240D Fall 2004 Department of Economics, U.C.-Davis

Assignment 1: Due 2.10 p.m. Wednesday October 13

Worth 3% of course grade. No credit for late assignments. Keep Answers as brief as possible.

Use Stata for question 4 and provide Stata program and output.

1. OLS. Suppose the dgp is

 $y_i = \beta x_i + u_i$ $u_i = x_i \varepsilon_i$ $x_i \sim \mathcal{N}[0, 1]$ $\varepsilon_i \sim \mathcal{N}[0, 1]$

Data are independent over *i* and x_i is independent of ε_i . Note that the first-four central moments of $\mathcal{N}[0, \sigma^2]$ are 0, σ^2 , 0 and $3\sigma^4$. In usual matrix notation $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$.

(a) Show that the error term u_i is heteroskedastic.

(b) Obtain plim $N^{-1}\mathbf{X}'\mathbf{X}$. [Hint: Obtain $\mathbb{E}[x_i^2]$ and apply a law of large numbers.]

(c) Obtain $\sigma_0^2 = V[u_i]$ where the expectation is with respect to all stochastic variables in the model.

(d) Obtain plim $N^{-1}\mathbf{X}'\mathbf{\Omega}\mathbf{X} = \lim N^{-1}\mathbf{E}[\mathbf{X}'\mathbf{\Omega}\mathbf{X}]$, where $\mathbf{\Omega} = \text{Diag}[\mathbf{V}[u_i|x_i]]$.

(e) Using answers to the preceding parts give the usual OLS result for the limit distribution of $\sqrt{N}(\hat{\beta}_{\text{OLS}} - \beta)$, ignoring potential heteroskedasticity. Your ultimate answer should be a number.

(f) Now give the variance in the limit distribution of $\sqrt{N}(\hat{\beta}_{OLS} - \beta)$, taking account of any heteroskedasticity. Your ultimate answer should be a number.

(g) Do any differences between answers to parts (e) and (f) accord with your prior beliefs?

2. Let $\{X_i, i = 1, ..., N\}$ be a sample of independent not identically distributed random variables with mean μ and variance σ_i^2 . Note that here there is heteroskedasticity. Let $\overline{X}_N = N^{-1} \sum_{i=1}^N X_i$ denote the sample average.

Use Chebyshev's inequality for any random variable Z

$$\Pr[(Z - \mu)^2 > k] \le \operatorname{Var}[Z]/k, \text{ for any } k > 0,$$

to prove that \overline{X}_N converges in probability to μ . State any additional assumptions, if any, you need to make. [Hint: Make an assumption about $N^{-1}\sum_i \sigma_i^2$.

You are to use the basic definition of convergence in probability, i.e. a (δ, ε) argument.

3. Consider the linear regression model:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u},$$

where **X** is an $N \times K$ stochastic matrix of regressors and β is a $K \times 1$ parameter vector.

(a) Obtain the (lengthy) formula for the estimator that minimizes:

$$Q(\boldsymbol{\beta}) = \mathbf{u}' \mathbf{Z} (\mathbf{Z}' \mathbf{Z})^{-1} \mathbf{Z}' \mathbf{u},$$

where **Z** is an $N \times r$ matrix of rank r and r > K. [Hint: Chain rule for matrix differentiation. $\partial f(\mathbf{x})/\partial \mathbf{x} = \partial \mathbf{z}'/\partial \mathbf{x} \times \partial f(\mathbf{z})/\partial \mathbf{z}$, for $f(\mathbf{x}) = f(g(\mathbf{x})) = f(\mathbf{z})$ where $\mathbf{z} = g(\mathbf{x})$ where \mathbf{x} and \mathbf{z} are column vectors].

(b) Obtain the probability limit of this estimator, stating clearly any assumptions you have made. [Your derivation does not have to have all details. In particular there is no need to verify that a law of large numbers can be applied].

4. This question uses Stata. Details on running Stata are given at http://www.econ.ucdavis.edu/faculty/cameron/stata/staccess.html

The OLS program for this question and the data are given under the heading Econ 240D / Assignment 1 Stata OLS program and Assignment 1 data. You should not need to modify the program. The program is long as in addition to the easy regress command I do OLS using matrix algebra.

Jaggia and Thosar (1993) model the number of bids received by 126 U.S. firms that were targets of tender offers during the period 1978-1985, and were actually taken over within 52 weeks of the initial offer. The dependent count variable is the number of bids after the initial bid (NUMBIDS) received by the target firm. The regressors measure

- Defensive actions taken by management of the target firm indicator variables for legal defense by lawsuit (LEGLREST), proposed changes in asset structure (REALREST), proposed change in ownership structure (FINREST) and management invitation for friendly third party bid (WHITEKNT). These are expected to decrease the number of bids, aside from WHITEKNT which may increase bids as it is itself a bid.
- Firm specific characteristics bid price divided by price 14 working days before bid (BID-PREM), percentage of stock held by institutions (INSTHOLD), total book value of assets in billion of dollars (SIZE) and book value squared (SIZESQ). A high value of BIDPREM indicates a bid so attractive that additional bids are unlikely. The greater institutional holdings the more likely outside offers are to be favorably received which will encourage more bids. As size of the firm increases there are expected to be more bids, up to a point where the firm gets so large that few others are capable of making a credible bid.
- Intervention by federal regulators an indicator variable for Department of Justice intervention (REGULATN). Regulator intervention is likely to discourage bids.

(a) Give the OLS estimates and state what regressors are statistically significant at 5%.

(b) What is the estimated magnitude of the effect of an increase in the bid premium of 10 percentage points? Express your answer in a meaningful sentence in layman's terms – don't just report parameter estimates. You need to think about this.