BUNDLING

OPTION 1. Offer only one type of package (Q,V) which will be bought only by type H, because $V > W_L(Q)$ but $V \le W_H(Q)$.

Profit-maximizing bundle has Q_1^* as the solution to $P_H(Q) = c$ and $V_1^* = W_H(Q_1^*)$

OPTION 2. Offer only one package (Q,V) which will be bought by both types, because $V \le W_L(Q)$ (which implies that $V < W_H(Q)$ since $W_L(Q) < W_H(Q)$ when Q > 0).

Profit-maximizing bundle has Q_2^* as the solution to $P_L(Q) = c$ and $V_2^* = W_L(Q_2^*)$

OPTION 2. Offer two packages: (Q_H, V_H) targeted to the high type and (Q_L, V_L) targeted to the low type. Constraints:

$$(IR_L) V_L \leq W_L(Q_L)$$

$$(IC_L) W_L(Q_L) - V_L \geq W_L(Q_H) - V_H$$

$$(IR_H) V_H \leq W_H(Q_H)$$

$$(IC_H) W_H(Q_H) - V_H \geq W_H(Q_L) - V_L$$

Observation 1: (IR_H) is redundant.

Observation 2: (IC_H) must be satisfied as an equality. Thus

$$V_H = W_H(Q_H) - W_H(Q_L) + V_L$$

Replacing this in (IC_L) and simplifying, we get that (IC_L) is satisfied if

$$W_H(Q_H) - W_H(Q_L) \ge W_L(Q_H) - W_L(Q_L)$$
 which is true if $Q_H \ge Q_L$

Strategy: solve maximization problem without (IC_L) and then check if the solution is such that $Q_H \ge Q_L$