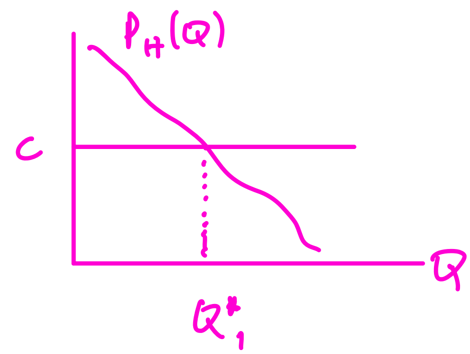
BUNDLING

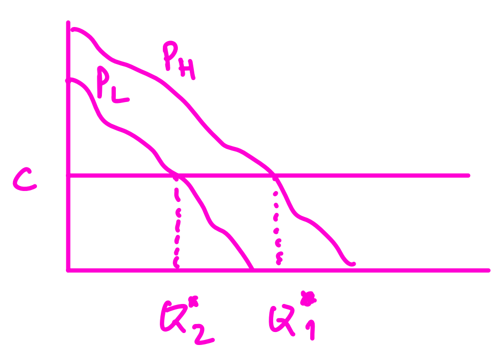
OPTION 1. Offer only one type of package (Q,V) which will be bought only by type H, because $V > W_L(Q)$ but $V \leq W_H(Q)$.

Profit-maximizing bundle has Q_1^* as the solution to $P_H(Q) = c$ and $V_1^* = W_H(Q_1^*)$



OPTION 2. Offer only one package (Q,V) which will be bought by both types, because $V \leq W_L(Q)$ (which implies that $V < W_H(Q)$ since $W_L(Q) < W_H(Q)$ when Q > 0).

Profit-maximizing bundle has Q_2^* as the solution to $P_L(Q) = c$ and $V_2^* = W_L(Q_2^*)$



$\boxed{\textbf{OPTION } \cancel{1}\ 3.}$ Offer two packages: (Q_H, V_H) targeted to the high type and (Q_L, V_L) targeted to the low type. Constraints:

$$(IR_L)\ \ V_L \leq W_L(Q_L)$$

$$(IC_L)\ \ W_L(Q_L) - V_L \geq W_L(Q_H) - V_H$$

$$\cancel{(IR_H)\ \ V_H \leq W_H(Q_H)}$$

$$(IC_H)\ \ W_H(Q_H) - V_H \geq W_H(Q_L) - V_L$$

Observation 1: (IR_H) is redundant.

Observation 2: (IC_H) must be satisfied as an equality. Thus

$$\underline{V_H = W_H(Q_H) - W_H(Q_L) + V_L}$$

Replacing this in (IC_L) and simplifying, we get that (IC_L) is satisfied if

$$\underline{\overbrace{W_H(Q_H) - W_H(Q_L)}^{\int_{Q_L}^{Q_H} P_H(x)\,dx} \geq \overbrace{W_L(Q_H) - W_L(Q_L)}^{\int_{Q_L}^{Q_H} P_L(x)\,dx}}$$ which is true if $Q_H \geq Q_L$

Strategy: solve maximization problem without (IC_L) and then check if the solution is such that $Q_H \geq Q_L$

$$\max_{V_L, Q_L, Q_H} \quad \pi = q_H N [W_H(Q_H) - W_H(Q_L) + V_L - cQ_H]$$
$$+ (1-q_H) N [V_L - cQ_L]$$

$$\text{s.t.} \quad (IR_L) \quad V_L \le W_L(Q_L)$$

π increasing in V_L

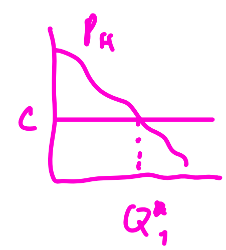
$\Rightarrow V_L = W_L(Q_L)$

$$\max_{V_L, Q_L, Q_H} \quad \pi = q_H N [W_H(Q_H) - W_H(Q_L) + W_L(Q_L) - cQ_H]$$
$$+ (1-q_H) N [W_L(Q_L) - cQ_L]$$

$$\frac{\partial \pi}{\partial Q_H} = q_H N [P_H(Q_H) - c] = 0 \Rightarrow P_H(Q_H) = c$$

$$Q_H^* = Q_1^*$$

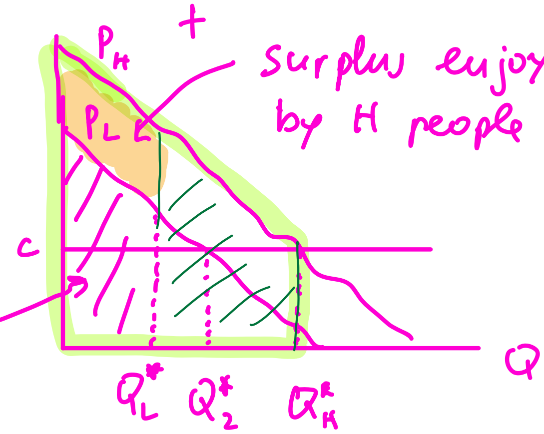
$$\frac{\partial \pi}{\partial Q_L} = q_H \cancel{N} [-P_H(Q_L) + P_L(Q_L)] +$$
$$(1-q_H) \cancel{N} [P_L(Q_L) - c] = 0$$



$$(1-q_H) P_L(Q_L) = (1-q_H) c + q_H [P_H(Q_L) - P_L(Q_2)]$$

divide by $(1-q_H)$

$$P_L(Q_L) = c + \underbrace{\frac{q_H}{1-q_H}}_{+} \underbrace{[P_H(Q_L) - P_L(Q_L)]}_{+} > c$$



surplus enjoyed by H people

$Q_H^* > Q_L^*$

(

$$V_L = W_L(Q_L^*)$$

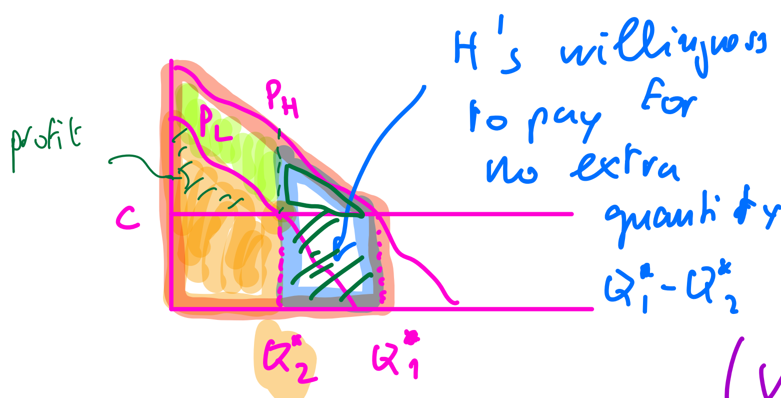
$$V_H^* = \overbrace{W_H(Q_H^*) - W_H(Q_L^*)} + \cancel{W_L}\ \underbrace{W_L(Q_L^*)}$$

Option 1: special case of Option 3

$$(V_H, Q_H) = (W_H(Q_H^*), Q_H^*)$$

$$(V_L, Q_L) = (V_L = 0, Q_L = 0)$$

Option 2: never optimal



$$(V_L, Q_L) = (W_L(Q_2^*), Q_2^*)$$

add new package

$$(V_H, Q_H) = (\qquad Q_1^*)$$

$$V_H = W_L(Q_2^*) + A \qquad c < A < \int_{Q_2^*}^{Q_1^*} P_H(x)\,dx$$

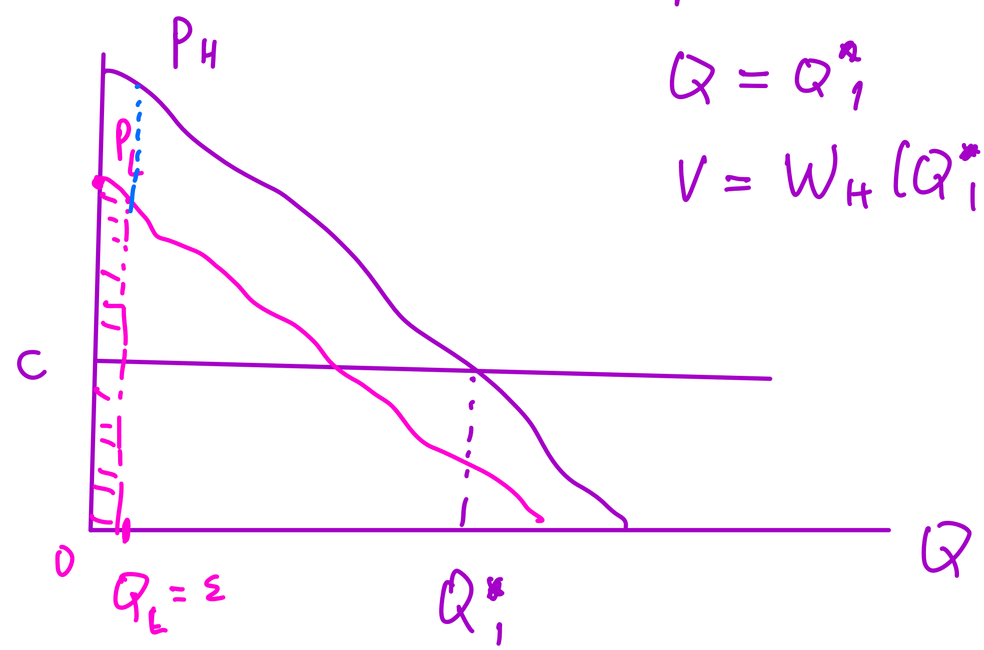
extra cost for monopolist: $c(Q_1^* - Q_2^*)$

Sufficient condition for Option 3 to be better than Option 1

Option 1:

$Q = Q_1^*$

$V = W_H(Q_1^*)$



Add a small package $Q_L = \varepsilon$ $\quad V_L = W_L(\varepsilon)$

if ε small $\quad W_L(\varepsilon) \simeq \frac{d}{dq} W_L(0) \cdot (\varepsilon - 0)$

$$= P_L(0) \cdot \varepsilon > \underbrace{c \cdot \varepsilon}$$

monopolist; cost of new package

Problem: H people now have this second option which gives them a surplus of $W_H(\varepsilon) - W_L(\varepsilon) \simeq$

$$P_H(0)\varepsilon - P_L(0)\cdot\varepsilon = [P_H(0) - P_L(0)]\varepsilon$$

need to reduce V_H from $W_H(Q_1^*)$ to

$$W_H(Q_1^*) - [P_H(0) - P_L(0)]\varepsilon$$

gain $(1-q_H) N [(P_L(0) - c)\varepsilon]$

loss $q_H N [(P_H(0) - P_L(0))\varepsilon]$

worth introducing new package if

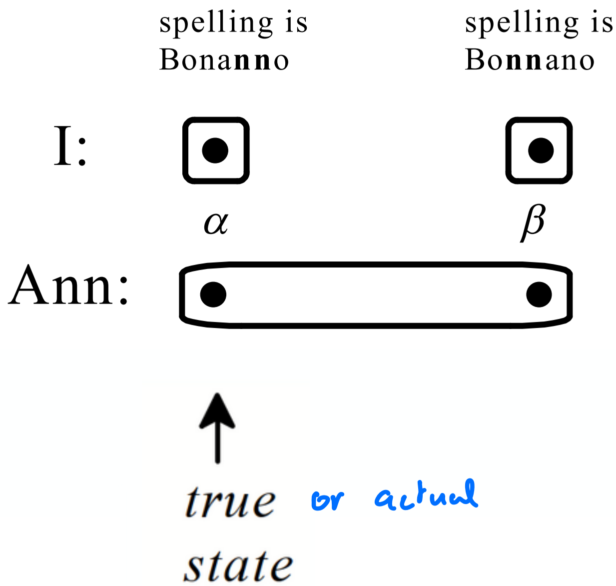
$$(1-q_H)\cancel{N}(P_L(0)-c)\cancel{\varepsilon} > q_H \cancel{N}[P_H(0)-P_L(0)]\cancel{\varepsilon}$$

$$P_L(0) - c - q_H P_L(0) + q_H c > q_H [P_H(0)-P_L(0)]$$

$$q_H [P_H(0)-P_L(0)] + q_H [P_L(0)-c] < P_L(0)-c$$

$$q_H [P_H(0)-c] < P_L(0)-c$$

$$\boxed{q_H < \frac{P_L(0)-c}{P_H(0)-c}}$$


spelling is Bonanno
spelling is Bonnano
I:
α
β
Ann:
true state
or actual

G_1

1 \ 2	L	R
T	0, 6	9, 18
B	9, 6	0, 0

G_2

1 \ 2	L	R
T	18, 6	0, 18
B	9, 6	9, 0

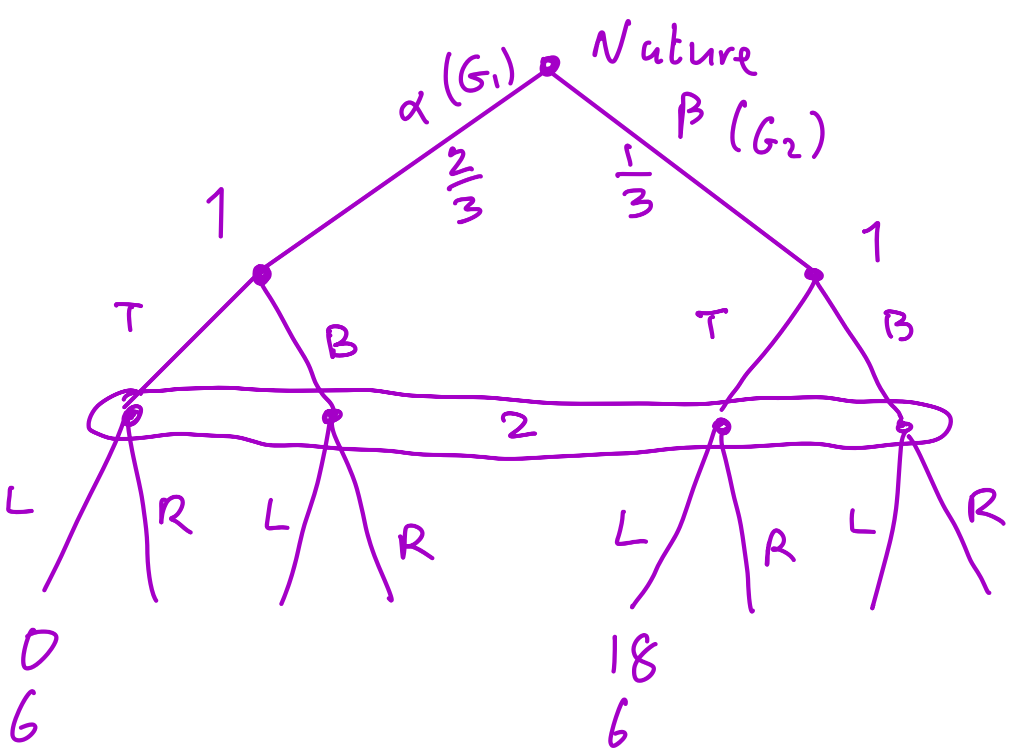
1: α β

2: α 2/3 1/3 β

HARSANYI TRANSFORMATIO

true state
or
actual

ONE-SIDED SITUATION OF INCOMLETE INFORMATION



$S_1 = \{TT, TB, BT, BB\}$

$S_2 = \{L, R\}$

		2	
			R
1			
	TB		$\frac{2}{3}x + \frac{1}{3}w, \ \frac{2}{3}y + \frac{1}{3}z$