

Additional Exercises

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"Microeconometrics: Methods and Applications"

1. Chapter 1: Introduction

No exercises.

2. Chapter 2: Causal and Noncausal Models

No exercises.

3. Chapter 3: Microeconomic Data Structures

No exercises.

4. Chapter 4: Linear Models

4-7 THIS QUESTION HAD SEVERAL ERRORS (notable (d)-(f)).

USE THE FOLLOWING REVISED QUESTION INSTEAD.

(Adapted from Nelson and Startz, 1990). Consider the three equation model, $y = \beta x + u$; $x = \lambda u + \varepsilon$; $z = \gamma \varepsilon + v$, where the mutually independent errors u , ε and v are iid normal with mean 0 and variances, respectively, σ_u^2 , σ_ε^2 and σ_v^2 .

(a) Show that $\text{plim}(\hat{\beta}_{\text{OLS}} - \beta) = \lambda \sigma_u^2 / (\lambda^2 \sigma_u^2 + \sigma_\varepsilon^2)$.

(b) Show that $\rho_{XZ}^2 = [\gamma \sigma_\varepsilon^2]^2 / [(\lambda^2 \sigma_u^2 + \sigma_\varepsilon^2)(\gamma^2 \sigma_\varepsilon^2 + \sigma_v^2)]$.

(c) Show that $\hat{\beta}_{\text{IV}} - \beta = m_{zu} / (\lambda m_{zu} + m_{z\varepsilon}) \xrightarrow{p} 0$, where, for example, $m_{zu} = N^{-1} \sum_i z_i u_i$.

(d) Show that $\hat{\beta}_{\text{IV}} - \beta$ is not defined if $m_{zu} = -m_{z\varepsilon} / \lambda$. Nelson and Startz (1990) argue that this region is visited often enough that the mean of $\hat{\beta}_{\text{IV}}$ does not exist.

(e) Show that $\hat{\beta}_{\text{IV}} - \beta = 1 / (\lambda + m_{z\varepsilon} / m_{zu})$ equals $1 / \lambda$ if m_{zu} is large relative to $m_{z\varepsilon} / \lambda$. Nelson and Startz (1990) conclude that if m_{zu} is large relative to $m_{z\varepsilon} / \lambda$ then $\hat{\beta}_{\text{IV}} - \beta$ is concentrated around $1 / \lambda$, rather than the probability limit of zero from part (c).

(f) Nelson and Startz (1990) argue that $\widehat{\beta}_{IV} - \beta$ concentrates on $1/\lambda$ more rapidly the smaller is γ , the smaller is σ_ε^2 , and the larger is λ . Given your answer in part (c), what do you conclude about the small sample distribution of $\widehat{\beta}_{IV}$ when ρ_{XZ}^2 is small?

4-10 Consider weighted least squares estimation of household medical expenditures on household total expenditure using the data of section 4.6.4 where again only those with positive medical expenditures are included, but in this question the regression is in levels and not logs. [Use program `mma04p2qreg` and generate new variables `med = exp(lnmed)` and `total = exp(lntotal)`.]

(a) Perform OLS regression of `med` on `total`. You should obtain slope coefficient of 0.0938.

(b) Do you think that the errors in regression of `med` on `total` are likely to be heteroskedastic? Explain.

(c) Compare the default OLS standard errors for the OLS slope coefficient estimate with heteroskedastic-robust standard errors. Comment.

(d) OLS packages have an option for weighting. By appropriate use of weights option in your package, perform GLS regression of `med` on `total` under the assumption that the error has variance $\sigma^2 \times (\text{total})^2$.

(e) Compare the default standard errors for the weighted LS slope coefficient estimate with heteroskedastic-robust standard errors for the weighted LS slope coefficient estimate. Comment.

(f) Compare the default standard errors for the weighted LS slope coefficient estimate with heteroskedastic-robust standard errors for the weighted LS slope coefficient estimate. Comment.

(g) Obtain the LS estimates in part (d) manually by (unweighted) OLS regression by first appropriately transforming `med`, `total` and the intercept.

4-11 Consider least squares estimation of the model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$ where $\boldsymbol{\Sigma} = \sigma^2(\mathbf{I} + \mathbf{A}\mathbf{A}')$ where \mathbf{A} is an $N \times m$ matrix with $k < m < N$ and for simplicity we assume that σ^2 and \mathbf{A} are known.

(a) Obtain the variance of the OLS estimator using (4.19).

(b) Compare your answer in (a) to the default variance estimate $\sigma^2(\mathbf{X}'\mathbf{X})^{-1}$. Will default OLS standard errors be biased / inconsistent in any particular direction?

(c) Give the variance of the GLS estimator of $\boldsymbol{\beta}$, using the result that $(\mathbf{I} + \mathbf{A}\mathbf{A}')^{-1} =$

$$\mathbf{I}_N - \mathbf{A}(\mathbf{I}_m + \mathbf{A}'\mathbf{A})^{-1}\mathbf{A}'.$$

(d) In general GLS is more efficient than OLS. But what if we instead compare the (incorrect) default variance $\sigma^2(\mathbf{X}'\mathbf{X})^{-1}$ of OLS with the true variance of GLS? Comment.

(e) GLS requires matrix inversion. For this problem what is the largest size matrix that needs to be inverted to perform GLS?

4-12 Consider regression of y on x when data (y, x) take values $(-1, -2)$, $(0, -1)$, $(0, 0)$, $(0, 1)$ and $(1, 2)$.

(a) Using an appropriate statistical package obtain the OLS estimate of the slope coefficient and the least absolute deviations regression estimate of the slope coefficient. Compare the two estimates of the slope coefficient and the precision of the estimates.

(b) From part (a) you should find that the intercept is 0. We will find the least absolute deviations regression estimate of the slope coefficient by grid search. For the given data compute (4.34) with $q = 0.5$ and $\mathbf{x}'_i\boldsymbol{\beta} = \beta x_i$ (one regressor and no intercept) for $\beta = 0.3, 0.35, 0.4, 0.45, 0.5, 0.55$ and 0.6 . Which value of β minimizes (4.34)? Compare your answer to that in part (a).

4-13 Consider dgp $y = 1 + 1 \times x$ and $u = \varepsilon$ where $x \sim \mathcal{N}[0, 25]$ and $\varepsilon \sim \mathcal{N}[0, 4]$. This is the same dgp as in Section 4.5.3 page 84 except that u is homoskedastic.

(a) Using the general result at bottom page 86 give the true slope and intercept coefficient for the p^{th} quantile of y conditional on \mathbf{x} for quantiles $q = 0.1, 0.2, \dots, 0.9$. [Hint: $F_\varepsilon^{-1}(0.1)$ is that value ε^* such that $\Pr[\varepsilon \leq \varepsilon^*] = 0.1$ for $\varepsilon \sim \mathcal{N}[0, 4]$].

(b) Generate a sample of size 10,000 from this dgp. This requires minor modification of the Section 4.5.3 program given on the book website. Estimate quantile regressions of y on x for $q = 0.1, 0.2, \dots, 0.9$. Compare your answers to the theoretical answers in part (a).

(c) Redo part (b) for dgp with the modification that $u = \sqrt{x}\varepsilon$ rather than ε . Comment on any changes compared to (b).

(d) Redo part (b) for dgp with the modifications that x is the square of a $\mathcal{N}[0, 25]$ variate and $u = x\varepsilon$. [This setup ensures that $\mathbf{x}'\boldsymbol{\alpha} > 1$ as assumed on page 86]. Comment on any changes compared to (b).

4-14 If y is exponential distributed with mean $\exp(\mathbf{x}'\boldsymbol{\beta})$ then the variance is $[\exp(\mathbf{x}'\boldsymbol{\beta})]^2$. This can be written as the regression model $y = \exp(\mathbf{x}'\boldsymbol{\beta}) + u$ where $u = \exp(\mathbf{x}'\boldsymbol{\beta})\varepsilon$ and $\varepsilon \sim \text{iid}[0, 1]$.

- (a) Show by argument similar to that in Section 4.6.1 that the population q^{th} quantile of y conditional on \mathbf{x} is $\mu_q(\mathbf{x}, \boldsymbol{\beta}) = \exp(\mathbf{x}'\boldsymbol{\beta}) + \exp(\mathbf{x}'\boldsymbol{\beta}) \times F_{\varepsilon}^{-1}(q)$.
- (b) Hence state what happens for this model to the intercept and slope coefficients of the population q^{th} quantile of y conditional on \mathbf{x} as q varies.

4-15 Consider the same data set as that in example 4.9.6 and use the program given at the book website. Note that wage76 is log hourly wage, grade76 is years of schooling and col4 is proximity to college.

(a) Regress wage76 on an intercept and grade76, with estimation by OLS and by IV where col4 is the instrument for grade76. Compare the size and precision of the OLS and IV slope coefficients, where heteroskedasticity-robust standard errors are used.

(b) Perform OLS regression of wage76 on an intercept and col4 and perform OLS regression of grade76 on an intercept and col4. Obtain the ratio of the two slope coefficient estimates as in (4.46) and compare to your answer in (a).

(c) The instrument here is a binary variable and there is one regressor. Compute the Wald estimate (see (4.48)) and compare it to the IV estimate from part (a).

(d) Do you think there might be a weak instrument problem here? Provide appropriate statistical measures.

(e) Redo parts (a) and (b), with all regressions now including as additional (exogenous) regressors age76, agesq76, black, south76, smsa76, reg2-reg9, smsa66, momdad14, sinmom14, nodaded, nomomed, dadad, momed, and famed1-famed8. Does result (4.46) still hold?

4-16 Consider the linear regression model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$ and the IV estimator

$$\hat{\boldsymbol{\beta}} = (\mathbf{Z}'\mathbf{X})^{-1}\mathbf{Z}'\mathbf{y}$$

where \mathbf{Z} and \mathbf{X} are $N \times k$ full rank matrices of constants (i.e. are nonstochastic).

- (a) Suppose $\mathbf{u} \sim \mathcal{N}[\mathbf{0}, \boldsymbol{\Omega}]$. Obtain the finite sample distribution of $\hat{\boldsymbol{\beta}}$.
- (b) Suppose $\mathbf{u} \sim [\mathbf{0}, \boldsymbol{\Omega}]$, the probability limits of $N^{-1}\mathbf{X}'\mathbf{X}$, $N^{-1}\mathbf{Z}'\mathbf{Z}$ and $N^{-1}\mathbf{Z}'\mathbf{X}$ all exist and are finite nonsingular and $N^{-1/2}\mathbf{Z}'\mathbf{u} \xrightarrow{d} \mathcal{N}[\mathbf{0}, \text{plim } N^{-1}\mathbf{Z}'\boldsymbol{\Omega}\mathbf{Z}]$. Obtain the limit distribution of $\sqrt{N}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})$.
- (c) Hence obtain the asymptotic distribution of $\hat{\boldsymbol{\beta}}$ and compare to your answer in (a).

4-17 Consider the same three equation model as in Exercise 4-7, with $y = \beta x + u$; $x = \lambda u + \varepsilon$; $z = \gamma \varepsilon + v$, where the mutually independent errors u , ε and v are iid normal with mean 0 and variances 1, and $\beta = 1$, $\lambda = 1$ and $\gamma = 0.01$. Perform 1000 simulations with $N = 100$ where in each simulation data (y, x, z) is generated and we obtain (1) $\hat{\beta}_{OLS}$ from OLS regression of y on x without intercept; (2) $\hat{\beta}_{IV}$ from IV regression of y on x without intercept with instrument z ; (3) R^2 and F from OLS regression (with intercept) of z on x .

(a) Compare the average across simulations of $\hat{\beta}_{OLS}$ with the probability limit given in Exercise 4-7 part (b). Comment.

(b) Compare the average across simulations of $\hat{\beta}_{IV}$ with the probability limit given in Exercise 4-7 part (c). Comment.

(c) Obtain percentiles and quartiles of the observed values across simulations of $\hat{\beta}_{IV}$. Comment in the light of Exercise 4-7 part (e).

(d) Do the R^2 and F statistics across simulations from OLS regression (with intercept) of z on x indicate a likely weak instruments problem?