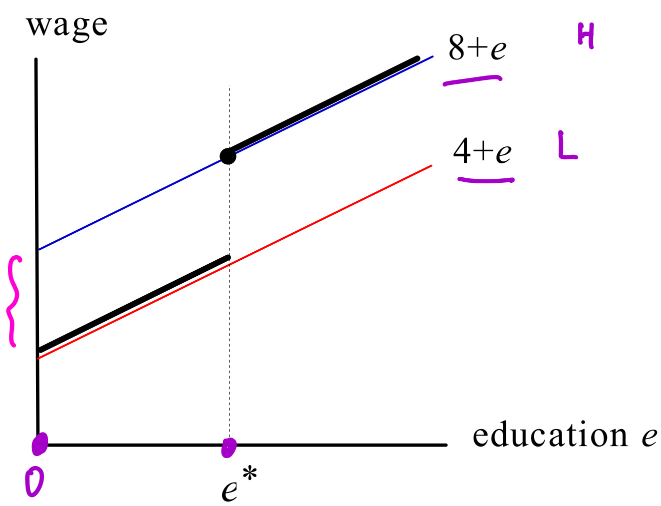
Example of a signaling equilibrium when education does increase productivity

Type L: $\begin{cases} \text{productivity: } \underline{4+e} & MB = 1 \\ \text{cost: } C_L(e) = \underline{4}e & MC = 4 \end{cases}$ and **Type H**: $\begin{cases} \text{productivity: } \underline{8+e} & MB = 1 \\ \text{cost: } C_H(e) = \underline{2}e & MC = 2 \end{cases}$



For a signaling equilibrium we need:

for Type L: $e=0$ better than $e=e^*$

$4 > 8 + e^* - 4e^*$ $\qquad e^* > \frac{4}{3}$

for Type H: $e=e^*$ better than $e=0$

$8 + e^* - 2e^* > 4$ $\qquad e^* < 4$

any $\frac{4}{3} < e^* < 4$ gives a signaling equilibrium

Suppose that 50% of the population is Type L and 50% is Type H.

Consider a signaling equilibrium with $e^* = 3$.

Then Type L have a net wage of 4

Type H a net wage of $\underbrace{8+3} - 2 \times 3 = 5$

Force everybody to choose $e = 0$ and force employers to pay

everybody w = average productivity: $\frac{1}{2} 4 + \frac{1}{2} 8 = 6$

An example with three types

Type A: productivity 10, cost $C_A(y) = ay$

Type B: productivity 15, cost $C_B(y) = by$

Type C: productivity 20, cost $C_C(y) = cy$

$y = 0$

$y = y_1$

$y = y_2$

$$0 < c < b < a$$

$$Wage\ offer: \begin{cases} 10 & \text{if } y < y_1 \\ 15 & \text{if } y_1 \le y < y_2 \\ 20 & \text{if } y_2 \le y \end{cases}$$

For a separating signaling equilibrium we need:

Type A to choose $y = 0$ $\quad 10 > 15 - ay_1 \quad 10 > 20 - ay_2$

Type B to choose $y = y_1$ $\quad 15 - by_1 > 10 \quad 15 - by_1 > 20 - by_2$

Type C to choose $y = y_2$ $\quad 20 - cy_2 > 10 \quad 20 - cy_2 > 15 - cy_1$

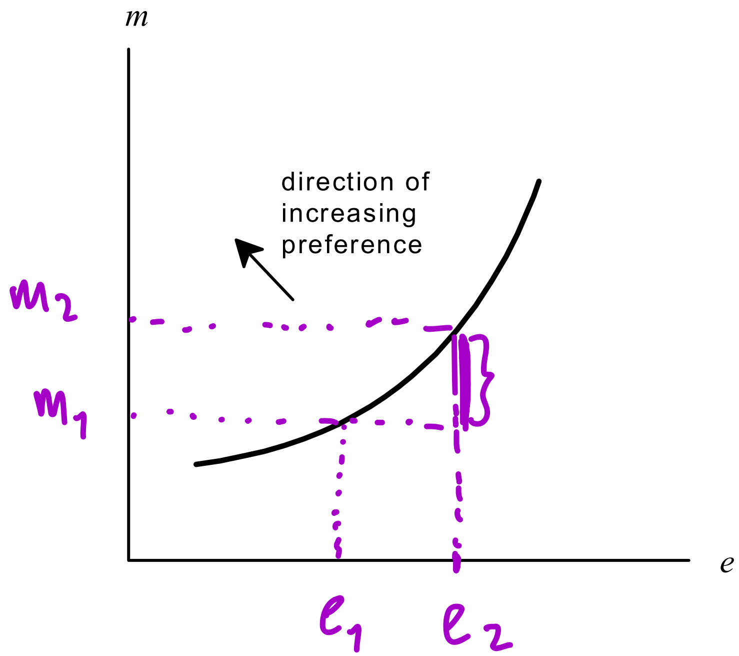
if $y_1 = 5$, $y_2 = 12$, $a = 2$, $b = \frac{3}{4}$, $c = \frac{1}{4}$

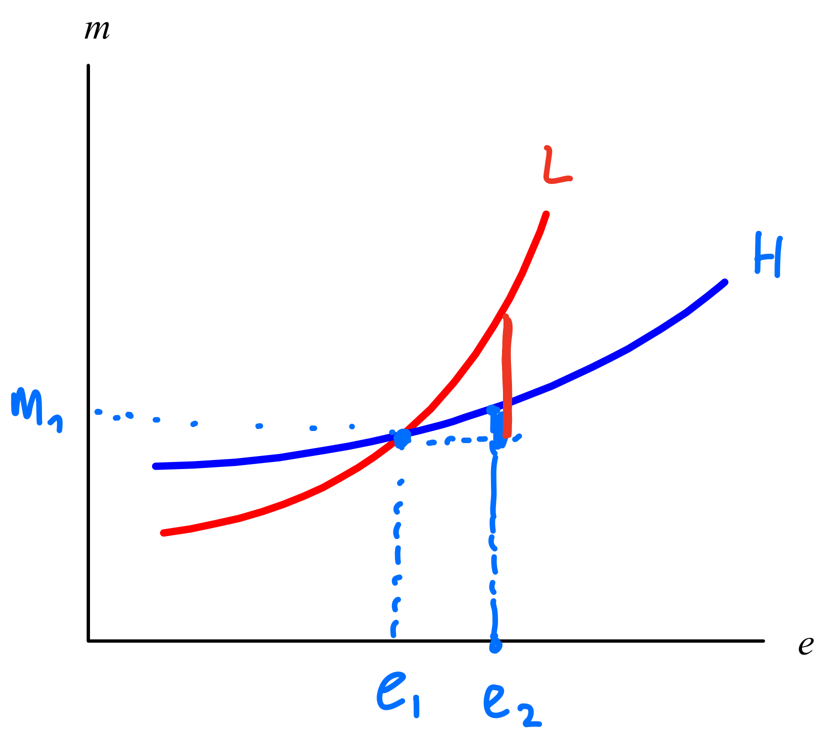
all six inequalities are satisfied

Psychological costs of education

$U(m, e)$

$$\frac{\partial U}{\partial m} > 0 \qquad \underbrace{\frac{\partial U}{\partial e} < 0}$$

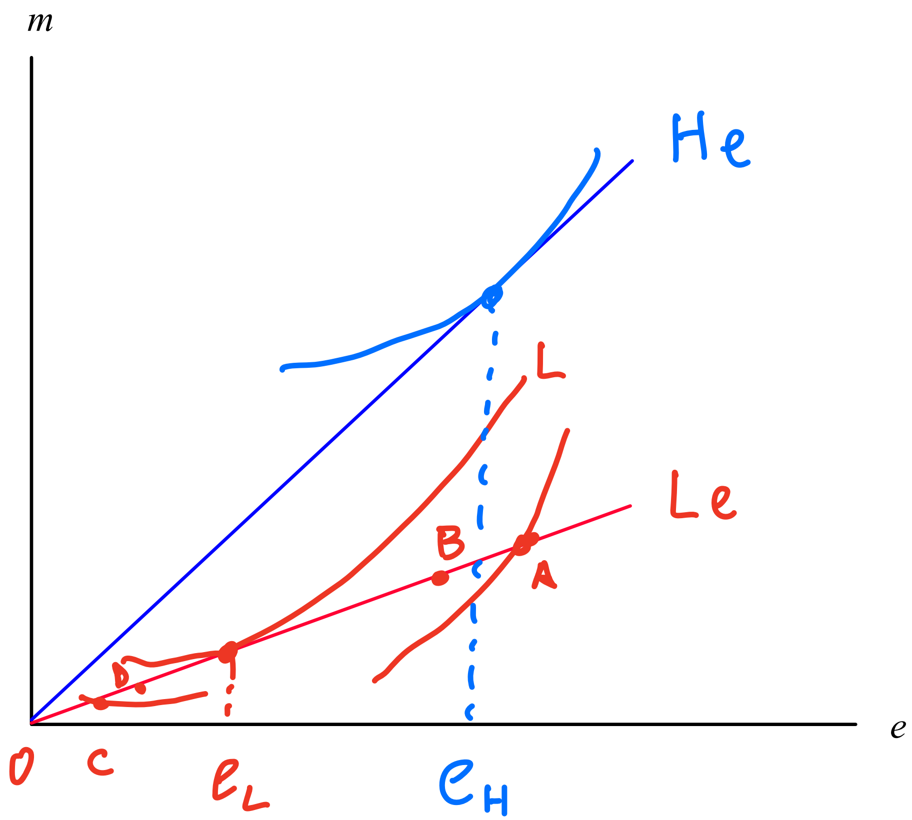




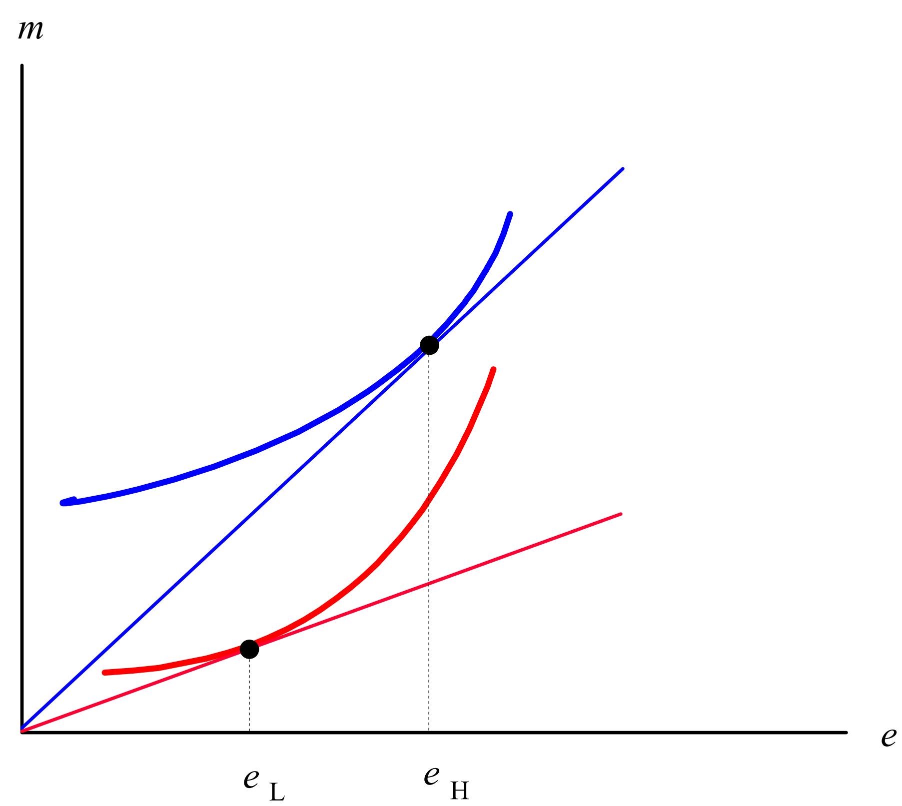
captures different
"cost of education"

Productivity of Type H: *He*   Productivity of Type L: *Le*

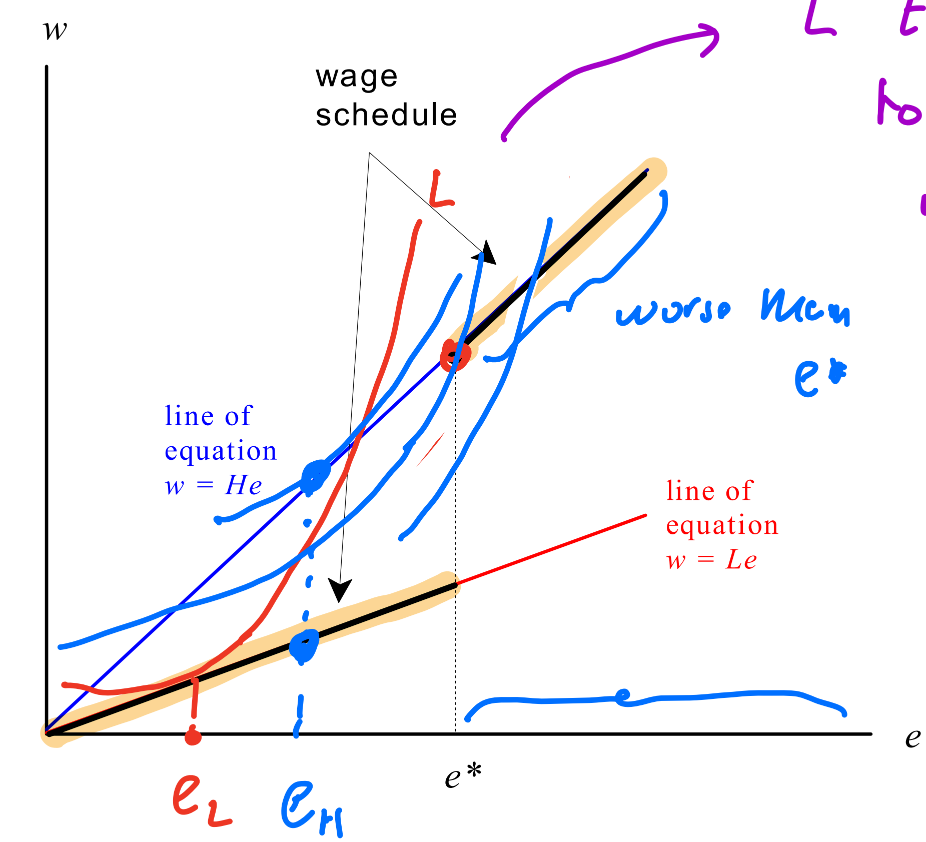
$$0 < L < H$$



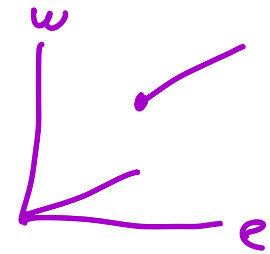
First-best choices when employer can tell types apart.


m
e
e L
e H

Asymmetric information

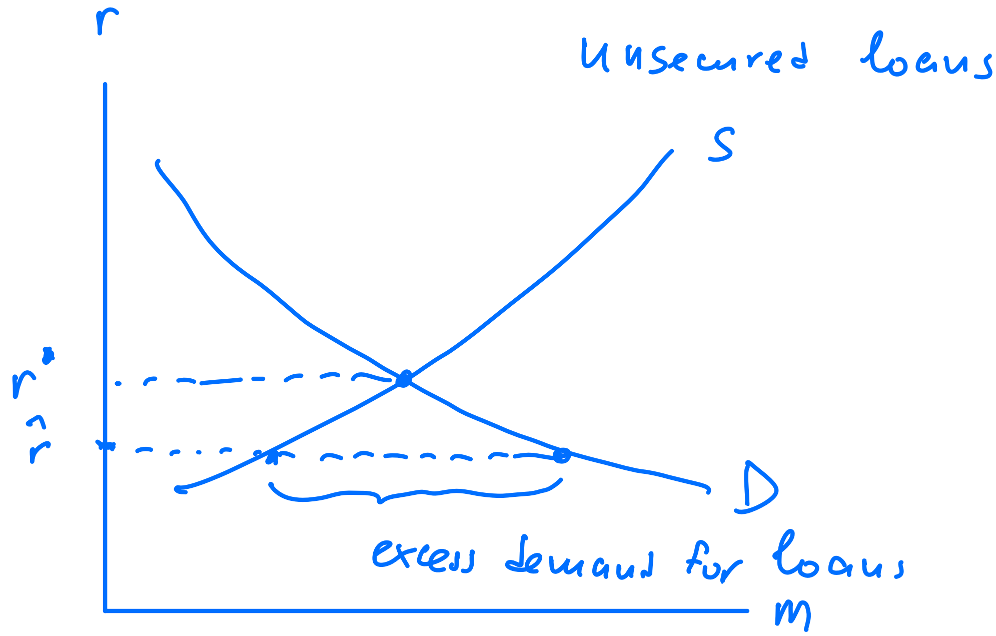


L types prefer e_L to any other point on the



H types prefer e^* to any other point

Relative to first best H types over-spend on education


r
unsecured loans
S
r*
r̂
D
excess demand for loans
m

Adverse Selection

You have just opened an all-you-can-eat buffet restaurant. The capacity of your restaurant is $K = 100$. Not quite sure how much you can charge. It costs \$2 to provide one standard-size serving of food. **You are risk neutral**.

Day	Price	More than ~~50~~ 10 people show up (= excess demand)?	Total profits
1	\$6	Yes	\$150
2	\$6.50	Yes	\$160
3	\$7	Yes	\$104

What happened? Revenue:

price	revenue
\$6	$\$(6 \times 100) = \$600$
\$6.50	$\$(6.50 \times 100) = \$650$
\$7	$\$(7 \times 100) = \$700$

$N = 1{,}200$ potential customers. Each can be described by a pair (r, c) where r is the reservation price and c is the amount of food he/she consumes.

Customer type	$(6,1)$	$(6,1.5)$	$(6.50,1.5)$	$(6.50,2.2)$	$(7,2.2)$	$(7,3.5)$
Number	100	100	200	300	200	300
Fraction	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{3}{12}$

Cost per dish = \$2

- If you charge \$6:

consumption	1	1.5	2.2	3.5
probability	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{5}{12}$	$\frac{3}{12}$

expected cost $\frac{1}{12}2 + \frac{3}{12}3 + \frac{5}{12}(4.4) + \frac{3}{12}7$

- If you charge \$6.50:

Customer type	~~(6,1)~~	~~(6,1.5)~~	$(6.50,1.5)$	$(6.50,2.2)$	$(7,2.2)$	$(7,3.5)$
			$\frac{2}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{3}{10}$
Fraction	~~$\frac{1}{12}$~~	~~$\frac{1}{12}$~~	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{3}{12}$

consumption	1	1.5	2.2	3.5
probability	0			

expected cost $\frac{2}{10}3 + \frac{5}{10}(4.4) + \frac{3}{10}7$

- If you charge $7:

Customer type	~~(6,1)~~	~~(6,1.5)~~	~~(6.50,1.5)~~	~~(6.50,2.2)~~	(7,2.2)	(7,3.5)
					$\frac{2}{5}$	$\frac{3}{5}$
Fraction	$\cancel{\frac{1}{12}}$	$\cancel{\frac{1}{12}}$	$\cancel{\frac{2}{12}}$	$\cancel{\frac{3}{12}}$	$\frac{2}{12}$	$\frac{3}{12}$

consumption	1	1.5	2.2	3.5
probability	0	0	$\frac{2}{5}$	$\frac{3}{5}$

Expected cost : $\frac{2}{5}(4.4) + \frac{3}{5}7$

Akerlof: market for second-hand cars

Quality	A	B	C	D	E	F	
Number of cars	10	20	10	40	30	10	Total: 120
fraction	$p_A = \frac{10}{120} = \frac{1}{12}$	$p_B = \frac{2}{12}$	$p_C = \frac{1}{12}$	$p_D = \frac{4}{12}$	$p_E = \frac{3}{12}$	$p_F = \frac{1}{12}$	
Buyer's value	\$6,000	\$5,000	\$4,000	\$3,000	\$2,000	\$1,000	
Seller's value	\$5,400	\$4,500	\$3,600	\$2,700	\$1,800	\$900	

one price P if $P = 4{,}200$

naive buyer

$$L = \begin{pmatrix} 6000-P & 5000-P & 4000-P & 3000-P & 2000-P & 1000-P \\ \frac{1}{12} & \frac{2}{12} & \frac{1}{12} & \frac{4}{12} & \frac{3}{12} & \frac{1}{12} \end{pmatrix}$$

$$M = \begin{pmatrix} 6000 & 5000 & 4000 & 3000 & 2000 & 1000 \\ \frac{1}{12} & \frac{2}{12} & \frac{1}{12} & \frac{4}{12} & \frac{3}{12} & \frac{1}{12} \end{pmatrix}$$

$$E[L] = \underbrace{E[M]} - P$$

if $E[M] > P$ I should buy