H - cQ_H] + (1-q_H) N[V_L - cQ_L]$$

$IR_L \quad V_L \leq W_L(Q_L)$ independent of V_H

$IC_L \quad W_L(Q_L) - V_L \geq W_L(Q_H) - V_H$ reinforced by $\uparrow V_H$

$IC_H \quad W_H(Q_H) - V_H \geq W_H(Q_L) - V_L$

$\uparrow$ this must be satisfied as an equality

$$\frac{\partial \Pi_3}{\partial V_H} > 0$$

$$\Downarrow$$

$$V_H = W_H(Q_H) - W_H(Q_L) + V_L$$

$$\max_{Q_H, V_L, Q_L} \quad \Pi_3 = q_H N[W_H(Q_H) - W_H(Q_L) + V_L - cQ_H] + (1-q_H) N[V_L - cQ_L]$$

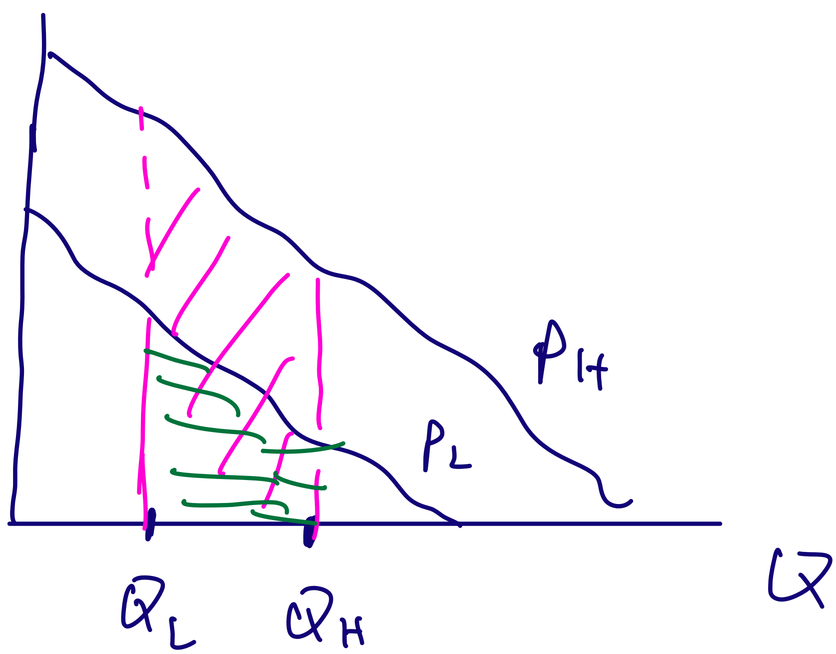
s.t. $IR_L \quad V_L \leq W_L(Q_L)$

$IC_L \quad W_L(Q_L) - V_L \geq W_L(Q_H) - W_H(Q_H) + W_H(Q_L) - V_L$

$IC_L \quad W_L(Q_L) - \cancel{V_L} \geq W_L(Q_H) - W_H(Q_H) + W_H(Q_L) - \cancel{V_L}$

$$W_H(Q_H) - W_H(Q_L) \geq W_L(Q_H) - W_L(Q_L)$$

will be satisfied if $Q_L < Q_H$



Strategy: solve max problem without IC_L constraint

if the solution is such that $Q_H > Q_L$ then ignoring IC_L was OK